(c) We need to find $t$ when $S = 80$. Using the given equation we have:

$$80 = 90 - 20\log_{10}(t+1) \Leftrightarrow 20\log_{10}(t+1) = 10$$
$$\Leftrightarrow \log_{10}(t+1) = \frac{1}{2}$$
$$\Leftrightarrow t+1 = \sqrt{10}$$
$$\Leftrightarrow t = \sqrt{10} - 1$$

That is, $t \approx 2.16$

Therefore, the students should be retested in approximately 2 months time.

## EXERCISES 7.5

**1.** The loudness of a sound, as experienced by the human ear, is based on its intensity level. This intensity level is modelled by the logarithmic function $d = 10\log_{10}\left(\frac{I}{I_0}\right)$ where $d$ is measured in decibels and corresponds to a sound intensity $I$ and $I_0$ (known as the threshold intensity) is the value of $I$ that corresponds to be the weakest sound that can be detected by the ear under certain conditions.

(a) Find the value of $d$ when $I$ is 10 times as great as $I_0$ (i.e., $I = 10I_0$).

(b) Find the value of $d$ when $I$ is 1000 times as great as $I_0$.

(c) Find the value of $d$ when $I$ is 10000 times as great as $I_0$.

**2.** A model, for the relationship between the average weight $W$ kilograms and the height $h$ metres for children aged 5 through to 13 years has been closely approximated by the function $\log_{10}W = \log_{10}2.4 + 0.80h$

(a) Based on this model, determine the average weight of a 10-year-old child who is 1.4 metres tall.

(b) How tall would an 8 year old child weighing 50 kg be?

(c) Find an expression for the weight, $W$, as a function of $h$.

(d) Sketch the graph of $W$ kg versus $h$ m.

(e) Hence, or otherwise, sketch the graph of $h$ m versus $W$ kg .

**3.** A measure of the 'energy' of a star can be related to its brightness. To determine this 'energy' stars are classified into categories of brightness called magnitudes. Those considered to be the least 'energetic' are labelled as the faintest stars. Such stars have a light flux given by $L_0$, and are assigned a magnitude 6. Other brighter stars having a light flux $L$ are assigned a magnitude $m$ by means of the formula $m = 6 - 2.5\log_{10}\left(\frac{L}{L_0}\right)$.

(a) Find the magnitude $m$ of a star, if relative to the faintest star, its light flux $L$ is such that $L = 10^{0.5}L_0$.

(b) Find an equation for $L$ in terms of $m$ and $L_0$.

(c) Sketch the general shape of the function for $L$ (as a function of $m$).

(d) Hence, or otherwise, sketch the graph of the function $m = 6 - 2.5\log_{10}\left(\frac{L}{L_0}\right)$.

**4.** For some manufacturers, it is important to consider the failure time of their computer chips. For Multi–Chips Pty Ltd, the time taken before a fraction $x$ of their computer chips fail has been approximated by the logarithmic function $t = -\frac{1}{c}\log_{10}(1-x)$, where $c$ is some positive constant and time $t$ is measured in years.

(a) Define the domain for this function.

(b) Determine how long will it be before 40% of the chips fail, when

i. $c = 0.1$ ii. $c = 0.2$ iii. $c = 0.3$

(c) How does the value of $c$ affect the reliability of a chip?

(d) Find an expression for the fraction $x$ of chips that will fail after $t$ years.

(e) For the case where c = 0.10, sketch the graph of $x$ versus $t$. Hence, sketch the graph of $t = -\frac{1}{c}\log_{10}(1-x)$ where $c = 0.10$.

**5.** Logarithms have been found useful in modelling economic situations in some countries. Pareto's law for capitalist countries states that the relationship between annual income, \$ $I$ and the number, $n$, of individuals whose income exceeds \$ $I$ is approximately modelled by the function $\log_{10}I = \log_{10}a - k\log_{10}n$ where $a$ and $k$ are real positive constants.

(a) Find and expression for $I$ that does not involve logarithms.

(b) By varying the values of $a$ and $b$, describe their effects on

i. the income \$ $I$.

ii. the number of people whose income exceeds \$ $I$.

**6.** After prolonged observations of our environment, it became obvious that the thickness of the ozone layer had being affected by the production of waste that had taken place over many years. The thickness of the ozone layer has been estimated by making use of the function $\log_{10}\lambda_0 - \log_{10}\lambda = kx$, where $\lambda_0$ is the intensity of a particular wavelength of light from the sun before it reaches the atmosphere, $\lambda$ is the intensity of the same wavelength after passing through a layer of ozone $x$ centimetres thick, and $k$ is the absorption constant of ozone for that wavelength.
The following table has some results based on available data for one region of the Earth's atmosphere:

| $\lambda_0$ | $k$ | $\frac{\lambda_0}{\lambda}$ |
|---|---|---|
| $3200 \times 10^{-8}$ | $k \approx 0.40$ | 1.10 |

(a) Based on the above table, find the approximate thickness of the ozone layer in this region of the atmosphere, giving your answer to the nearest hundredth of a centimetre.

(b) Obtain an expression for the intensity $\lambda$, in terms of $k$, $\lambda_0$ and $x$.

(c) What would the percentage decrease in the intensity of light with a wavelength of $3200 \times 10^{-8}$ cm be, if the ozone layer is 0.20 centimetre thick?

(d) For a fixed value of $\lambda_0$, how does $k$ relate to the intensity $\lambda$ ?