Logs AI *[23 marks]*

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m^{-2}). The intensity level is given by the following formula.

$L = 10\log_{10}\left(S \times 10^{12}\right)$, $S \geq 0$.

1a. An orchestra has a sound intensity of $6.4 \times 10^{-3}\,\text{W m}^{-2}$. Calculate the intensity level, L of the orchestra. *[2 marks]*

Markscheme

$10\log_{10}\left(6.4 \times 10^{-3} \times 10^{12}\right)$ ***(M1)***

= 98.1(dB) (98.06179...) ***A1***

[2 marks]

1b. A rock concert has an intensity level of 112 dB. Find the sound intensity, S. *[2 marks]*

Markscheme

$112 = 10\log_{10}\left(S \times 10^{12}\right)$ ***(M1)***

$0.158\,(\text{W m}^{-2})$ ($0.158489...\,(\text{W m}^{-2})$) ***A1***

[2 marks]

The pH of a solution is given by the formula $pH = -\log_{10} C$ where C is the hydrogen ion concentration in a solution, measured in moles per litre (Ml^{-1}).

2a. Find the pH value for a solution in which the hydrogen ion concentration is 5.2×10^{-8}. *[2 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$pH = -\log_{10}\left(5.2 \times 10^{-8}\right) = 7.29 (7.28399\ldots)$ **(M1)A1**

[2 marks]

2b. Write an expression for C in terms of pH. *[2 marks]*

Markscheme

$C = 10^{-pH}$ **(M1)A1**

Note: Award **M1** for an exponential equation with 10 as the base.

[2 marks]

2c. Find the hydrogen ion concentration in a solution with $pH 4.2$. Give your answer in the form $a \times 10k$ where $1 \leq a < 10$ and k is an integer. *[2 marks]*

Markscheme

$C = 10^{-4.2} = 6.30957\ldots \times 10^{-5}$ **(M1)**

6.31×10^{-5} **A1**

[2 marks]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$\log_{10} N = a - M$, for some $a \in \mathbb{R}$.

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

3a. Find the value of a. *[2 marks]*

Markscheme

$\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

3b. Find the value of b. *[2 marks]*

Markscheme

EITHER

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M}\right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \ \left(= 10^5\right)$ ***A1***

[2 marks]

3c. Given $0 < M < 8$, find the range for N. *[2 marks]*

Markscheme

$0.001 < N < 100000 \quad \left(10^{-3} < N < 10^{5}\right)$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequalities/interval notation.

[2 marks]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

3d. Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year. *[2 marks]*

Markscheme

$N = \frac{10^{5}}{10^{7.2}} \quad (= 0.0063095\ldots)$ ***(M1)***

length of time $= \frac{1}{0.0063095\ldots} = 10^{2.2}$

$= 158$ years ***A1***

[2 marks]

The pH of a solution measures its acidity and can be determined using the formula $\text{pH} = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

4a. Calculate the pH of the coffee. *[2 marks]*

Markscheme

(pH =) $-\log_{10}\left(1.3 \times 10^{-5}\right)$ ***(M1)***

4.89 $(4.88605\ldots)$ ***A1***

[2 marks]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

4b. Determine whether the unknown liquid is more or less acidic than the coffee. Justify your answer mathematically. *[3 marks]*

Markscheme

EITHER

calculating pH

(pH =) $-\log_{10}\left(10 \times 1.3 \times 10^{-5}\right)$ ***(M1)***

3.89 $(3.88605\ldots)$ ***A1***

($3.89 < 4.89$, therefore) the unknown liquid is more acidic (than coffee). ***A1***

Note: Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the C-value to the pH level to earn the final ***A1***; a comparison of C-values only earns ***M0A0A0***.

OR

referencing the graph

The graph of $y = -\log_{10}(x)$ shows that as the value of x increases, the value of y decreases. ***M1***

Since the C-value (x-value) of the unknown liquid is larger than that of the coffee, the pH level (y-value) is lower. ***R1***

The unknown liquid is more acidic (than coffee). ***A1***

Note: Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the C-value to the pH level to earn the final ***A1***; a comparison of C-values only earns ***M0R0A0***.

[3 marks]