# Logs practice *[48 marks]*

1. Solve the equation $2\ln x = \ln 9 + 4$. Give your answer in the form $x = p\mathrm{e}^q$ where $p,\ q \in \mathbb{Z}^+$. *[5 marks]*

# Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

**METHOD 1**

$2\ln x - \ln 9 = 4$

uses $m\ln x = \ln x^m$ **(M1)**

$\ln x^2 - \ln 9 = 4$

uses $\ln a - \ln b = \ln \frac{a}{b}$ **(M1)**

$\ln \frac{x^2}{9} = 4$

$\frac{x^2}{9} = \mathrm{e}^4$ **A1**

$x^2 = 9\mathrm{e}^4 \Rightarrow x = \sqrt{9\mathrm{e}^4}(x > 0)$ **A1**

$x = 3\mathrm{e}^2(p = 3, q = 2)$ **A1**

**METHOD 2**

expresses $4$ as $4\ln \mathrm{e}$ and uses $\ln x^m = m\ln x$ **(M1)**

$2\ln x = 2\ln 3 + 4\ln \mathrm{e}(\ln x = \ln 3 + 2\ln \mathrm{e})$ **A1**

uses $2\ln \mathrm{e} = \ln \mathrm{e}^2$ and $\ln a + \ln b = \ln ab$ **(M1)**

$\ln x = \ln\left(3\mathrm{e}^2\right)$ **A1**

$x = 3\mathrm{e}^2(p = 3, q = 2)$ **A1**

**METHOD 3**

expresses $4$ as $4\ln \mathrm{e}$ and uses $m\ln x = \ln x^m$ **(M1)**

$\ln x^2 = \ln 3^2 + \ln \mathrm{e}^4$ **A1**

uses $\ln a + \ln b = \ln ab$ **(M1)**

$\ln x^2 = \ln\left(3^2\,\mathrm{e}^4\right)$

$x^2 = 3^2\,\mathrm{e}^4 \Rightarrow x = \sqrt{3^2\,\mathrm{e}^4}(x > 0)$ **A1**

so $x = 3\mathrm{e}^2(x > 0)(p = 3, q = 2)$ **A1**

**[5 marks]**

2. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3\left(4x^3\right)$, where $x > 0$. *[5 marks]*

# Markscheme

attempt to use change the base ***(M1)***

$\log_3 \sqrt{x} = \frac{\log_3 2}{2} + \log_3 (4x^3)$

attempt to use the power rule ***(M1)***

$\log_3 \sqrt{x} = \log_3 \sqrt{2} + \log_3 (4x^3)$

attempt to use product or quotient rule for logs, $\ln a + \ln b = \ln ab$ ***(M1)***

$\log_3 \sqrt{x} = \log_3 \left(4\sqrt{2}x^3\right)$

**Note:** The ***M*** marks are for attempting to use the relevant log rule and may be applied in any order and at any time during the attempt seen.

$\sqrt{x} = 4\sqrt{2}x^3$

$x = 32x^6$

$x^5 = \frac{1}{32}$ ***(A1)***

$x = \frac{1}{2}$ ***A1***

***[5 marks]***

Let $f(x) = a \log_3 (x - 4)$, for $x > 4$, where $a > 0$.

Point $\text{A}(13, 7)$ lies on the graph of $f$.

3a. Find the value of $a$. *[3 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to substitute coordinates (in any order) into $f$ ***(M1)***

*eg* $a\log_3(13-4)=7$ , $a\log_3(7-4)=13$ , $a\log 9=7$

finding $\log_3 9=2$ (seen anywhere) ***(A1)***

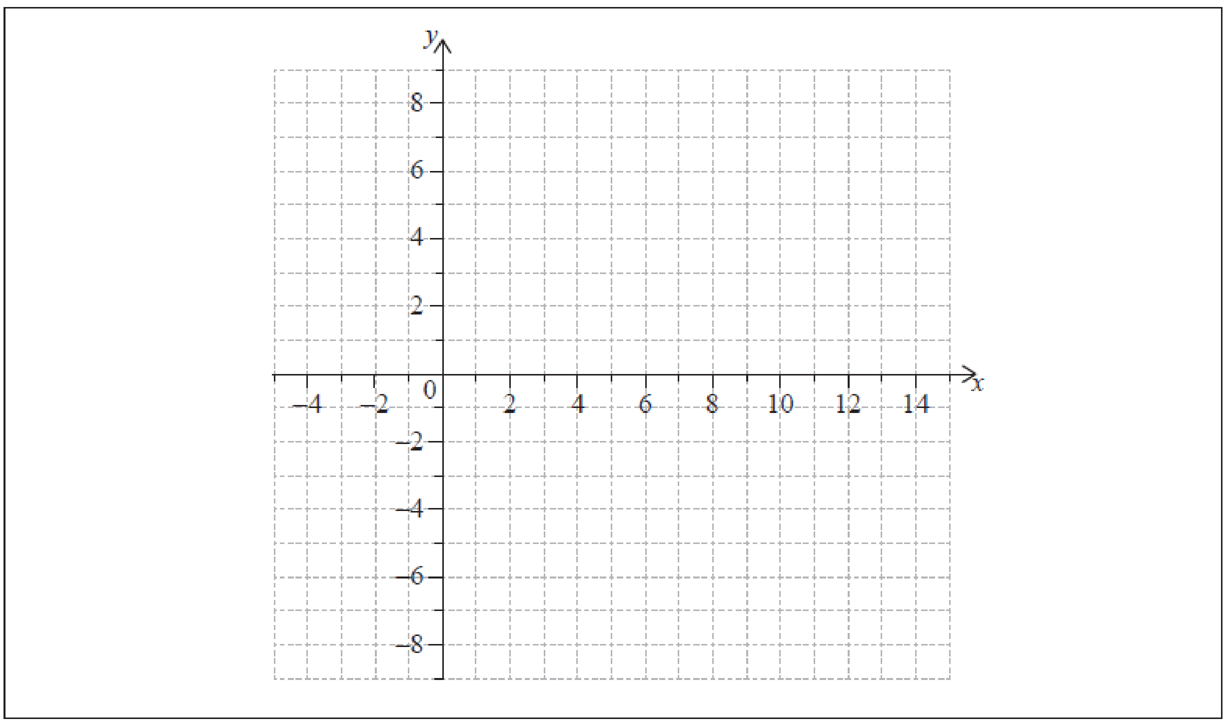
*eg* $\log_3 9=2$ , $2a=7$

$a=\frac{7}{2}$ ***A1 N2***

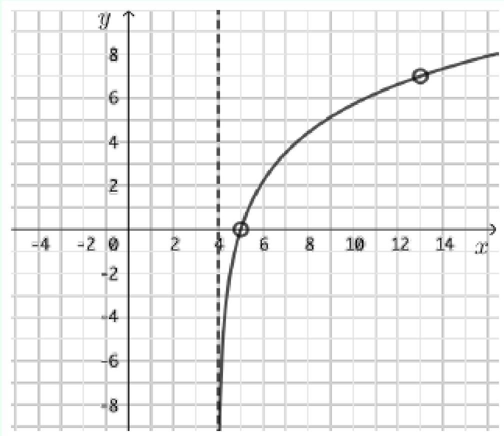
***[3 marks]***

3b. The $x$-intercept of the graph of $f$ is $(5,\ 0)$. *[3 marks]*

On the following grid, sketch the graph of $f$.



## Markscheme




***A1A1A1 N3***

**Note:** Award ***A1*** for correct shape of logarithmic function (must be increasing and concave down).
**Only** if the shape is correct, award the following:
***A1*** for being asymptotic to $x = 4$
***A1*** for curve including both points in circles.

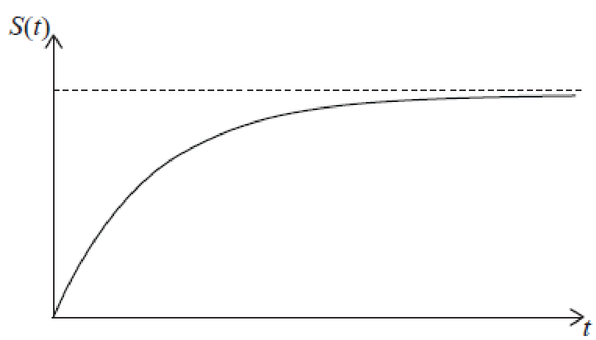
***[3 marks]***

Jean-Pierre jumps out of an airplane that is flying at constant altitude. Before opening his parachute, he goes through a period of freefall.

Jean-Pierre's vertical speed during the time of freefall, $S$, in $\mathrm{m\,s^{-1}}$, is modelled by the following function.

$$S(t) = K - 60\left(1.2^{-t}\right), t \geq 0$$

where $t$, is the number of seconds after he jumps out of the airplane, and $K$ is a constant. A sketch of Jean-Pierre's vertical speed against time is shown below.




Jean-Pierre's initial vertical speed is $0\ \mathrm{m\,s^{-1}}$.

4a. Find the value of $K$. *[2 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

$0 = K - 60(1.2^0)$ ***(M1)***

**Note:** Award ***(M1)*** for correctly substituted function equated to zero.

$(K =)\ 60$ ***(A1) (C2)***

***[2 marks]***

4b. In the context of the model, state what the horizontal asymptote represents. *[1 mark]*

# Markscheme

the (vertical) speed that Jean-Pierre is approaching (as $t$ increases) ***(A1) (C1)***

**OR**

the limit of the (vertical) speed of Jean-Pierre ***(A1) (C1)***

**Note:** Accept "maximum speed" or "terminal speed".

***[1 mark]***

4c. Find Jean-Pierre's vertical speed after $10$ seconds. Give your answer in $\mathrm{km\,h^{-1}}$ . *[3 marks]*

# Markscheme

$(S =)\ 60 - 60\left(1.2^{-10}\right)$ ***(M1)***

**Note:** Award ***(M1)*** for correctly substituted function.

$(S =)\ 50.3096\ldots\ \left(\mathrm{m\,s^{-1}}\right)$ ***(A1)*(ft)**

**Note:** Follow through from part (a).

$181\ \left(\mathrm{km\,h^{-1}}\right)\ \left(181.114\ldots\left(\mathrm{km\,h^{-1}}\right)\right)$ ***(A1)*(ft)** ***(C3)***

**Note:** Award the final ***(A1)*(ft)** for correct conversion of their speed to $\mathrm{km\ h^{-1}}$.

***[3 marks]***

5. Solve the simultaneous equations *[7 marks]*

$\log_2 6x = 1 + 2\log_2 y$

$1 + \log_6 x = \log_6 (15y - 25)$.

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of at least one "log rule" applied correctly for the first equation ***M1***

$\log_2 6x = \log_2 2 + 2\log_2 y$

$= \log_2 2 + \log_2 y^2$

$= \log_2 \left(2y^2\right)$

$\Rightarrow 6x = 2y^2$ ***A1***

use of at least one "log rule" applied correctly for the second equation ***M1***

$\log_6 \left(15y - 25\right) = 1 + \log_6 x$

$= \log_6 6 + \log_6 x$

$= \log_6 6x$

$\Rightarrow 15y - 25 = 6x$ ***A1***

attempt to eliminate $x$ (or $y$) from their two equations ***M1***

$2y^2 = 15y - 25$

$2y^2 - 15y + 25 = 0$

$\left(2y - 5\right)\left(y - 5\right) = 0$

$x = \frac{25}{12}$, $y = \frac{5}{2}$, ***A1***

or $x = \frac{25}{3}$, $y = 5$ ***A1***

**Note:** $x$, $y$ values do not have to be "paired" to gain either of the final two ***A*** marks.

***[7 marks]***

6. Solve the equation $\log_2(x+3) + \log_2(x-3) = 4$. *[5 marks]*

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\log_2(x+3)+\log_2(x-3)=4$

$\log_2(x^2-9)=4$ ***(M1)***

$x^2-9=2^4\ (=16)$ ***M1A1***

$x^2=25$

$x=\pm 5$ ***(A1)***

$x=5$ ***A1***

***[5 marks]***

7. Find the solution of $\log_2 x - \log_2 5 = 2 + \log_2 3$. *[4 marks]*

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\log_2 x - \log_2 5 = 2 + \log_2 3$

collecting at least two log terms ***(M1)***

*eg* $\log_2 \frac{x}{5} = 2 + \log_2 3$ or $\log_2 \frac{x}{15} = 2$

obtaining a correct equation without logs ***(M1)***

*eg* $\frac{x}{5} = 12$ **OR** $\frac{x}{15} = 2^2$ ***(A1)***

$x = 60$ ***A1***

***[4 marks]***

8. Given that $\log_{10}\left(\frac{1}{2\sqrt{2}}(p+2q)\right)=\frac{1}{2}(\log_{10}p+\log_{10}q)\ ,\ p>0,\ q>0,$ find $p$ in terms of $q$. *[5 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\log_{10} \frac{1}{2\sqrt{2}}(p + 2q) = \frac{1}{2}(\log_{10} p + \log_{10} q)$

$\log_{10} \frac{1}{2\sqrt{2}}(p + 2q) = \frac{1}{2}\log_{10} pq$ ***(M1)***

$\log_{10} \frac{1}{2\sqrt{2}}(p + 2q) = \log_{10}(pq)^{\frac{1}{2}}$ ***(M1)***

$\frac{1}{2\sqrt{2}}(p + 2q) = (pq)^{\frac{1}{2}}$ ***(A1)***

$(p + 2q)^2 = 8pq$

$p^2 + 4pq + 4q^2 = 8pq$

$p^2 - 4pq + 4q^2 = 0$

$(p - 2q)^2 = 0$ ***M1***

hence $p = 2q$ ***A1***

***[5 marks]***

9. Solve the equation $4^x + 2^{x+2} = 3$. *[5 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to form a quadratic in $2^x$ ***M1***

$(2^x)^2 + 4 \bullet 2^x - 3 = 0$ ***A1***

$2^x = \frac{-4 \pm \sqrt{16+12}}{2} \left(= -2 \pm \sqrt{7}\right)$ ***M1***

$2^x = -2 + \sqrt{7} \left(\text{as } -2 - \sqrt{7} < 0\right)$ ***R1***

$x = \log_2\left(-2 + \sqrt{7}\right) \left(x = \frac{\ln\left(-2+\sqrt{7}\right)}{\ln 2}\right)$ ***A1***

**Note:** Award ***R0 A1*** if final answer is $x = \log_2\left(-2 + \sqrt{7}\right)$.

***[5 marks]***