Logs practice *[48 marks]*

1. Solve the equation $2 \ln x = \ln 9 + 4$. Give your answer in the form $x = pe^q$ where $p, \ q \in \mathbb{Z}^+$.   *[5 marks]*

2. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3 \left(4x^3\right)$, where $x > 0$.   *[5 marks]*
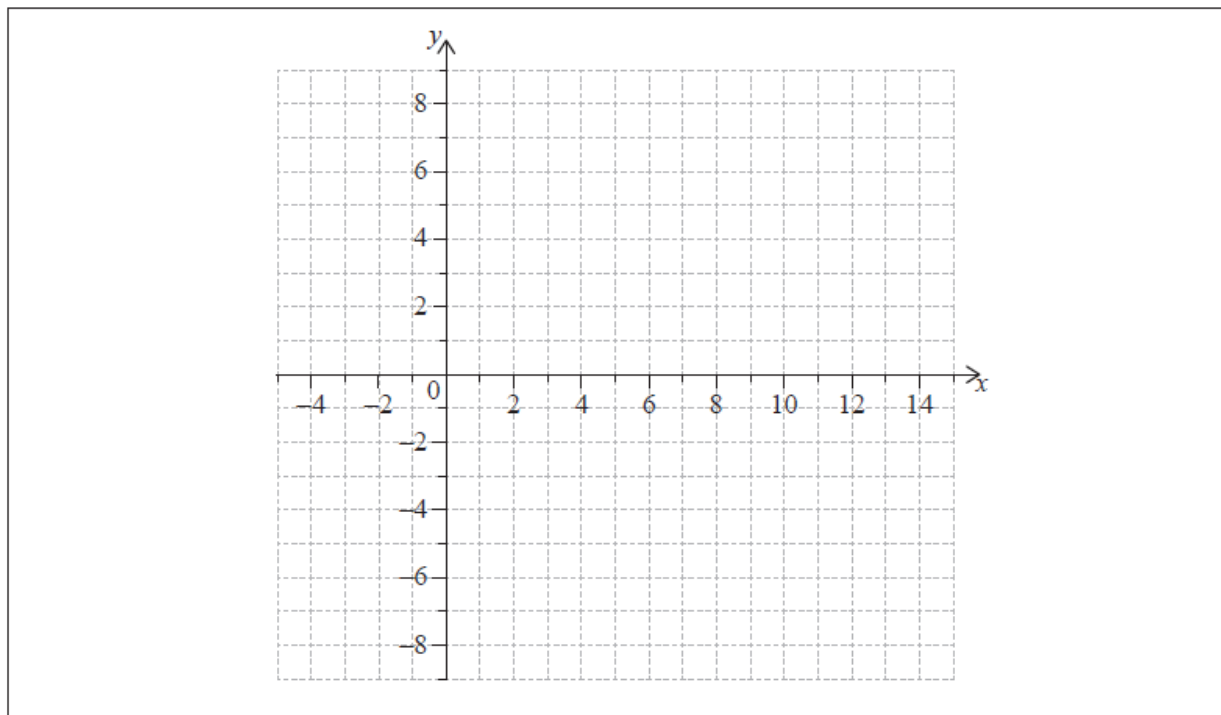
Let $f(x) = a \log_3 (x - 4)$, for $x > 4$, where $a > 0$.

Point $A(13, 7)$ lies on the graph of f.

3a. Find the value of a.   *[3 marks]*

3b. The x-intercept of the graph of f is $(5, \ 0)$.   *[3 marks]*

On the following grid, sketch the graph of f.

Jean-Pierre jumps out of an airplane that is flying at constant altitude. Before opening his parachute, he goes through a period of freefall.

Jean-Pierre's vertical speed during the time of freefall, S, in ms^{-1}, is modelled by the following function.

$$S(t) = K - 60\left(1.\,2^{-t}\right), t \geq 0$$

where t, is the number of seconds after he jumps out of the airplane, and K is a constant. A sketch of Jean-Pierre's vertical speed against time is shown below.
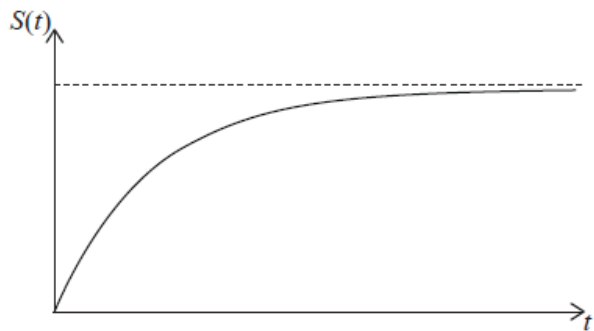
Jean-Pierre's initial vertical speed is $0 \mathrm{\,ms}^{-1}$.

4a. Find the value of K. [2 marks]

4b. In the context of the model, state what the horizontal asymptote represents. [1 mark]

4c. Find Jean-Pierre's vertical speed after 10 seconds. Give your answer in $\mathrm{km\,h}^{-1}$. [3 marks]

5. Solve the simultaneous equations [7 marks]

$\log_2 6x = 1 + 2\log_2 y$

$1 + \log_6 x = \log_6 (15y - 25)$.

6. Solve the equation $\log_2(x + 3) + \log_2(x - 3) = 4$. [5 marks]

7. Find the solution of $\log_2 x - \log_2 5 = 2 + \log_2 3$. [4 marks]

8. Given that $\log_{10}\left(\frac{1}{2\sqrt{2}}(p + 2q)\right) = \frac{1}{2}\left(\log_{10}p + \log_{10}q\right)$, $p > 0$, $q > 0$, find p in terms of q. [5 marks]

9. Solve the equation $4^x + 2^{x+2} = 3$. *[5 marks]*