Normal distribution 22.03 *[104 marks]*

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

1a. By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. *[4 marks]*

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Use $\mu = 32.29$ in the remainder of the question.

1b. Find the 86th percentile time to complete the jigsaw puzzle. *[2 marks]*

1c. Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. *[2 marks]*

Six randomly chosen people complete the jigsaw puzzle.

1d. Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. *[3 marks]*

1e. Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle. *[4 marks]*

Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle.

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

2a. Find the probability that a randomly selected chocolate muffin weighs less than 61 g. *[2 marks]*

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2b. In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. *[2 marks]*

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The weights, B grams, of the banana muffins are normally distributed with a mean of $68\ \mathrm{g}$ and standard deviation of $3.4\ \mathrm{g}$.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

2c. Find the probability that the randomly selected muffin weighs less than $61\ \mathrm{g}$. *[4 marks]*

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2d. Given that a randomly selected muffin weighs less than $61 \mathrm{~g}$, find the probability that it is chocolate. *[3 marks]*

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

2e. Find the value of σ. *[5 marks]*

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

3a. Find the value of σ. *[4 marks]*

3b. On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. *[2 marks]*

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

3c. Find the probability that she will be late to work at least one day next week. *[3 marks]*

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3d. Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. *[5 marks]*

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

3e. Find the probability that Suzi will receive a bonus. *[4 marks]*

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The random variable X follows a normal distribution with mean μ and standard deviation σ.

4a. Find $\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$. *[3 marks]*

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The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ. Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

4b. Find the value of μ and of σ. *[5 marks]*

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A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

Find the probability that an avocado chosen at random from this purchase is categorized as

4c. medium. *[2 marks]*

4d. large. *[1 mark]*

4e. premium. *[1 mark]*

4f. The selling prices of the different categories of avocado at this supermarket are shown in the following table: *[4 marks]*

Category	Medium	Large	Premium
Selling price ($) per avocado	1.10	1.29	1.96

The supermarket pays the farm $\$ 200$ for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least $\$ 438$.

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

5a. Sketch the probability density function for X, and shade the region representing $P(\mu - 2\sigma < X < \mu + \sigma)$. *[2 marks]*

5b. Find the value of $P(\mu - 2\sigma < X < \mu + \sigma)$. *[2 marks]*

5c. Find the value of k for which $P(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. *[2 marks]*

6. It is given that one in five cups of coffee contain more than 120 mg of caffeine. *[6 marks]*
It is also known that three in five cups contain more than 110 mg of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.
Find the mean and standard deviation of the caffeine content of coffee.

A random variable X is normally distributed with mean μ and standard deviation σ, such that $\mathrm{P}(X < 30.31) = 0.1180$ and $\mathrm{P}(X > 42.52) = 0.3060$.

7a. Find μ and σ. *[6 marks]*

7b. Find $\mathrm{P}\left(|X - \mu| < 1.2\sigma\right)$. *[2 marks]*

Let X and Y be normally distributed with $X \sim \mathrm{N}\left(14, a^2\right)$ and $Y \sim \mathrm{N}\left(22, a^2\right)$, $a > 0$.

8a. Find b so that $\mathrm{P}\left(X > b\right) = \mathrm{P}\left(Y < b\right)$. *[2 marks]*

8b. It is given that $\mathrm{P}\left(X > 20\right) = 0.112$. *[4 marks]*

Find $\mathrm{P}\left(16 < Y < 28\right)$.

Let X be a random variable which follows a normal distribution with mean μ.
Given that $\mathrm{P}\left(X < \mu - 5\right) = 0.2$, find

9a. $\mathrm{P}\left(X > \mu + 5\right)$. *[2 marks]*

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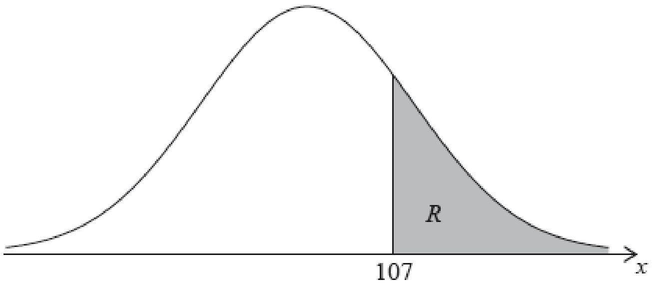
9b. $\mathrm{P}\left(X < \mu + 5 \mid X > \mu - 5\right)$. *[5 marks]*

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

10a. Write down $\mathrm{P}(X > 107)$. *[1 mark]*

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10b. Find $\mathrm{P}(100 < X < 107)$. *[3 marks]*

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10c. Find $\mathrm{P}(93 < X < 107)$. *[2 marks]*

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