Normal distribution 22.03 *[104 marks]*

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

1a. By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. *[4 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$T \sim \mathrm{N}\left(\mu,\ 8.6^2\right)$

$\mathrm{P}(T \le 36.8) = 0.7$ **(A1)**

states a correct equation, for example, $\frac{36.8-\mu}{8.6} = 0.5244\ldots$ **A1**

attempts to solve their equation **(M1)**

$\mu = 36.8 - (0.5244\ldots)(8.6)(= 32.2902\ldots)$ **A1**

the solution to the equation is $\mu = 32.29$, correct to two decimal places **AG**

[4 marks]

Use $\mu = 32.29$ in the remainder of the question.

1b. Find the 86th percentile time to complete the jigsaw puzzle. *[2 marks]*

Markscheme

let $t_{0.86}$ be the 86th percentile

attempts to use the inverse normal feature of a GDC to find $t_{0.86}$ **(M1)**

$t_{0.86} = 41.6$ (mins) **A1**

[2 marks]

1c. Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. *[2 marks]*

Markscheme

evidence of identifying the correct area under the normal curve **(M1)**

Note: Award **M1** for a clearly labelled sketch.

$\mathrm{P}(T > 30) = 0.605$ **A1**

[2 marks]

Six randomly chosen people complete the jigsaw puzzle.

1d. Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. *[3 marks]*

Markscheme

let X represent the number of people out of the six who take more than 30 minutes to complete the jigsaw puzzle

$X \sim \mathrm{B}(6, 0.6049\ldots)$ **(M1)**

for example, $\mathrm{P}(X = 5) + \mathrm{P}(X = 6)$ or $1 - \mathrm{P}(X \leq 4)$ **(A1)**

$\mathrm{P}(X \geq 5) = 0.241$ **A1**

[3 marks]

1e. Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle. *[4 marks]*

Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle.

Markscheme

recognizes that $\mathrm{P}\left(T > 30 \middle| T \geq 25\right)$ is required **(M1)**

Note: Award **M1** for recognizing conditional probability.

$= \frac{\mathrm{P}(T>30 \cap T\geq 25)}{\mathrm{P}(T\geq 25)}$ **(A1)**

$= \frac{\mathrm{P}(T>30)}{\mathrm{P}(T\geq 25)} = \frac{0.6049\ldots}{0.8016\ldots}$ **M1**

$= 0.755$ **A1**

[4 marks]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

2a. Find the probability that a randomly selected chocolate muffin weighs less than 61 g. *[2 marks]*

Markscheme

$\mathrm{P}(C < 61)$ ***(M1)***

$= 0.365112\ldots$

$= 0.365$ ***A1***

[2 marks]

2b. In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. *[2 marks]*

Markscheme

recognition of binomial eg $X \sim \text{B}(12,\ 0.365\ldots)$ ***(M1)***

$\text{P}(X=5) = 0.213666\ldots$

$= 0.214$ ***A1***

[2 marks]

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

2c. Find the probability that the randomly selected muffin weighs less than 61 g. *[4 marks]*

Markscheme

Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$\text{P}(B < 61) = 0.0197555\ldots$ ***(A1)***

EITHER

$\text{P}(CM) \times \text{P}\left(C < 61 \mid CM\right) + \text{P}(BM) \times \text{P}\left(B < 61 \mid BM\right)$ (or equivalent in words) ***(M1)***

OR

tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN

$(0.6 \times 0.365\ldots) + (0.4 \times 0.0197\ldots)$ ***(A1)***

$= 0.226969\ldots$

$= 0.227$ ***A1***

[4 marks]

2d. Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. *[3 marks]*

Markscheme

recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $\text{P}\left(A \mid B\right)$

$\frac{0.6 \times 0.365112...}{0.226969...}$ ***(A1)***

$= 0.965183...$

$= 0.965$ ***A1***

[3 marks]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to $\sigma\text{ g}$. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

2e. Find the value of σ. *[5 marks]*

Markscheme

METHOD 1

$\mathrm{P}(CM) \times \mathrm{P}\left(C < 61 \mid CM\right) \times \mathrm{P}(BM) \times \mathrm{P}\left(B < 61 \mid BM\right) = 0.157$ ***(M1)***

$(0.6 \times \mathrm{P}(C < 61)) + (0.4 \times 0.0197555\ldots) = 0.157$

$\mathrm{P}(C < 61) = 0.248496\ldots$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}(C < 61)$ with a variable standard deviation.

$\sigma = 1.47225\ldots$

$\sigma = 1.47\,(\mathrm{g})$ ***A2***

METHOD 2

$\mathrm{P}(CM) \times \mathrm{P}\left(C < 61 \mid CM\right) \times \mathrm{P}(BM) \times \mathrm{P}\left(B < 61 \mid BM\right) = 0.157$ ***(M1)***

$(0.6 \times \mathrm{P}(C < 61)) + (0.4 \times 0.0197555\ldots) = 0.157$

$\mathrm{P}(C < 61) = 0.248496\ldots$ ***(A1)***

use of inverse normal to find z score of their $\mathrm{P}(C < 61)$ ***(M1)***

$z = -0.679229\ldots$

correct substitution ***(A1)***

$\frac{61-62}{\sigma} = -0.679229\ldots$

$\sigma = 1.47225\ldots$

$\sigma = 1.47\,(\mathrm{g})$ ***A1***

[5 marks]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

3a. Find the value of σ. *[4 marks]*

Markscheme

METHOD 1

$T \sim \mathrm{N}\left(35, \sigma^2\right)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

attempt to solve for σ graphically or numerically using the GDC ***(M1)***

graph of normal curve $T \sim \mathrm{N}\left(35, \sigma^2\right)$ for $\mathrm{P}(T > 40)$ and $y = 0.25$ OR $\mathrm{P}(T < 40)$ and $y = 0.75$
OR table of values for $\mathrm{P}(T < 40)$ or $\mathrm{P}(T > 40)$

$\sigma = 7.413011\ldots$

$\sigma = 7.41$ (min) ***A2***

METHOD 2

$T \sim \mathrm{N}\left(35, \sigma^2\right)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

$z = 0.674489\ldots$ ***(A1)***

valid equation using their z-score (clearly identified as z-score and not a probability) ***(M1)***

$\frac{40-35}{\sigma} = 0.674489\ldots$ OR $5 = 0.674489\ldots\sigma$

$7.413011\ldots$

$\sigma = 7.41$ (min) ***A1***

[4 marks]

3b. On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. *[2 marks]*

Markscheme

$\mathrm{P}(T > 45)$ ***(M1)***

$= 0.0886718\ldots$

$= 0.0887$ ***A1***

[2 marks]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

3c. Find the probability that she will be late to work at least one day next week. *[3 marks]*

Markscheme

recognizing binomial probability ***(M1)***

$L \sim \mathrm{B}(5,\ 0.0886718\ldots)$

$\mathrm{P}(L \geq 1) = 1 - \mathrm{P}(L = 0)$ OR

$\mathrm{P}(L \geq 1) = \mathrm{P}(L = 1) + \mathrm{P}(L = 2) + \mathrm{P}(L = 3) + \mathrm{P}(L = 4) + \mathrm{P}(L = 5)$ ***(M1)***

$0.371400\ldots$

$\mathrm{P}(L \geq 1) = 0.371$ ***A1***

[3 marks]

3d. Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. *[5 marks]*

Markscheme

recognizing conditional probability in context ***(M1)***

finding $\{L<3\}\cap\{L\geq 1\}=\{L=1,\ L=2\}$ (may be seen in conditional probability) ***(A1)***

$\mathrm{P}(L=1)+\mathrm{P}(L=2)=0.36532\ldots$ (may be seen in conditional probability) ***(A1)***

$P\left(L<3\,\middle|\,L\geq 1\right)=\frac{0.36532\ldots}{0.37140\ldots}$ ***(A1)***

$0.983636\ldots$

0.984 ***A1***

[5 marks]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

3e. Find the probability that Suzi will receive a bonus. *[4 marks]*

Markscheme

METHOD 1

recognizing that Suzi can be late no more than once (in the remaining six days) ***(M1)***

$\mathrm{X \sim B}(6,\ 0.0886718\ldots)$, where X is the number of days late ***(A1)***

$\mathrm{P}(X \leq 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$ ***(M1)***

$= 0.907294\ldots$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

METHOD 2

recognizing that Suzi must be on time at least five times (of the remaining six days) ***(M1)***

$\mathrm{X \sim B}(6,\ 0.911328\ldots)$, where X is the number of days on time ***(A1)***

$\mathrm{P}(X \geq 5) = 1 - \mathrm{P}(X \leq 4)$ OR $1 - 0.0927052\ldots$ OR $\mathrm{P}(X = 5) + (X = 6)$ OR $0.334434\ldots + 0.572860\ldots$ ***(M1)***

$= 0.907294\ldots$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

[4 marks]

The random variable X follows a normal distribution with mean μ and standard deviation σ.

4a. Find $\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$. *[3 marks]*

Markscheme

$\mathrm{P}\left(\frac{\mu-1.5\sigma-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{\mu+1.5\sigma-\mu}{\sigma}\right)$ ***(M1)***

$P(-1.5 < Z < 1.5)$ OR $1-2\times\mathrm{P}(Z < -1.5)$ ***(A1)***

$\mathrm{P}(-1.5 < Z < 1.5) = 0.866385\ldots$

$\mathrm{P}(\mu-1.5\sigma < X < \mu+1.5\sigma) = 0.866$ ***A1***

Note: Do not award any marks for use of their answers from part (b).

[3 marks]

The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ. Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

4b. Find the value of μ and of σ. *[5 marks]*

Markscheme

$z_1 = -1.75068\ldots$ and $z_2 = 1.30468\ldots$ (seen anywhere) ***(A1)***

correct equations ***(A1)(A1)***

$\frac{106.2-\mu}{\sigma} = -1.75068\ldots,\ \ \mu + 1.30468\ldots\sigma = 182.6$

attempt to solve their equations involving z values ***(M1)***

$\mu = 149.976\ldots,\ \sigma = 25.0051\ldots$

$\mu = 150,\ \sigma = 25.0$ ***A1***

[5 marks]

A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

Find the probability that an avocado chosen at random from this purchase is categorized as

4c. medium. *[2 marks]*

Markscheme

new sample space is 96% (may be seen in (ii) or (iii)) ***(M1)***

$\text{P}(\text{medium}|\text{not small})$ OR $\frac{0.576}{0.96}$

$\text{P}(\text{Medium}) = 0.6$ ***A1***

[2 marks]

4d. large. *[1 mark]*

Markscheme

$\text{P}(\text{Large}) = 0.3$ ***A1***

[1 mark]

4e. premium. *[1 mark]*

Markscheme

$\text{P}(\text{Premium}) = 0.1$ ***A1***

[1 mark]

4f. The selling prices of the different categories of avocado at this supermarket are shown in the following table: *[4 marks]*

Category	Medium	Large	Premium
Selling price ($) per avocado	1.10	1.29	1.96

The supermarket pays the farm $\$ 200$ for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least $\$ 438$.

Markscheme

attempt to express revenue from avocados ***(M1)***

$1.1 \times 0.6 + 1.29 \times 0.3 + 1.96 \times 0.1$ OR $1.243n$

correct inequality or equation for net profit in terms of n ***(A1)***

$1.1 \times 0.6n + 1.29 \times 0.3n + 1.96 \times 0.1n - 200 \geq 438$ OR
$1.243n - 200 = 438$

attempt to solve the inequality ***(M1)***

sketch OR $n = 513.274\ldots$

$n = 514$ ***A1***

Note: Only award follow through in part (d) for 3 probabilities which add up to 1. FT of probabilities from c) that do not add up to 1 should only be awarded ***M*** marks, where appropriate, in d).

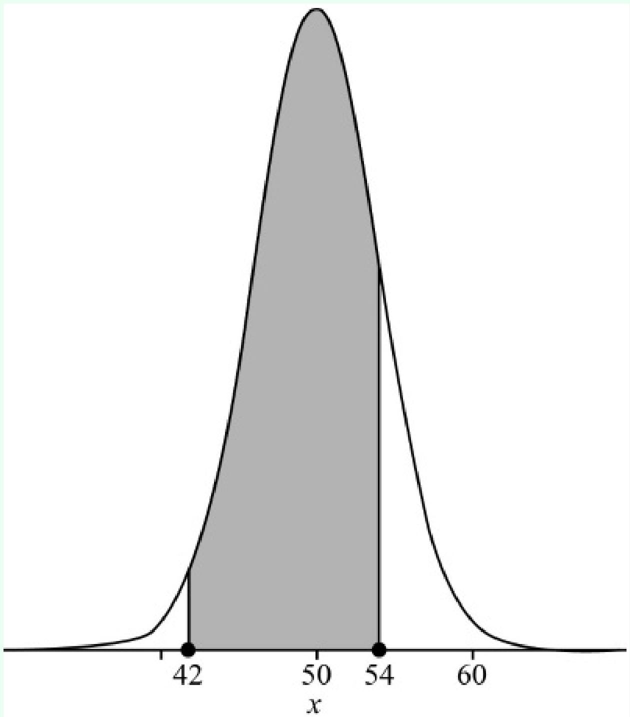
[4 marks]

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

5a. Sketch the probability density function for X, and shade the region representing $P(\mu - 2\sigma < X < \mu + \sigma)$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



normal curve centred on 50 ***A1***

vertical lines at $x = 42$ and $x = 54$, with shading in between ***A1***

[2 marks]

5b. Find the value of $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. *[2 marks]*

Markscheme

$\mathrm{P}(42 < X < 54)$ $(= \mathrm{P}(-2 < Z < 1))$ ***(M1)***

$= 0.819$ ***A1***

[2 marks]

5c. Find the value of k for which $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. *[2 marks]*

Markscheme

$P(\mu - k\sigma < X < \mu + k\sigma) = 0.5 \Rightarrow P(X < \mu + k\sigma) = 0.75$ ***(M1)***

$k = 0.674$ ***A1***

Note: Award ***M1A0*** for $k = -0.674$.

[2 marks]

6. It is given that one in five cups of coffee contain more than 120 mg of caffeine. *[6 marks]*
It is also known that three in five cups contain more than 110 mg of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.
Find the mean and standard deviation of the caffeine content of coffee.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

let X be the random variable "amount of caffeine content in coffee"

$\mathrm{P}(X > 120) = 0.2, \mathrm{P}(X > 110) = 0.6$ ***(M1)***

$(\Rightarrow \mathrm{P}(X < 120) = 0.8, \mathrm{P}(X < 110) = 0.4)$

Note: Award ***M1*** for at least one correct probability statement.

$\frac{120-\mu}{\sigma} = 0.84162\ldots, \frac{110-\mu}{\sigma} = -0.253347\ldots$ ***(M1)(A1)(A1)***

Note: Award ***M1*** for attempt to find at least one appropriate z-value.

$120 - \mu = 0.84162\sigma, 110 - \mu = -0.253347\sigma$

attempt to solve simultaneous equations ***(M1)***

$\mu = 112, \sigma = 9.13$ ***A1***

[6 marks]

A random variable X is normally distributed with mean μ and standard deviation σ, such that $\mathrm{P}(X < 30.31) = 0.1180$ and $\mathrm{P}(X > 42.52) = 0.3060$.

7a. Find μ and σ. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\mathrm{P}(X < 42.52) = 0.6940$ ***(M1)***

either $\mathrm{P}\left(Z < \frac{30.31-\mu}{\sigma}\right) = 0.1180$or $\mathrm{P}\left(Z < \frac{42.52-\mu}{\sigma}\right) = 0.6940$ ***(M1)***

$\frac{30.31-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.1180)}_{-1.1850...}$ ***(A1)***

$\frac{42.52-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.6940)}_{0.5072...}$ ***(A1)***

attempting to solve simultaneously ***(M1)***

$\mu = 38.9$ and $\sigma = 7.22$ ***A1***

[6 marks]

7b. Find $\mathrm{P}\left(|X - \mu| < 1.2\sigma\right)$. *[2 marks]*

Markscheme

$\mathrm{P}(\mu - 1.2\sigma < X < \mu + 1.2\sigma)$ (or equivalent *eg*. $2\mathrm{P}(\mu < X < \mu + 1.2\sigma)$) ***(M1)***

$= 0.770$ ***A1***

Note: Award ***(M1)A1*** for $\mathrm{P}(-1.2 < Z < 1.2) = 0.770$.

[2 marks]

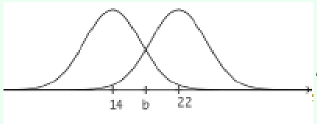
Let X and Y be normally distributed with $X \sim \mathrm{N}\left(14, a^2\right)$ and $Y \sim \mathrm{N}\left(22, a^2\right)$, $a > 0$.

8a. Find b so that $\mathrm{P}\left(X > b\right) = \mathrm{P}\left(Y < b\right)$. *[2 marks]*

Markscheme

METHOD 1

recognizing that b is midway between the means of 14 and 22. ***(M1)***

eg 

, $b = \frac{14+22}{2}$

$b = 18$ ***A1 N2***

METHOD 2

valid attempt to compare distributions ***(M1)***

eg $\frac{b-14}{a} = \frac{b-22}{a}$, $b - 14 = 22 - b$

$b = 18$ ***A1 N2***

[2 marks]

8b. It is given that $\mathrm{P}\left(X > 20\right) = 0.112$. *[4 marks]*

Find $\mathrm{P}\left(16 < Y < 28\right)$.

Markscheme

valid attempt to compare distributions (seen anywhere) ***(M1)***

eg Y is a horizontal translation of X of 8 units to the right,

$\mathrm{P}\left(16 < Y < 28\right) = \mathrm{P}\left(8 < X < 20\right), \mathrm{P}\left(Y > 22 + 6\right) = \mathrm{P}\left(X > 14 + 6\right)$

valid approach using symmetry ***(M1)***

eg

$1 - 2\mathrm{P}\left(X > 20\right), 1 - 2\mathrm{P}\left(Y < 16\right), 2 \times \mathrm{P}\left(14 < x < 20\right), \mathrm{P}\left(X < 8\right) = \mathrm{P}\left(X >\right.$

correct working ***(A1)***

eg $1 - 2\left(0.112\right), 2 \times \left(0.5 - 0.112\right), 2 \times 0.388, 0.888 - 0.112$

$\mathrm{P}\left(16 < Y < 28\right) = 0.776$ ***A1 N3***

[4 marks]

Let X be a random variable which follows a normal distribution with mean μ. Given that $\mathrm{P}\left(X < \mu - 5\right) = 0.2$, find

9a. $\mathrm{P}\left(X > \mu + 5\right)$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of symmetry *eg* diagram ***(M1)***

$\mathrm{P}\left(X > \mu + 5\right) = 0.2$ ***A1***

[2 marks]

9b. $\mathrm{P}\left(X < \mu + 5 \mid X > \mu - 5\right)$. *[5 marks]*

Markscheme

EITHER

$\mathrm{P}\left(X < \mu + 5 \mid X > \mu - 5\right) = \frac{\mathrm{P}(X > \mu - 5 \cap X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(M1)***

$= \frac{\mathrm{P}(\mu - 5 < X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(A1)***

$= \frac{0.6}{0.8}$ ***A1A1***

Note: ***A1*** for denominator is independent of the previous ***A*** marks.

OR

use of diagram ***(M1)***

Note: Only award ***(M1)*** if the region $\mu - 5 < X < \mu + 5$ is indicated and used.

$\mathrm{P}\left(X > \mu - 5\right) = 0.8$ $\mathrm{P}\left(\mu - 5 < X < \mu + 5\right) = 0.6$ ***(A1)***

Note: Probabilities can be shown on the diagram.

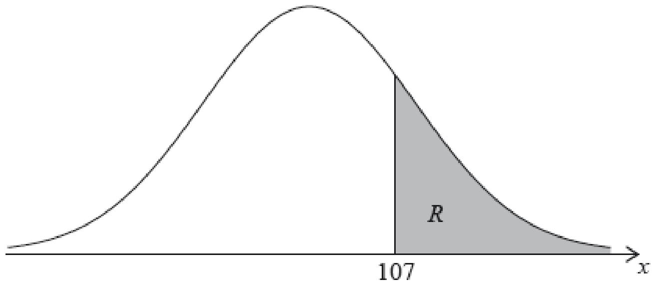
$= \frac{0.6}{0.8}$ ***M1A1***

THEN

$= \frac{3}{4}$ $= (0.75)$ ***A1***

[5 marks]

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

10a. Write down $\mathrm{P}(X > 107)$. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\mathrm{P}(X > 107) = 0.24 \quad \left(= \frac{6}{25},\ 24\%\right)$ ***A1 N1***

[1 mark]

10b. Find $\mathrm{P}(100 < X < 107)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(X > 100) = 0.5,\ \mathrm{P}(X > 100) - \mathrm{P}(X > 107)$

correct working ***(A1)***

eg $0.5 - 0.24,\ 0.76 - 0.5$

$\mathrm{P}(100 < X < 107) = 0.26 \quad \left(= \frac{13}{50},\ 26\%\right)$ ***A1 N2***

[3 marks]

10c. Find $\mathrm{P}(93 < X < 107)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $2 \times 0.26,\ 1 - 2(0.24),\ \mathrm{P}(93 < X < 100) = \mathrm{P}(100 < X < 107)$

$\mathrm{P}(93 < X < 107) = 0.52 \quad \left(= \frac{13}{25},\ 52\%\right)$ ***A1 N2***

[2 marks]