Related rates *[20 marks]*

A water trough which is 10 metres long has a uniform cross-section in the shape of a semicircle with radius 0.5 metres. It is partly filled with water as shown in the following diagram of the cross-section. The centre of the circle is O and the angle KOL is θ radians.

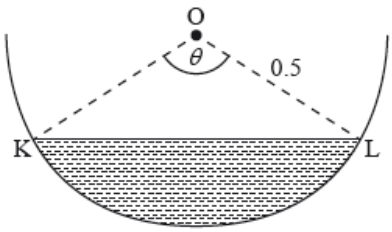

diagram not to scale

1a. Find an expression for the volume of water $V\ (\mathrm{m}^3)$ in the trough in terms of θ. *[3 marks]*

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The volume of water is increasing at a constant rate of $0.0008\text{m}^3\text{s}^{-1}$.

1b. Calculate $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ when $\theta = \frac{\pi}{3}$. *[4 marks]*

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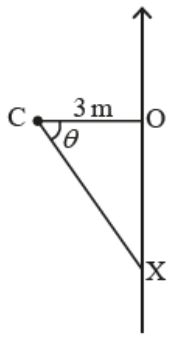
2. A camera at point C is 3 m from the edge of a straight section of road as shown in the following diagram. The camera detects a car travelling along the road at $t = 0$. It then rotates, always pointing at the car, until the car passes O, the point on the edge of the road closest to the camera. *[6 marks]*



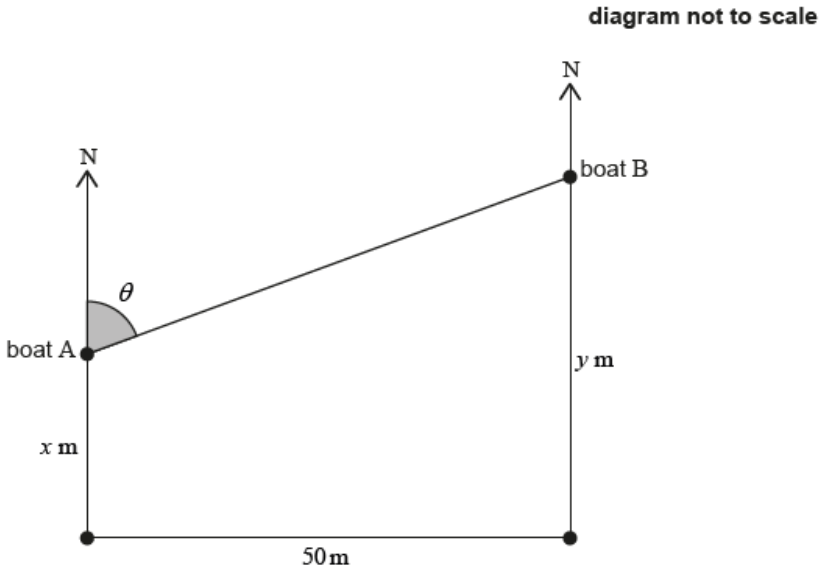
A car travels along the road at a speed of 24 ms^{-1}. Let the position of the car be X and let $\text{O}\hat{\text{C}}\text{X} = \theta$.

Find $\frac{\mathrm{d}\theta}{\mathrm{d}t}$, the rate of rotation of the camera, in radians per second, at the instant the car passes the point O .

Two boats A and B travel due north.

Initially, boat B is positioned 50 metres due east of boat A.

The distances travelled by boat A and boat B, after t seconds, are x metres and y metres respectively. The angle θ is the radian measure of the bearing of boat B from boat A. This information is shown on the following diagram.



3a. Show that $y = x + 50 \cot \theta$. *[1 mark]*

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3b. At time T, the following conditions are true. *[6 marks]*

Boat B has travelled 10 metres further than boat A.
Boat B is travelling at double the speed of boat A.
The rate of change of the angle θ is -0.1 radians per second.

Find the speed of boat A at time T.

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