

# Related rates [20 marks]

A water trough which is 10 metres long has a uniform cross-section in the shape of a semicircle with radius 0.5 metres. It is partly filled with water as shown in the following diagram of the cross-section. The centre of the circle is O and the angle KOL is  $\theta$  radians.

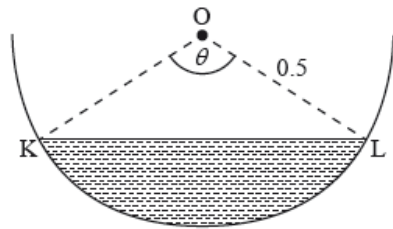


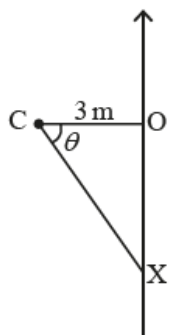
diagram not to scale

- 1a. Find an expression for the volume of water  $V$  ( $\text{m}^3$ ) in the trough in terms of  $\theta$ . [3 marks]

The volume of water is increasing at a constant rate of  $0.0008\text{m}^3\text{s}^{-1}$ .

- 1b. Calculate  $\frac{d\theta}{dt}$  when  $\theta = \frac{\pi}{3}$ . [4 marks]

2. A camera at point C is 3 m from the edge of a straight section of road as [6 marks] shown in the following diagram. The camera detects a car travelling along the road at  $t = 0$ . It then rotates, always pointing at the car, until the car passes O, the point on the edge of the road closest to the camera.



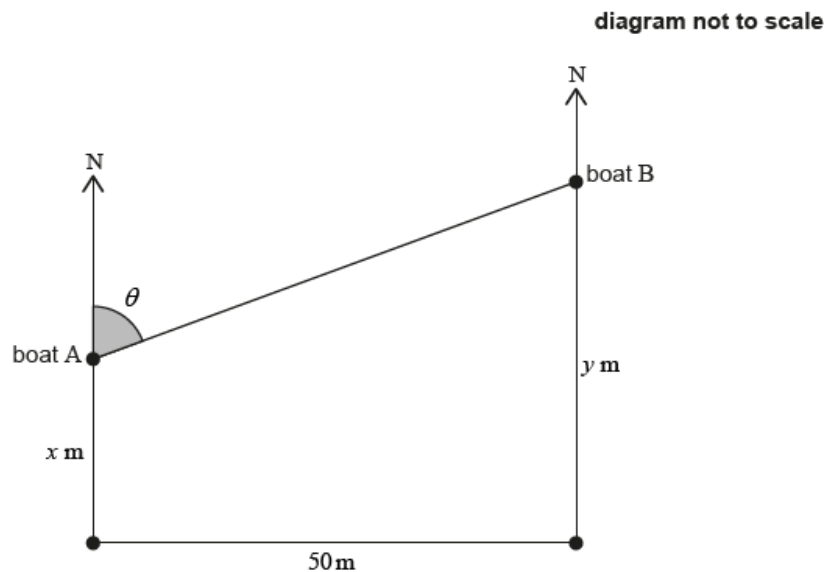
A car travels along the road at a speed of  $24 \text{ ms}^{-1}$ . Let the position of the car be X and let  $\widehat{OCX} = \theta$ .

Find  $\frac{d\theta}{dt}$ , the rate of rotation of the camera, in radians per second, at the instant the car passes the point O.

Two boats A and B travel due north.

Initially, boat B is positioned 50 metres due east of boat A.

The distances travelled by boat A and boat B, after  $t$  seconds, are  $x$  metres and  $y$  metres respectively. The angle  $\theta$  is the radian measure of the bearing of boat B from boat A. This information is shown on the following diagram.



3a. Show that  $y = x + 50 \cot \theta$ . [1 mark]

3b. At time  $T$ , the following conditions are true. [6 marks]

Boat B has travelled 10 metres further than boat A.

Boat B is travelling at double the speed of boat A.

The rate of change of the angle  $\theta$  is  $-0.1$  radians per second.

Find the speed of boat A at time  $T$ .