# Trig 29.03 Paper I *[198 marks]*

1. By using the substitution $u = \sec x$ or otherwise, find an expression for *[6 marks]*

   $\int_0^{\frac{\pi}{3}} \sec^n x \tan x \, \mathrm{d}x$ in terms of $n$, where $n$ is a non-zero real number.

# Markscheme

**METHOD 1**

$u = \sec x \Rightarrow \mathrm{d}\,u = \sec x \tan x \,\mathrm{d}\,x$ ***(A1)***

attempts to express the integral in terms of $u$ ***M1***

$\int_1^2 u^{n-1}\,\mathrm{d}\,u$ ***A1***

$= \frac{1}{n}[u^n]_1^2 \ \left(= \frac{1}{n}[\sec^n x]_0^{\frac{\pi}{3}}\right)$ ***A1***

**Note:** Condone the absence of or incorrect limits up to this point.

$= \frac{2^n - 1^n}{n}$ ***M1***

$= \frac{2^n - 1}{n}$ ***A1***

**Note:** Award ***M1*** for correct substitution of **<u>their</u>** limits for $u$ into their antiderivative for $u$ (or given limits for $x$ into their antiderivative for $x$).

**METHOD 2**

$\int \sec^n x \tan x \,\mathrm{d}\,x = \int \sec^{n-1} x \sec x \tan x \,\mathrm{d}\,x$ ***(A1)***

applies integration by inspection ***(M1)***

$= \frac{1}{n}[\sec^n x]_0^{\frac{\pi}{3}}$ ***A2***

**Note:** Award ***A2*** if the limits are not stated.

$= \frac{1}{n}\left(\sec^n \frac{\pi}{3} - \sec^n 0\right)$ ***M1***

**Note:** Award ***M1*** for correct substitution into their antiderivative.

$= \frac{2^n - 1}{n}$ ***A1***

***[6 marks]***

2a. Show that $2x-3-\frac{6}{x-1}=\frac{2x^2-5x-3}{x-1},\ x\in\mathbb{R},\ x\neq 1$. *[2 marks]*

# Markscheme

**METHOD 1**

attempt to write all LHS terms with a common denominator of $x-1$ ***(M1)***

$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1}$ OR $\frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$

$=\frac{2x^2-2x-3x+3-6}{x-1}$ OR $\frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$ ***A1***

$=\frac{2x^2-5x-3}{x-1}$ ***AG***

**METHOD 2**

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder $-6$ ***A1***

$=2x-3-\frac{6}{x-1}$ as required. ***AG***

***[2 marks]***

2b. Hence or otherwise, solve the equation $2\sin 2\theta-3-\frac{6}{\sin 2\theta-1}=0$ for $0\leq\theta\leq\pi,\ \theta\neq\frac{\pi}{4}$. *[5 marks]*

# Markscheme

consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$

**EITHER**

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ ***(M1)***

**Note:** Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

**OR**

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$

**THEN**

$\sin 2\theta = -\frac{1}{2}$ or $\sin 2\theta = 3$ ***(A1)***

**Note:** Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept $210$ or $330$) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

**Note:** Award ***A0*** if additional answers given.

***[5 marks]***

3a. Show that $\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$. *[2 marks]*

# Markscheme

**Note:** Do not award the final ***A1*** for proofs which work from both sides to find a common expression other than $2\sin x\cos x - 2\sin^2 x$.

**METHOD 1 (LHS to RHS)**

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

LHS $= 2\sin x\cos x + \cos 2x - 1$ OR

$\sin 2x + 1 - 2\sin^2 x - 1$ OR

$2\sin x\cos x + 1 - 2\sin^2 x - 1$

$= 2\sin x\cos x - 2\sin^2 x$ ***A1***

$\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x) =$ RHS ***AG***

**METHOD 2 (RHS to LHS)**

RHS $= 2\sin x\cos x - 2\sin^2 x$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

$= \sin 2x + 1 - 2\sin^2 x - 1$ ***A1***

$= \sin 2x + \cos 2x - 1 =$ LHS ***AG***

***[2 marks]***

3b. Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. *[6 marks]*

## Markscheme

attempt to factorise ***M1***

$(\cos x - \sin x)(2\sin x + 1) = 0$ ***A1***

recognition of $\cos x = \sin x \Rightarrow \frac{\sin x}{\cos x} = \tan x = 1$ OR $\sin x = -\frac{1}{2}$ ***(M1)***

one correct reference angle seen anywhere, accept degrees ***(A1)***

$\frac{\pi}{4}$ OR $\frac{\pi}{6}$ (accept $-\frac{\pi}{6},\ \frac{7\pi}{6}$)

**Note:** This ***(M1)(A1)*** is independent of the previous ***M1A1***.

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}$ ***A2***

**Note:** Award ***A1*** for any two correct (radian) answers.
Award ***A1A0*** if additional values given with the four correct (radian) answers.
Award ***A1A0*** for four correct answers given in degrees.

***[6 marks]***

4a. Show that the equation $2\cos^2 x + 5\sin x = 4$ may be written in the form $2\sin^2 x - 5\sin x + 2 = 0$. *[1 mark]*

## Markscheme

**METHOD 1**

correct substitution of $\cos^2 x = 1 - \sin^2 x$ ***A1***

$2(1 - \sin^2 x) + 5\sin x = 4$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

**METHOD 2**

correct substitution using double-angle identities ***A1***

$(2\cos^2 x - 1) + 5\sin x = 3$

$1 - 2\sin^2 x - 5\sin x = 3$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

***[1 mark]***

4b. Hence, solve the equation $2\cos^2 x + 5\sin x = 4, 0 \le x \le 2\pi$. *[5 marks]*

# Markscheme

**EITHER**

attempting to factorise ***M1***

$(2\sin x - 1)(\sin x - 2)$ ***A1***

**OR**

attempting to use the quadratic formula ***M1***

$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left(= \frac{5 \pm 3}{4}\right)$ ***A1***

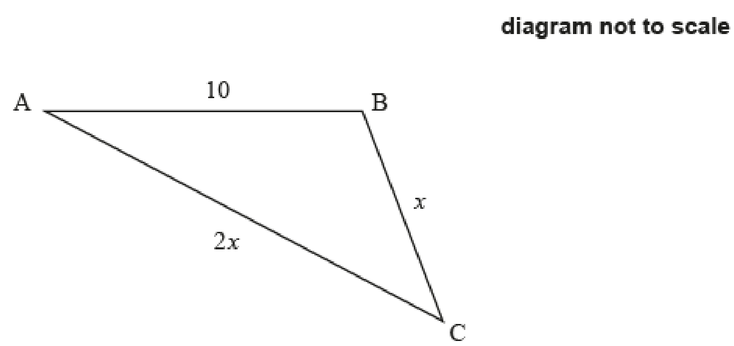
**THEN**

$\sin x = \frac{1}{2}$ ***(A1)***

$x = \frac{\pi}{6}, \ \frac{5\pi}{6}$ ***A1A1***

***[5 marks]***

5. The following diagram shows triangle $\mathrm{ABC}$, with $\mathrm{AB} = 10$, $\mathrm{BC} = x$ and $\mathrm{AC} = 2x$. *[7 marks]*




Given that $\cos \hat{\mathrm{C}} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

# Markscheme

**METHOD 1**

attempt to use the cosine rule to find the value of $x$ ***(M1)***

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ ***A1***

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}\left(= 5\sqrt{2}\right)$ ***A1***

attempt to find $\sin \widehat{C}$ (seen anywhere) ***(M1)***

$\sin^2 \widehat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side $3$ and hypotenuse $4$

$\sin \widehat{C} = \frac{\sqrt{7}}{4}$ ***(A1)***

**Note:** The marks for finding $\sin \widehat{C}$ may be awarded independently of the first three marks for finding $x$.

correct substitution into the area formula using their value of $x$ (or $x^2$) and their value of $\sin \widehat{C}$ ***(M1)***

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

**METHOD 2**

attempt to find the height, $h$, of the triangle in terms of $x$ ***(M1)***

$h^2 + \left(\frac{3}{4}x\right)^2 = x^2$ OR $h^2 + \left(\frac{5}{4}x\right)^2 = 10^2$ OR $h = \frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either $h^2$ or $h$ ***(M1)***

$x^2 - \left(\frac{3}{4}x\right)^2 = 10^2 - \left(\frac{5}{4}x\right)^2$ OR $\sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2 = 50$ OR $x = \sqrt{50}\left(= 5\sqrt{2}\right)$ ***A1***

correct substitution into the area formula using their value of $x$ (or $x^2$) ***(M1)***

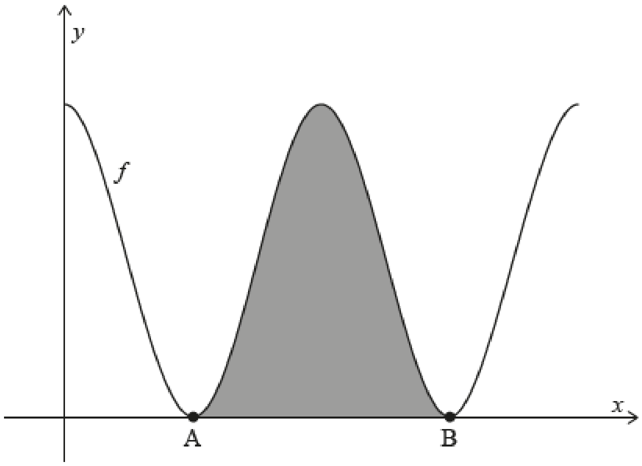
$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}\sqrt{50}$ OR $A = \frac{1}{2}\left(2 \times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

***[7 marks]***

Consider the function $f$ defined by $f(x) = 6 + 6\cos x$, for $0 \leq x \leq 4\pi$.

The following diagram shows the graph of $y = f(x)$.




The graph of $f$ touches the $x$-axis at points $\mathrm{A}$ and $\mathrm{B}$, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the $x$-axis, between the points $\mathrm{A}$ and $\mathrm{B}$.

6a. Find the $x$-coordinates of $\mathrm{A}$ and $\mathrm{B}$. *[3 marks]*

# Markscheme

$6 + 6\cos x = 0$ (or setting their $f'(x) = 0$) ***(M1)***

$\cos x = -1$ (or $\sin x = 0$)

$x = \pi, x = 3\pi$ ***A1A1***

***[3 marks]***

6b. Show that the area of the shaded region is $12\pi$. *[5 marks]*

## Markscheme

attempt to integrate $\int_{\pi}^{3\pi} (6 + 6\cos x)\,\mathrm{d}x$ ***(M1)***

$= [6x + 6\sin x]_{\pi}^{3\pi}$ ***A1A1***

substitute their limits into their integrated expression and subtract ***(M1)***

$=(18\pi + 6\sin 3\pi) - (6\pi + 6\sin \pi)$

$=(6(3\pi) + 0) - (6\pi + 0)(= 18\pi - 6\pi)$ ***A1***

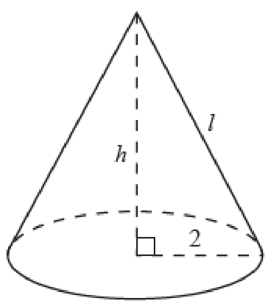
area $= 12\pi$ ***AG***

***[5 marks]***

The right cone in the following diagram has a total surface area of $12\pi$, equal to the shaded area in the previous diagram.

The cone has a base radius of $2$, height $h$, and slant height $l$.

**diagram not to scale**




6c. Find the value of $l$. *[3 marks]*

## Markscheme

attempt to substitute into formula for surface area (including base) ***(M1)***

$\pi(2^2) + \pi(2)(l) = 12\pi$ ***(A1)***

$4\pi + 2\pi l = 12\pi$

$2\pi l = 8\pi$

$l = 4$ ***A1***

***[3 marks]***

6d. Hence, find the volume of the cone. *[4 marks]*

# Markscheme

valid attempt to find the height of the cone ***(M1)***

e.g. $2^2 + h^2 = (\text{their} l)^2$

$h = \sqrt{12}\left(= 2\sqrt{3}\right)$ ***(A1)***

attempt to use $V = \frac{1}{3}\pi r^2 h$ with their values substituted ***M1***

$\left(\frac{1}{3}\pi\left(2^2\right)\left(\sqrt{12}\right)\right)$

$\text{volume} = \frac{4\pi\sqrt{12}}{3}\left(= \frac{8\pi\sqrt{3}}{3} = \frac{8\pi}{\sqrt{3}}\right)$ ***A1***

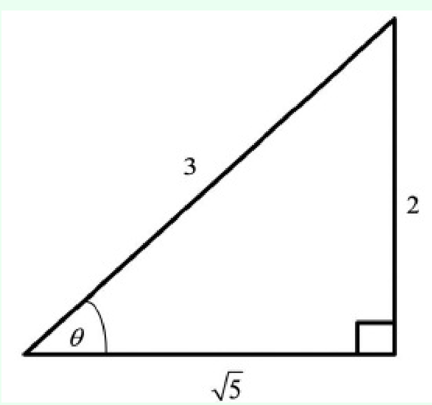
***[4 marks]***

7. It is given that $\operatorname{cosec}\theta = \frac{3}{2}$, where $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$. Find the exact value of $\cot\theta$. *[4 marks]*

# Markscheme

**METHOD 1**

attempt to use a right angled triangle ***M1***




correct placement of all three values and $\theta$ seen in the triangle ***(A1)***

$\cot\theta < 0$ (since $\operatorname{cosec}\theta > 0$ puts $\theta$ in the second quadrant) ***R1***

$\cot\theta = -\frac{\sqrt{5}}{2}$ ***A1***

**Note:** Award ***M1A1R0A0*** for $\cot\theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

**METHOD 2**

Attempt to use $1+\cot^2\theta=\operatorname{cosec}^2\theta$ ***M1***

$1+\cot^2\theta=\frac{9}{4}$

$\cot^2\theta=\frac{5}{4}$ ***(A1)***

$\cot\theta=\pm\frac{\sqrt{5}}{2}$

$\cot\theta<0$ (since $\operatorname{cosec}\theta>0$ puts $\theta$ in the second quadrant) ***R1***

$\cot\theta=-\frac{\sqrt{5}}{2}$ ***A1***

**Note:** Award ***M1A1R0A0*** for $\cot\theta=\frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

**METHOD 3**

$\sin\theta=\frac{2}{3}$

attempt to use $\sin^2\theta+\cos^2\theta=1$ ***M1***

$\frac{4}{9}+\cos^2\theta=1$

$\cos^2\theta=\frac{5}{9}$ ***(A1)***

$\cos\theta=\pm\frac{\sqrt{5}}{3}$

$\cos\theta<0$ (since $\operatorname{cosec}\theta>0$ puts $\theta$ in the second quadrant) ***R1***

$\cos\theta=-\frac{\sqrt{5}}{3}$

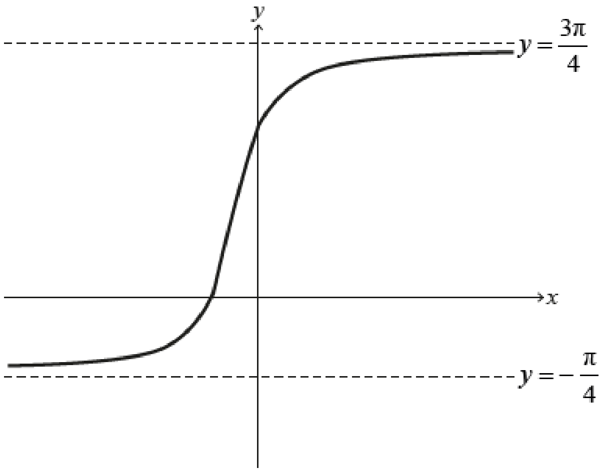
$\cot\theta=-\frac{\sqrt{5}}{2}$ ***A1***

**Note:** Award ***M1A1R0A0*** for $\cot\theta=\frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

***[4 marks]***

The following diagram shows the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$, with asymptotes at $y = -\frac{\pi}{4}$ and $y = \frac{3\pi}{4}$.




8a. Describe a sequence of transformations that transforms the graph of $y = \arctan x$ to the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$. *[3 marks]*

# Markscheme

**EITHER**

horizontal stretch/scaling with scale factor $\frac{1}{2}$

**Note:** Do not allow 'shrink' or 'compression'

followed by a horizontal translation/shift $\frac{1}{2}$ units to the left ***A2***

**Note:** Do not allow 'move'

***OR***

horizontal translation/shift $1$ unit to the left

followed by horizontal stretch/scaling with scale factor $\frac{1}{2}$ ***A2***

**THEN**

vertical translation/shift up by $\frac{\pi}{4}$ (or translation through $\begin{pmatrix} 0 \\ \frac{\pi}{4} \end{pmatrix}$ ***A1***

(may be seen anywhere)

***[3 marks]***

8b. Show that $\arctan p + \arctan q \equiv \arctan\left(\frac{p+q}{1-pq}\right)$ where $p, q > 0$ and $pq < 1$. *[4 marks]*

# Markscheme

let $\alpha = \arctan p$ and $\beta = \arctan q$ ***M1***

$p = \tan \alpha$ and $q = \tan \beta$ ***(A1)***

$\tan(\alpha + \beta) = \frac{p+q}{1-pq}$ ***A1***

$\alpha + \beta = \arctan\left(\frac{p+q}{1-pq}\right)$ ***A1***

so $\arctan p + \arctan q \equiv \arctan\left(\frac{p+q}{1-pq}\right)$ where $p, q > 0$ and $pq < 1$. ***AG***

***[4 marks]***

8c. Verify that $\arctan(2x+1) = \arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}$ for $x \in \mathbb{R}, x > 0$. *[3 marks]*

# Markscheme

**METHOD 1**

$\frac{\pi}{4} = \arctan 1$ (or equivalent) ***A1***

$\arctan\left(\frac{x}{x+1}\right) + \arctan 1 = \arctan\left(\frac{\frac{x}{x+1}+1}{1-\frac{x}{x+1}(1)}\right)$ ***A1***

$= \arctan\left(\frac{\frac{x+x+1}{x+1}}{\frac{x+1-x}{x+1}}\right)$ ***A1***

$= \arctan(2x+1)$ ***AG***

**METHOD 2**

$\tan\frac{\pi}{4} = 1$ (or equivalent) ***A1***

Consider $\arctan(2x+1) - \arctan\left(\frac{x}{x+1}\right) = \frac{\pi}{4}$

$\tan\left(\arctan(2x+1) - \arctan\left(\frac{x}{x+1}\right)\right)$

$= \arctan\left(\frac{2x+1-\frac{x}{x+1}}{1+\frac{x(2x+1)}{x+1}}\right)$ ***A1***

$= \arctan\left(\frac{(2x+1)(x+1)-x}{x+1+x(2x+1)}\right)$ ***A1***

$= \arctan 1$ ***AG***

**METHOD 3**

$\tan(\arctan(2x+1)) = \tan\left(\arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}\right)$

$\tan\frac{\pi}{4} = 1$ (or equivalent) ***A1***

$\text{LHS} = 2x+1$ ***A1***

$\text{RHS} = \frac{\frac{x}{x+1}+1}{1-\frac{x}{x+1}} (= 2x+1)$ ***A1***

***[3 marks]***

8d. Using mathematical induction and the result from part (b), prove that *[9 marks]*

$\sum_{r=1}^{n} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$.

# Markscheme

let $\mathrm{P}(n)$ be the proposition that $\sum_{r=1}^{n} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$

consider $\mathrm{P}(1)$

when $n = 1$, $\sum_{r=1}^{1} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{1}{2}\right) = \mathrm{RHS}$ and so $\mathrm{P}(1)$ is true ***R1***

assume $\mathrm{P}(k)$ is true, ie. $\sum_{r=1}^{k} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{k}{k+1}\right) (k \in \mathbb{Z}^+)$ ***M1***

**Note:** Award ***M0*** for statements such as "let $n = k$".
**Note:** Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

consider $\mathrm{P}(k+1)$:

$$\sum_{r=1}^{k+1} \arctan\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{k} \arctan\left(\frac{1}{2r^2}\right) + \arctan\left(\frac{1}{2(k+1)^2}\right)$$ ***(M1)***

$$= \arctan\left(\frac{k}{k+1}\right) + \arctan\left(\frac{1}{2(k+1)^2}\right)$$ ***A1***

$$= \arctan\left(\frac{\frac{k}{k+1} + \frac{1}{2(k+1)^2}}{1 - \left(\frac{k}{k+1}\right)\left(\frac{1}{2(k+1)^2}\right)}\right)$$ ***M1***

$$= \arctan\left(\frac{(k+1)(2k^2+2k+1)}{2(k+1)^3 - k}\right)$$ ***A1***

**Note:** Award ***A1*** for correct numerator, with $(\mathrm{k}+1)$ factored. Denominator does not need to be simplified

$$= \arctan\left(\frac{(k+1)(2k^2+2k+1)}{2k^3+6k^2+5k+2}\right)$$ ***A1***

**Note:** Award ***A1*** for denominator correctly expanded. Numerator does not need to be simplified. These two ***A*** marks may be awarded in any order

$$= \arctan\left(\frac{(k+1)(2k^2+2k+1)}{(k+2)(2k^2+2k+1)}\right) = \arctan\left(\frac{k+1}{k+2}\right)$$ ***A1***

**Note:** The word 'arctan' must be present to be able to award the last three A marks

$\mathrm{P}(k+1)$ is true whenever $\mathrm{P}(k)$ is true and $\mathrm{P}(1)$ is true, so

$\mathrm{P}(n)$ is true for for $n \in \mathbb{Z}^+$ ***R1***

**Note:** Award the final ***R1*** mark provided at least four of the previous marks have been awarded.
**Note:** To award the final ***R1***, the truth of $\mathrm{P}(k)$ must be mentioned. '$\mathrm{P}(k)$ implies $\mathrm{P}(k+1)$' is insufficient to award the mark.

***[9 marks]***

9. Solve the equation $2\cos^2 x + 5\sin x = 4, 0 \le x \le 2\pi$. *[7 marks]*

# Markscheme

attempt to use $\cos^2 x = 1 - \sin^2 x$ ***M1***

$2\sin^2 x - 5\sin x + 2 = 0$ ***A1***

**EITHER**

attempting to factorise ***M1***

$(2\sin x - 1)(\sin x - 2)$ ***A1***

**OR**

attempting to use the quadratic formula ***M1***

$\sin x = \frac{5 \pm \sqrt{5^2 - 4\times2\times2}}{4} \left(= \frac{5\pm3}{4}\right)$ ***A1***

**THEN**

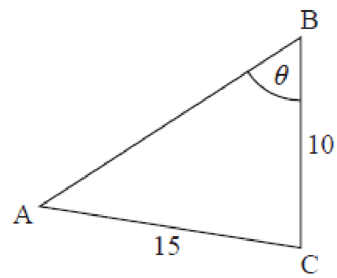
$\sin x = \frac{1}{2}$ ***(A1)***

$x = \frac{\pi}{6}, \ \frac{5\pi}{6}$ ***A1A1***

***[7 marks]***

The following diagram shows a triangle $\mathrm{ABC}$.

diagram not to scale




$\mathrm{AC} = 15\ \mathrm{cm}, \mathrm{BC} = 10\ \mathrm{cm}$, and $\mathrm{A\hat{B}C} = \theta$.

Let $\sin \mathrm{C\hat{A}B} = \frac{\sqrt{3}}{3}$.

10a. Given that $\mathrm{A\hat{B}C}$ is acute, find $\sin \theta$. *[3 marks]*

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1 – (sine rule)**

evidence of choosing sine rule ***(M1)***

*eg* $\frac{\sin\hat{A}}{a} = \frac{\sin\hat{B}}{b}$

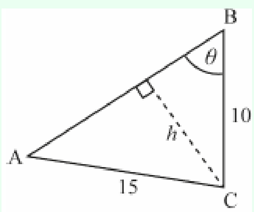
correct substitution ***(A1)***

*eg* $\frac{\sqrt{3}/3}{10} = \frac{\sin\theta}{15}$ , $\frac{\sqrt{3}}{30} = \frac{\sin\theta}{15}$ , $\frac{\sqrt{3}}{30} = \frac{\sin\mathrm{B}}{15}$

$\sin\,\theta = \frac{\sqrt{3}}{2}$ ***A1 N2***

**METHOD 2 – (perpendicular from vertex $\mathrm{C}$)**

valid approach to find perpendicular length (may be seen on diagram) ***(M1)***

*eg* 

, $\frac{h}{15} = \frac{\sqrt{3}}{3}$

correct perpendicular length ***(A1)***

*eg* $\frac{15\sqrt{3}}{3}$ , $5\sqrt{3}$

$\sin\,\theta = \frac{\sqrt{3}}{2}$ ***A1 N2***

**Note:** Do not award the final ***A*** mark if candidate goes on to state $\sin\,\theta = \frac{\pi}{3}$, as this demonstrates a lack of understanding.

***[3 marks]***

10b. Find $\cos\left(2 \times \mathrm{C\hat{A}B}\right)$. *[3 marks]*

# Markscheme

attempt to substitute into double-angle formula for cosine ***(M1)***

$1-2\left(\frac{\sqrt{3}}{3}\right)^2,\ 2\left(\frac{\sqrt{6}}{3}\right)^2-1,\ \left(\frac{\sqrt{6}}{3}\right)^2-\left(\frac{\sqrt{3}}{3}\right)^2,\ \cos(2\theta)=1-2\left(\frac{\sqrt{3}}{2}\right)^2,\ 1-2$ si

correct working ***(A1)***

*eg* $1-2\times\frac{3}{9},\ 2\times\frac{6}{9}-1,\ \frac{6}{9}-\frac{3}{9}$

$\cos\left(2\times \mathrm{C\hat{A}B}\right)=\frac{3}{9}\ \ \left(=\frac{1}{3}\right)$ ***A1 N2***

***[3 marks]***

11. Let $f(x)=4\cos\left(\frac{x}{2}\right)+1$, for $0\leqslant x\leqslant 6\pi$. Find the values of $x$ for which $f(x)>2\sqrt{2}+1$. *[8 marks]*

# Markscheme

**METHOD 1 – FINDING INTERVALS FOR *x***

$4\cos\left(\frac{x}{2}\right)+1>2\sqrt{2}+1$

correct working ***(A1)***

*eg* $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ ***(A1)***

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) ***(A1)***

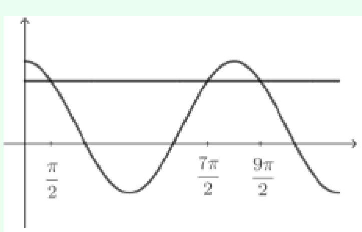
*eg* $-\frac{\pi}{4},\ \frac{7\pi}{4},\ 315^\circ,\ \frac{9\pi}{4},\ -45^\circ,\ \frac{15\pi}{4}$

three correct values for $x$ ***A1A1***

*eg* $\frac{\pi}{2},\ \frac{7\pi}{2},\ \frac{9\pi}{2}$

valid approach to find intervals ***(M1)***

*eg*




correct intervals (must be in radians) ***A1A1 N2***

$0 \leqslant x < \frac{\pi}{2},\ \frac{7\pi}{2} < x < \frac{9\pi}{2}$

**Note:** If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***.
If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***.
Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

**METHOD 2 – FINDING INTERVALS FOR $\frac{x}{2}$**

$4\cos\left(\frac{x}{2}\right)+1 > 2\sqrt{2}+1$

correct working ***(A1)***

*eg* $4\cos\left(\frac{x}{2}\right)= 2\sqrt{2},\ \cos\left(\frac{x}{2}\right)> \frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ ***(A1)***

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) ***(A1)***

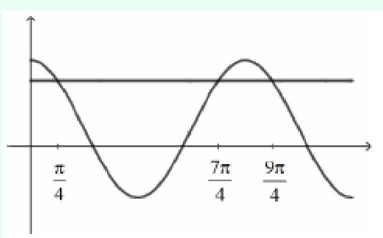
*eg* $-\frac{\pi}{4},\ \frac{7\pi}{4},\ 315^{\circ},\ \frac{9\pi}{4},\ -45^{\circ},\ \frac{15\pi}{4}$

three correct values for $\frac{x}{2}$ ***A1***

*eg* $\frac{\pi}{4},\ \frac{7\pi}{4},\ \frac{9\pi}{4}$

valid approach to find intervals ***(M1)***

*eg*




one correct interval for $\frac{x}{2}$ ***A1***

*eg* $0 \leqslant \frac{x}{2} < \frac{\pi}{4},\ \frac{7\pi}{4} < \frac{x}{2} < \frac{9\pi}{4}$

correct intervals (must be in radians) ***A1A1*** ***N2***

$0 \leqslant x < \frac{\pi}{2},\ \frac{7\pi}{2} < x < \frac{9\pi}{2}$

**Note:** If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***.
If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***.
Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

***[8 marks]***

12. $A$ and $B$ are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$. *[7 marks]*

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

# Markscheme

attempt to use $\cos(2A + B) = \cos 2A \cos B - \sin 2A \sin B$ (may be seen later) ***M1***

attempt to use any double angle formulae (seen anywhere) ***M1***

attempt to find either $\sin A$ or $\cos B$ (seen anywhere) ***M1***

$\cos A = \frac{2}{3} \Rightarrow \sin A \left(= \sqrt{1 - \frac{4}{9}}\right) = \frac{\sqrt{5}}{3}$ ***(A1)***

$\sin B = \frac{1}{3} \Rightarrow \cos B \left(= \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}\right) = \frac{2\sqrt{2}}{3}$ ***A1***

$\cos 2A \left(= 2\cos^2 A - 1\right) = -\frac{1}{9}$ ***A1***

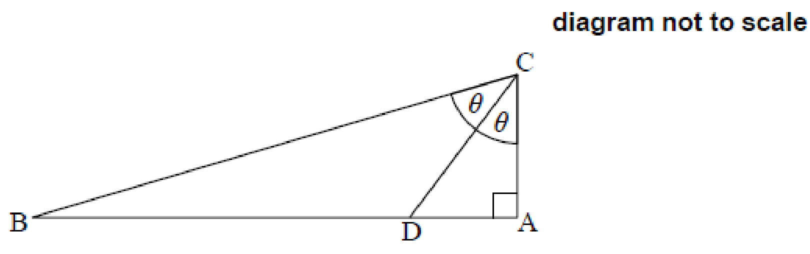
$\sin 2A \left(= 2 \sin A \cos A\right) = \frac{4\sqrt{5}}{9}$ ***A1***

So $\cos(2A + B) = \left(-\frac{1}{9}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{4\sqrt{5}}{9}\right)\left(\frac{1}{3}\right)$

$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$ ***AG***

***[7 marks]***

The following diagram shows a right triangle ABC. Point D lies on AB such that CD bisects AĈB.




AĈD = $\theta$ and AC = 14 cm

13a. Given that $\sin \theta = \frac{3}{5}$, find the value of $\cos \theta$. *[3 marks]*

## Markscheme

valid approach *(M1)*

*eg* labelled sides on separate triangle, $\sin^2 x + \cos^2 x = 1$

correct working *(A1)*

*eg* missing side is 4, $\sqrt{1 - \left(\frac{3}{5}\right)^2}$

$\cos\theta = \frac{4}{5}$ ***A1 N3***

***[3 marks]***

13b. Find the value of $\cos 2\theta$. *[3 marks]*

## Markscheme

correct substitution into $\cos 2\theta$ *(A1)*

*eg* $2\left(\frac{16}{25}\right) - 1,\ 1 - 2\left(\frac{3}{5}\right)^2,\ \frac{16}{25} - \frac{9}{25}$

$\cos 2\theta = \frac{7}{25}$ ***A1 N2***

***[2 marks]***

13c. Hence or otherwise, find $\mathrm{BC}$. *[2 marks]*

## Markscheme

correct working *(A1)*

*eg* $\frac{7}{25} = \frac{14}{\mathrm{BC}},\ \mathrm{BC} = \frac{14 \times 25}{7}$

$\mathrm{BC} = 50$ (cm) ***A1 N2***

***[2 marks]***

Let $\theta$ be an **obtuse** angle such that $\sin\theta = \frac{3}{5}$.

14a. Find the value of $\tan\theta$. *[4 marks]*

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of valid approach ***(M1)***

*eg* sketch of triangle with sides 3 and 5, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

*eg* missing side is 4 (may be seen in sketch), $\cos\theta = \frac{4}{5},\ \cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$ ***A2 N4***

***[4 marks]***

14b. Line $L$ passes through the origin and has a gradient of $\tan\theta$. Find the equation of $L$. *[2 marks]*

## Markscheme

correct substitution of either gradient **or** origin into equation of line ***(A1)***

(do not accept $y = mx + b$)

*eg* $y = x\tan\theta,\ \ y - 0 = m(x - 0),\ \ y = mx$

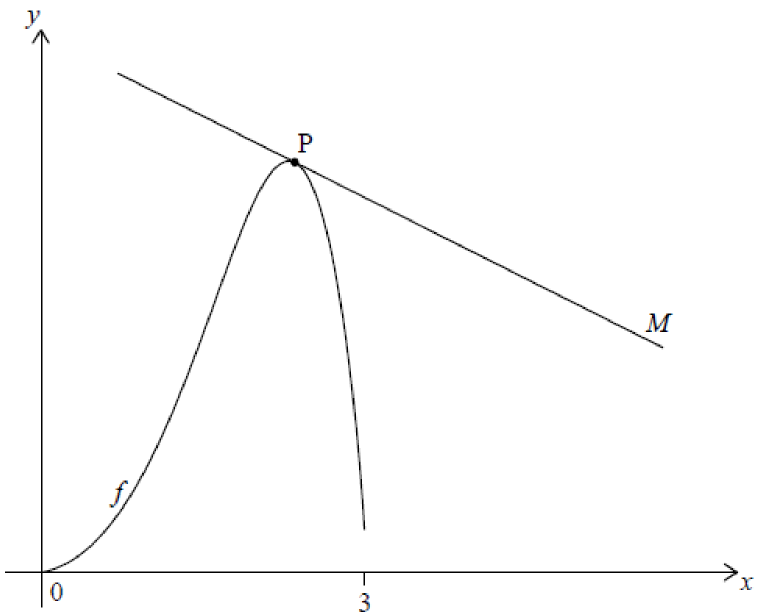
$y = -\frac{3}{4}x$ ***A2 N4***

**Note:** Award ***A1A0*** for $L = -\frac{3}{4}x$.

***[2 marks]***

Let $f(x) = \mathrm{e}^x \sin x - \frac{3x}{4}$.

14c. The following diagram shows the graph of $f$ for $0 \leq x \leq 3$. Line $M$ is a tangent to the graph of $f$ at point P. *[4 marks]*




Given that $M$ is parallel to $L$, find the $x$-coordinate of P.

# Markscheme

valid approach to equate **their** gradients ***(M1)***

*eg* $f' = \tan\theta$, $f' = -\frac{3}{4}$, $\mathrm{e}^x \cos x + \mathrm{e}^x \sin x - \frac{3}{4} = -\frac{3}{4}$, $\mathrm{e}^x\left(\cos x + \sin x\right) - \frac{3}{4} = -\frac{3}{4}$

correct equation without $\mathrm{e}^x$ ***(A1)***

*eg* $\sin x = -\cos x$, $\cos x + \sin x = 0$, $\frac{-\sin x}{\cos x} = 1$

correct working ***(A1)***

*eg* $\tan\theta = -1$, $x = 135^\circ$

$x = \frac{3\pi}{4}$ (do not accept $135^\circ$) ***A1 N1***

**Note:** Do not award the final ***A1*** if additional answers are given.

***[4 marks]***

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let $\theta$ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm$^2$.

15a. Show that $\sin\theta = \frac{\sqrt{15}}{4}$. *[1 mark]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**EITHER**

$\frac{5\sqrt{15}}{2} = \frac{1}{2} \times 4 \times 5\sin\theta$ ***A1***

**OR**

height of triangle is $\frac{5\sqrt{15}}{4}$ if using 4 as the base or $\sqrt{15}$ if using 5 as the base ***A1***

**THEN**

$\sin\theta = \frac{\sqrt{15}}{4}$ ***AG***

***[1 mark]***

15b. Find the two possible values for the length of the third side. *[6 marks]*

# Markscheme

let the third side be $x$

$x^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\theta$ ***M1***

valid attempt to find $\cos\theta$ ***(M1)***

**Note:** Do not accept writing $\cos\left(\arcsin\left(\frac{\sqrt{15}}{4}\right)\right)$ as a valid method.

$\cos\theta = \pm\sqrt{1 - \frac{15}{16}}$

$= \frac{1}{4},\ -\frac{1}{4}$ ***A1A1***

$x^2 = 16 + 25 - 2 \times 4 \times 5 \times \pm\frac{1}{4}$

$x = \sqrt{31}$ or $\sqrt{51}$ ***A1A1***

***[6 marks]***

16a. Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. *[2 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$ ***M1A1***

**Note:** Do not award the ***M1*** for just $\sin^2 x + \cos^2 x$.

**Note:** Do not award ***A1*** if correct expression is followed by incorrect working.

$= 1 + \sin 2x$ ***AG***

***[2 marks]***

16b. Show that $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$. *[4 marks]*

# Markscheme

$\sec 2x + \tan 2x = \frac{1}{\cos 2x} + \frac{\sin 2x}{\cos 2x}$ ***M1***

**Note:** ***M1*** is for an attempt to change both terms into sine and cosine forms (with the same argument) or both terms into functions of $\tan x$.

$= \frac{1 + \sin 2x}{\cos 2x}$

$= \frac{(\sin x + \cos x)^2}{\cos^2 x - \sin^2 x}$ ***A1A1***

**Note:** Award ***A1*** for numerator, ***A1*** for denominator.

$= \frac{(\sin x + \cos x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ ***M1***

$= \frac{\cos x + \sin x}{\cos x - \sin x}$ ***AG***

**Note:** Apply MS in reverse if candidates have worked from RHS to LHS.

**Note:** Alternative method using $\tan 2x$ and $\sec 2x$ in terms of $\tan x$.

***[4 marks]***

16c. Hence or otherwise find $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x)\,dx$ in the form $\ln\left(a + \sqrt{b}\right)$ where $a, b \in \mathbb{Z}$. *[9 marks]*

# Markscheme

**METHOD 1**

$\int_0^{\frac{\pi}{6}} \left(\frac{\cos x+\sin x}{\cos x-\sin x}\right) \mathrm{d}x$ ***A1***

**Note:** Award ***A1*** for correct expression with or without limits.

**EITHER**

$= \left[-\ln\left(\cos x-\sin x\right)\right]_0^{\frac{\pi}{6}}$ or $\left[\ln\left(\cos x-\sin x\right)\right]_{\frac{\pi}{6}}^0$ ***(M1)A1A1***

**Note:** Award ***M1*** for integration by inspection or substitution, ***A1*** for $\ln\left(\cos x-\sin x\right)$, ***A1*** for completely correct expression including limits.

$= -\ln\left(\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\right)+\ln\left(\cos 0-\sin 0\right)$ ***M1***

**Note:** Award ***M1*** for substitution of limits into their integral and subtraction.

$= -\ln\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$ ***(A1)***

**OR**

let $u=\cos x-\sin x$ ***M1***

$\frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x-\cos x=-\left(\sin x+\cos x\right)$

$-\int_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}}\left(\frac{1}{u}\right)\mathrm{d}u$ ***A1A1***

**Note:** Award ***A1*** for correct limits even if seen later, ***A1*** for integral.

$= \left[-\ln u\right]_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}}$ or $\left[\ln u\right]_{\frac{\sqrt{3}}{2}-\frac{1}{2}}^1$ ***A1***

$= -\ln\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(+\ln 1\right)$ ***M1***

**THEN**

$= \ln\left(\frac{2}{\sqrt{3}-1}\right)$

**Note:** Award ***M1*** for both putting the expression over a common denominator and for correct use of law of logarithms.

$= \ln\left(1+\sqrt{3}\right)$ ***(M1)A1***

**METHOD 2**

$\left[\frac{1}{2}\ln\left(\tan 2x+\sec 2x\right)-\frac{1}{2}\ln\left(\cos 2x\right)\right]_0^{\frac{\pi}{6}}$ ***A1A1***

$= \frac{1}{2}\ln\left(\sqrt{3}+2\right)-\frac{1}{2}\ln\left(\frac{1}{2}\right)-0$ ***A1A1(A1)***

$= \frac{1}{2}\ln\left(4+2\sqrt{3}\right)$ ***M1***

$= \frac{1}{2}\ln\left(\left(1+\sqrt{3}\right)^2\right)$ ***M1A1***

$= \ln\left(1+\sqrt{3}\right)$ ***A1***

***[9 marks]***

17a. Use integration by parts to show that *[5 marks]*
$\int e^x \cos 2x dx = \frac{2e^x}{5}\sin 2x + \frac{e^x}{5}\cos 2x + c,\ c \in \mathbb{R}$.

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1**

attempt at integration by parts with $u = e^x,\ \frac{dv}{dx} = \cos 2x$ ***M1***

$\int e^x \cos 2x\, dx = \frac{e^x}{2}\sin 2x\, dx - \int \frac{e^x}{2}\sin 2x\, dx$ ***A1***

$= \frac{e^x}{2}\sin 2x - \frac{1}{2}\left(-\frac{e^x}{2}\cos 2x + \int \frac{e^x}{2}\cos 2x\right)$ ***M1A1***

$= \frac{e^x}{2}\sin 2x + \frac{e^x}{4}\cos 2x - \frac{1}{4}\int e^x \cos 2x\, dx$

$\therefore \frac{5}{4}\int e^x \cos 2x\, dx = \frac{e^x}{2}\sin 2x + \frac{e^x}{4}\cos 2x$ ***M1***

$\int e^x \cos 2x\, dx = \frac{2e^x}{5}\sin 2x + \frac{e^x}{5}\cos 2x\ (+c)$ ***AG***

**METHOD 2**

attempt at integration by parts with $u = \cos 2x,\ \frac{dv}{dx} = e^x$ ***M1***

$\int e^x \cos 2x\, dx = e^x \cos 2x + 2\int e^x \sin 2x\, dx$ ***A1***

$= e^x \cos 2x + 2\left(e^x \sin 2x - 2\int e^x \cos 2x\, dx\right)$ ***M1A1***

$= e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x\, dx$

$\therefore 5\int e^x \cos 2x\, dx = e^x \cos 2x + 2e^x \sin 2x$ ***M1***

$\int e^x \cos 2x \, dx = \frac{2e^x}{5} \sin 2x + \frac{e^x}{5} \cos 2x \, (+c)$ ***AG***

**METHOD 3**

attempt at use of table ***M1***

*eg*

| | |
|---|---|
| $\cos 2x$ | $e^x$ |
| $-2\sin 2x$ | $e^x$ |
| $-4\cos 2x$ | $e^x$ |

***A1A1***

**Note:** ***A1*** for first 2 lines correct, ***A1*** for third line correct.

$\int e^x \cos 2x \, dx = e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x \, dx$ ***M1***

$\therefore 5\int e^x \cos 2x \, dx = e^x \cos 2x + 2e^x \sin 2x$ ***M1***

$\int e^x \cos 2x \, dx = \frac{2e^x}{5} \sin 2x + \frac{e^x}{5} \cos 2x \, (+c)$ ***AG***

***[5 marks]***

17b. Hence, show that $\int e^x \cos^2 x dx = \frac{e^x}{5} \sin 2x + \frac{e^x}{10} \cos 2x + \frac{e^x}{2} + c$, $c \in \mathbb{R}$. *[3 marks]*

# Markscheme

$\int e^x \cos^2 x dx = \int \frac{e^x}{2}(\cos 2x + 1) \, dx$ ***M1A1***

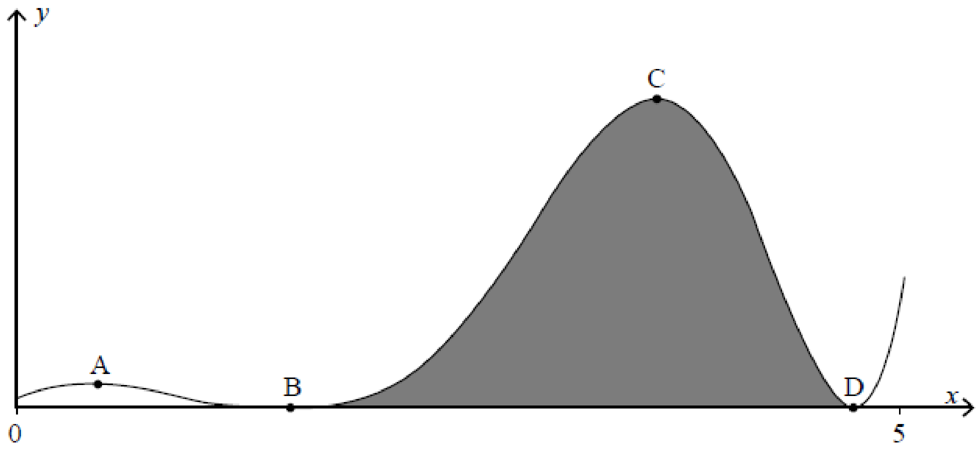
$= \frac{1}{2}\left(\frac{2e^x}{5} \sin 2x + \frac{e^x}{5} \cos 2x\right) + \frac{e^x}{2}$ ***A1***

$= \frac{e^x}{5} \sin 2x + \frac{e^x}{10} \cos 2x + \frac{e^x}{2} (+c)$ ***AG***

**Note:** Do not accept solutions where the RHS is differentiated.

***[3 marks]***

The function $f$ is defined by $f(x) = \mathrm{e}^x \cos^2 x$, where $0 \leq x \leq 5$. The curve $y = f(x)$ is shown on the following graph which has local maximum points at A and C and touches the $x$-axis at B and D.




17c. Find the $x$-coordinates of A and of C , giving your answers in the form $a + \arctan b$, where $a, b \in \mathbb{R}$. *[6 marks]*

# Markscheme

$f'(x) = \mathrm{e}^x \cos^2 x - 2\mathrm{e}^x \sin x \cos x$ ***M1A1***

**Note:** Award ***M1*** for an attempt at both the product rule and the chain rule.

$\mathrm{e}^x \cos x (\cos x - 2\sin x) = 0$ ***(M1)***

**Note:** Award ***M1*** for an attempt to factorise $\cos x$ or divide by $\cos x$ $(\cos x \neq 0)$.

discount $\cos x = 0$ (as this would also be a zero of the function)

$\Rightarrow \cos x - 2\sin x = 0$

$\Rightarrow \tan x = \frac{1}{2}$ ***(M1)***

$\Rightarrow x = \arctan\left(\frac{1}{2}\right)$ (at A) and $x = \pi + \arctan\left(\frac{1}{2}\right)$ (at C) ***A1A1***

**Note:** Award ***A1*** for each correct answer. If extra values are seen award ***A1A0***.

***[6 marks]***

17d. Find the area enclosed by the curve and the $x$-axis between B and D, as shaded on the diagram. *[5 marks]*

## Markscheme

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ ***A1***

**Note:** The ***A1*** may be awarded for work seen in part (c).

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(\mathrm{e}^x \cos^2 x\right)\,\mathrm{d}x = \left[\frac{\mathrm{e}^x}{5}\sin 2x + \frac{\mathrm{e}^x}{10}\cos 2x + \frac{\mathrm{e}^x}{2}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$ ***M1***

$= \left(-\frac{\mathrm{e}^{\frac{3\pi}{2}}}{10} + \frac{\mathrm{e}^{\frac{3\pi}{2}}}{2}\right) - \left(-\frac{\mathrm{e}^{\frac{\pi}{2}}}{10} + \frac{\mathrm{e}^{\frac{\pi}{2}}}{2}\right) \left(= \frac{2\mathrm{e}^{\frac{3\pi}{2}}}{5} - \frac{2\mathrm{e}^{\frac{\pi}{2}}}{5}\right)$ ***M1(A1)A1***

**Note:** Award ***M1*** for substitution of the end points and subtracting, ***(A1)*** for $\sin 3\pi = \sin \pi = 0$ and $\cos 3\pi = \cos \pi = -1$ and ***A1*** for a completely correct answer.

***[5 marks]***

18a. Find the roots of $z^{24} = 1$ which satisfy the condition $0 < \arg(z) < \frac{\pi}{2}$, expressing your answers in the form $re^{\mathrm{i}\theta}$, where $r, \theta \in \mathbb{R}^+$. *[5 marks]*

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\left(r(\cos\theta + \mathrm{i}\sin\theta)\right)^{24} = 1(\cos 0 + \mathrm{i}\sin 0)$

use of De Moivre's theorem ***(M1)***

$r^{24} = 1 \Rightarrow r = 1$ ***(A1)***

$24\theta = 2\pi n \Rightarrow \theta = \frac{\pi n}{12}, (n \in \mathbb{Z})$ ***(A1)***

$0 < \arg(z) < \frac{\pi}{2} \Rightarrow n = 1, 2, 3, 4, 5$

$z = \mathrm{e}^{\frac{\pi\mathrm{i}}{12}}$ or $\mathrm{e}^{\frac{2\pi\mathrm{i}}{12}}$ or $\mathrm{e}^{\frac{3\pi\mathrm{i}}{12}}$ or $\mathrm{e}^{\frac{4\pi\mathrm{i}}{12}}$ or $\mathrm{e}^{\frac{5\pi\mathrm{i}}{12}}$ ***A2***

**Note:** Award ***A1*** if additional roots are given or if three correct roots are given with no incorrect (or additional) roots.

***[5 marks]***

Let *S* be the sum of the roots found in part (a).

18b. Show that Re *S* = Im *S*. *[4 marks]*

## Markscheme

Re $S = \cos\frac{\pi}{12} + \cos\frac{2\pi}{12} + \cos\frac{3\pi}{12} + \cos\frac{4\pi}{12} + \cos\frac{5\pi}{12}$

Im $S = \sin\frac{\pi}{12} + \sin\frac{2\pi}{12} + \sin\frac{3\pi}{12} + \sin\frac{4\pi}{12} + \sin\frac{5\pi}{12}$ ***A1***

**Note:** Award ***A1*** for both parts correct.

but $\sin\frac{5\pi}{12} = \cos\frac{\pi}{12}$, $\sin\frac{4\pi}{12} = \cos\frac{2\pi}{12}$, $\sin\frac{3\pi}{12} = \cos\frac{3\pi}{12}$, $\sin\frac{2\pi}{12} = \cos\frac{4\pi}{12}$ and $\sin\frac{\pi}{12} = \cos\frac{5\pi}{12}$ ***M1A1***

⇒ Re $S$ = Im $S$ ***AG***

**Note:** Accept a geometrical method.

***[4 marks]***

18c. By writing $\frac{\pi}{12}$ as $\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$, find the value of cos $\frac{\pi}{12}$ in the form $\frac{\sqrt{a}+\sqrt{b}}{c}$, where $a$, $b$ and $c$ are integers to be determined. *[3 marks]*

## Markscheme

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$ ***M1A1***

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ ***A1***

***[3 marks]***

18d. Hence, or otherwise, show that $S = \frac{1}{2}\left(1+\sqrt{2}\right)\left(1+\sqrt{3}\right)(1+\mathrm{i})$. *[4 marks]*

# Markscheme

$\cos \frac{5\pi}{12} = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$ ***(M1)***

**Note:** Allow alternative methods *eg* $\cos\frac{5\pi}{12} = \sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$.

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$ ***(A1)***

Re $S$ = $\cos\frac{\pi}{12} + \cos\frac{2\pi}{12} + \cos\frac{3\pi}{12} + \cos\frac{4\pi}{12} + \cos\frac{5\pi}{12}$

Re $S$ = $\frac{\sqrt{2}+\sqrt{6}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{\sqrt{6}-\sqrt{2}}{4}$ ***A1***

$= \frac{1}{2}\left(\sqrt{6} + 1 + \sqrt{2} + \sqrt{3}\right)$ ***A1***

$= \frac{1}{2}\left(1 + \sqrt{2}\right)\left(1 + \sqrt{3}\right)$

$S$ = Re($S$)(1 + i) since Re $S$ = Im $S$, ***R1***

$S$ = $\frac{1}{2}\left(1 + \sqrt{2}\right)\left(1 + \sqrt{3}\right)(1 + \mathrm{i})$ ***AG***

***[4 marks]***

19. Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$. *[5 marks]*

Find, in terms of *b*, the solutions of $\sin 2x = -a,\ 0 \leqslant x \leqslant \pi$.

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

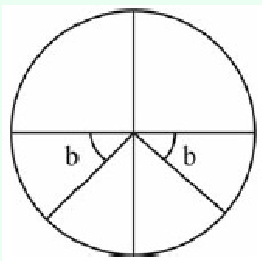
$\sin 2x = -\sin b$

**EITHER**

$\sin 2x = \sin(-b)$ or $\sin 2x = \sin(\pi + b)$ or $\sin 2x = \sin(2\pi - b)$ … ***(M1)(A1)***

**Note:** Award ***M1*** for any one of the above, ***A1*** for having final two.

**OR**




***(M1)(A1)***

**Note:** Award ***M1*** for one of the angles shown with b clearly labelled, ***A1*** for both angles shown. Do not award ***A1*** if an angle is shown in the second quadrant and subsequent ***A1*** marks not awarded.

**THEN**

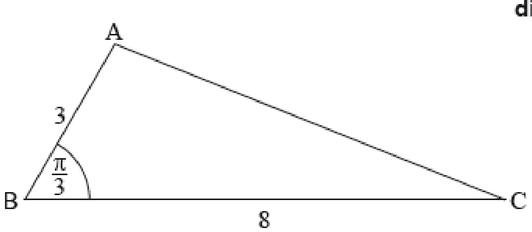
$2x = \pi + b$ or $2x = 2\pi - b$ ***(A1)(A1)***

$x = \frac{\pi}{2} + \frac{b}{2},\ x = \pi - \frac{b}{2}$ ***A1***

***[5 marks]***

The following diagram shows triangle ABC, with $\mathrm{AB} = 3\mathrm{cm}$, $\mathrm{BC} = 8\mathrm{cm}$, and $\mathrm{A\hat{B}C} = \frac{\pi}{3}$.

diagram not to scale




20a. Show that $\mathrm{AC} = 7$ cm. *[4 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of choosing the cosine rule ***(M1)***

$eg c^2 = a^2 + b^2 - ab\cos C$

correct substitution into RHS of cosine rule ***(A1)***

$eg 3^2 + 8^2 - 2 \times 3 \times 8 \times \cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

$eg \cos\frac{\pi}{3} = \frac{1}{2},\ \mathrm{AC}^2 = 9 + 64 - \left(48 \times \frac{1}{2}\right),\ 9 + 64 - 24$

correct working clearly leading to answer ***A1***

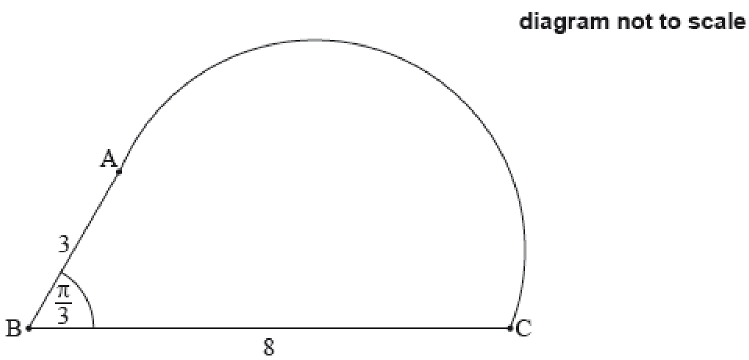
e$g \mathrm{AC}^2 = 49,\ b = \sqrt{49}$

$\mathrm{AC} = 7\ (\mathrm{cm})$ ***AG*** ***N0***

**Note:** Award no marks if the only working seen is $\mathrm{AC}^2 = 49$ or $\mathrm{AC} = \sqrt{49}$ (or similar).

***[4 marks]***

20b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. *[3 marks]*




Find the exact perimeter of this shape.

## Markscheme

correct substitution for semicircle ***(A1)***

*eg* $\text{semicircle} = \frac{1}{2}(2\pi \times 3.5),\ \frac{1}{2} \times \pi \times 7,\ 3.5\pi$

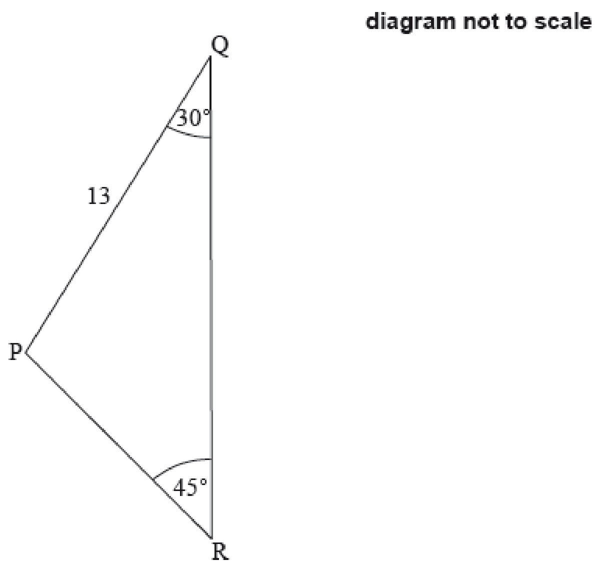
valid approach (seen anywhere) ***(M1)***

*eg*
$\text{perimeter} = \text{AB} + \text{BC} + \text{semicircle}, 3 + 8 + \left(\frac{1}{2} \times 2 \times \pi \times \frac{7}{2}\right),\ 8 + 3 + 3.5\pi$

$11 + \frac{7}{2}\pi\ (= 3.5\pi + 11)\ (\text{cm})$ ***A1*** ***N2***

***[3 marks]***

21. The following diagram shows triangle PQR. *[6 marks]*




$\text{P}\hat{\text{Q}}\text{R} = 30°$, $\text{Q}\hat{\text{R}}\text{P} = 45°$ and $\text{PQ} = 13\ \text{cm}$.

Find PR.

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1**

evidence of choosing the sine rule ***(M1)***

*eg* $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***A1***

*eg* $\frac{x}{\sin 30} = \frac{13}{\sin 45},\ \frac{13 \sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2},\ \sin 45 = \frac{1}{\sqrt{2}}$ ***(A1)(A1)***

correct working ***A1***

*eg* $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}},\ \frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}},\ 13 \times \frac{1}{2} \times \sqrt{2}$

correct answer ***A1 N3***

*eg* $\text{PR} = \frac{13\sqrt{2}}{2},\ \frac{13}{\sqrt{2}}\ (\text{cm})$

**METHOD 2 (using height of ΔPQR)**

valid approach to find height of ΔPQR ***(M1)***

*eg* $\sin 30 = \frac{x}{13},\ \cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2}$ or $\cos 60 = \frac{1}{2}$ ***(A1)***

$\text{height} = 6.5$ ***A1***

correct working ***A1***

*eg* $\sin 45 = \frac{6.5}{\text{PR}},\ \sqrt{6.5^2 + 6.5^2}$

correct working ***(A1)***

*eg* $\sin 45 = \frac{1}{\sqrt{2}},\ \cos 45 = \frac{1}{\sqrt{2}},\ \sqrt{\frac{169 \times 2}{4}}$

correct answer ***A1 N3***

*eg* $\text{PR} = \frac{13\sqrt{2}}{2},\ \frac{13}{\sqrt{2}}\ (\text{cm})$

***[6 marks]***

22. Solve the equation $\sec^2 x + 2\tan x = 0,\ 0 \leqslant x \leqslant 2\pi$. *[5 marks]*

# Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1**

use of $\sec^2 x = \tan^2 x + 1$ ***M1***

$\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)^2 = 0$ ***(M1)***

$\tan x = -1$ ***A1***

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ ***A1A1***

**METHOD 2**

$\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos x} = 0$ ***M1***

$1 + 2\sin x\cos x = 0$

$\sin 2x = -1$ ***M1A1***

$2x = \frac{3\pi}{2}, \frac{7\pi}{2}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ ***A1A1***

**Note:** Award ***A1A0*** if extra solutions given or if solutions given in degrees (or both).

***[5 marks]***