- 9. Let f and g be two functions. Given that  $f \circ g(x) = \frac{x+2}{3}$  and g(x) = 2x + 5, find f(x-1). [6 marks]
- 15. A function is called *self-inverse* if  $f(x) = f^{-1}(x)$  for all x in the domain.
  - (a) Show that  $f(x) = \frac{1}{x}$ ,  $x \ne 0$  is a self-inverse function.
  - (b) Find the value of the constant k so that  $g(x) = \frac{3x-5}{x+k}$ ,  $x \ne k$  is a self-inverse function. [8 marks]
- 5. If  $h(x) = x^2 6x + 2$ :
  - (a) Write h(x) in the form  $(x-p)^2 + q$ .
  - (b) Hence or otherwise find the range of h(x).
  - (c) By using the largest possible domain of the form x > k where, find the inverse function  $h^{-1}(x)$ . [7 marks]
- 6. The function f(x) is defined by  $f(x) = \frac{3-x}{x+1}, x \neq -1$ .
  - (a) Find the range of *f*.
  - (b) Sketch the graph of y = f(x).
  - (c) Find the inverse function of f in the form  $f^{-1}(x) = \frac{ax + b}{cx + d}$ . State its domain and range. [11 marks]
- 8. The functions f(x) and g(x) are given by  $f(x) = \sqrt{x-2}$  and  $g(x) = x^2 + x$ . The function  $f \circ g(x)$  is defined for  $x \in \mathbb{R}$  except for the interval ] a, b[.
  - (a) Calculate the value of *a* and of *b*.
  - (b) Find the range of  $f \circ g$ .

[7 marks]

- 3. The functions f and g are defined over the domain of all real numbers,  $g(x) = e^x$ .
  - (a) Write  $f(x) = x^2 + 4x + 9$   $x \in \mathbb{R}$  in the form  $f(x) = (x + p)^2 + q$ .
  - (b) Hence sketch the graph of  $y = x^2 + 4x + 9$ , labelling carefully all axes intercepts and the coordinates of the turning point.
  - (c) State the range of f(x) and g(x).
  - d Hence or otherwise find the range of  $h(x) = e^{2x} + 4e^x + 9$ . [10 marks]
- 6. A bowl of soup is served at a temperature of 55 °C in a room with a constant air temperature of 20 °C. Every 5 minutes, the temperature difference between the soup and the room air decreases by 30%. Assuming the room air temperature is constant, at what temperature will the soup be seven minutes after serving?

  [7 marks]
- 8. A cup of tea is poured at 98 °C. After two minutes it has reached 94 °C. The difference between the temperature of the tea and the room temperature (22 °C) falls exponentially. Find the time it takes for the tea to cool to 78 °C.

  [5 marks]
- Given  $\log_a b^2 = c$  and  $\log_b a = c 1$  for some value c, where 0 < a < b, express a in terms of b. [6 marks]
- Solve the equation  $9 \log_5 x = 25 \log_x 5$ , expressing your answers in the form  $5^{\frac{p}{q}}$ , where  $p, q \in \mathbb{Z}$ . [6 marks]

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8. Find the exact solution to the equation  $\ln x = 4 \log_x e$ . [5 marks]

7. The speed (V metres per second) of a parachutist t seconds after jumping from an aeroplane is modelled by the expression:

$$V = 40(1 - 3^{-0.1t})$$

- (a) Find his initial speed.
- (b) What speed does the model predict that he will eventually reach?

[6 marks]

5. If  $a = \ln 2$  and  $b = \ln 5$ , find in terms of a and b:

[6 marks]

(a) ln 50

- (b) ln 0.16
- 6. Solve  $\log_2 x = \log_x 2$ .

[5 marks]

- Prove that if  $a^x = b^y = (ab)^{xy}$  where a, b > 1 then x + y = 1or x = y = 0. [5 marks]
- 8. Evaluate  $\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \log \frac{4}{5} + \log \frac{8}{9} + \log \frac{9}{10}$ . [4 marks]
- Given that  $\log_a b = \log_b a$ , and that  $a, b \neq 1$  and  $a \neq b$ , find *b* in terms of *a*. [5 marks]
- If  $a = \log x$ ,  $b = \log y$  and  $c = \log z$  (all logs base 10) find in terms of a, b, c and integers:
- (a)  $\log \frac{x^2 \sqrt{y}}{z}$  (b)  $\log \sqrt{0.1x}$  (c)  $\log_{100} \left(\frac{y}{z}\right)$  [6 marks]

3. Solve the simultaneous equations:

$$ln x + ln y^2 = 8$$

$$\ln x^2 + \ln y = 6$$

[6 marks]

If  $y = \ln x - \ln(x+2) + \ln(4-x^2)$ , express x in terms of y.

[6 marks]

## **EXAM-STYLE QUESTION**

**3** Let  $p = \log_a x$  and  $q = \log_a y$ . Express  $\log_x a$  in terms of pand  $\log_{\nu} a$  in terms of q. Hence, show that:

$$\mathbf{a} \quad \log_{xy} a = \frac{1}{p+q}$$

$$\mathbf{b} \quad \log_{\frac{x}{y}} a = \frac{1}{p - q}$$

## EXAM-STULE QUESTIONS

**5** Solve the equation 
$$5^{x+1} + \frac{4}{5^x} - 21 = 0$$

Solve the equation  $\log_3 x + \log_x 9 - 3 = 0$ 

Solve the equation  $3 \times 9^x - 2 \times 4^x = 5 \times 6^x$  giving your answer to three significant figures.

**8** Solve these simultaneous equations.

$$6\log_2 x + 6\log_8 y = 7$$

$$4\log_4 x + 4\log_2 y = 9$$

**9** Solve these simultaneous equations.

$$2\log_{x} y = 1$$

$$xy = 125$$

**10** Solve these simultaneous equations.

$$y\log_2 8 = x$$

$$y\log_2 8 = x$$
  $2^x + 8^y = 64$ 

**10** Find the value of x which satisfies the equation  $e^x - e^{-x} = 4$ Hence, show that for this value of x

$$e^x + e^{-x} = 2\sqrt{5}$$