

9. Let f and g be two functions. Given that $f \circ g(x) = \frac{x+2}{3}$ and $g(x) = 2x + 5$, find $f(x-1)$. [6 marks]

15. A function is called *self-inverse* if $f(x) = f^{-1}(x)$ for all x in the domain.

(a) Show that $f(x) = \frac{1}{x}$, $x \neq 0$ is a self-inverse function.

(b) Find the value of the constant k so that

$$g(x) = \frac{3x-5}{x+k}, x \neq k \text{ is a self-inverse function. [8 marks]}$$

5. If $h(x) = x^2 - 6x + 2$:

(a) Write $h(x)$ in the form $(x-p)^2 + q$.

(b) Hence or otherwise find the range of $h(x)$.

(c) By using the largest possible domain of the form $x > k$ where, find the inverse function $h^{-1}(x)$. [7 marks]

6. The function $f(x)$ is defined by $f(x) = \frac{3-x}{x+1}$, $x \neq -1$.

(a) Find the range of f .

(b) Sketch the graph of $y = f(x)$.

(c) Find the inverse function of f in the form $f^{-1}(x) = \frac{ax+b}{cx+d}$. State its domain and range. [11 marks]

8. The functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + x$. The function $f \circ g(x)$ is defined for $x \in \mathbb{R}$ except for the interval $] a, b[$.

(a) Calculate the value of a and of b .

(b) Find the range of $f \circ g$. [7 marks]

3. The functions f and g are defined over the domain of all real numbers, $g(x) = e^x$.
- (a) Write $f(x) = x^2 + 4x + 9$ $x \in \mathbb{R}$ in the form $f(x) = (x + p)^2 + q$.
 - (b) Hence sketch the graph of $y = x^2 + 4x + 9$, labelling carefully all axes intercepts and the coordinates of the turning point.
 - (c) State the range of $f(x)$ and $g(x)$.
 - (d) Hence or otherwise find the range of $h(x) = e^{2x} + 4e^x + 9$. [10 marks]
6. A bowl of soup is served at a temperature of 55°C in a room with a constant air temperature of 20°C . Every 5 minutes, the temperature difference between the soup and the room air decreases by 30%. Assuming the room air temperature is constant, at what temperature will the soup be seven minutes after serving? [7 marks]
8. A cup of tea is poured at 98°C . After two minutes it has reached 94°C . The difference between the temperature of the tea and the room temperature (22°C) falls exponentially. Find the time it takes for the tea to cool to 78°C . [5 marks]
6. Given $\log_a b^2 = c$ and $\log_b a = c - 1$ for some value c , where $0 < a < b$, express a in terms of b . [6 marks]
7. Solve the equation $9 \log_5 x = 25 \log_x 5$, expressing your answers in the form $\frac{p}{5^q}$, where $p, q \in \mathbb{Z}$. [6 marks]
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8. Find the exact solution to the equation $\ln x = 4 \log_x e$. [5 marks]

EXAM-STYLE QUESTION

3 Let $p = \log_a x$ and $q = \log_a y$. Express $\log_x a$ in terms of p and $\log_y a$ in terms of q . Hence, show that:

a $\log_{xy} a = \frac{1}{p+q}$

b $\log_{\frac{x}{y}} a = \frac{1}{p-q}$

EXAM-STYLE QUESTIONS

5 Solve the equation $5^{x+1} + \frac{4}{5^x} - 21 = 0$

6 Solve the equation $\log_3 x + \log_x 9 - 3 = 0$

7 Solve the equation $3 \times 9^x - 2 \times 4^x = 5 \times 6^x$ giving your answer to three significant figures.

8 Solve these simultaneous equations.

$$6\log_2 x + 6\log_8 y = 7 \qquad 4\log_4 x + 4\log_2 y = 9$$

9 Solve these simultaneous equations.

$$2\log_x y = 1 \qquad xy = 125$$

10 Solve these simultaneous equations.

$$y\log_2 8 = x \qquad 2^x + 8^y = 64$$

10 Find the value of x which satisfies the equation $e^x - e^{-x} = 4$
Hence, show that for this value of x

$$e^x + e^{-x} = 2\sqrt{5}$$