Practice exam papers

Mathematics: analysis and approaches
Higher level
Paper 1 Practice Set A

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 *[Maximum mark: 5]*

On his way to school, Suresh stops for coffee with probability 0.8. If he stops for coffee, the probability that he is late for school is 0.4; otherwise, the probability that he is late is 0.1. Given that on a particular day Suresh is late for school, what is the probability that he did not stop for coffee?

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2 *[Maximum mark: 7]*

Use the substitution $u = x - 3$ to find the exact value of $\int_3^7 5x\sqrt{x-3}\,dx$.

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3 *[Maximum mark: 6]*

z is the complex number which satisfies the equation $3z - 4z^* = 18 + 21i$. Find $\left|\frac{z}{3}\right|$.

4 *[Maximum mark: 6]*

a Show that $(2x + 1)$ is a factor of $f(x) = 2x^3 - 13x^2 + 17x + 12$. *[2]*

b Solve the inequality $2x^3 - 13x^2 + 17x + 12 > 0$. *[4]*

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5 *[Maximum mark: 6]*

Given the functions

$f(x) = \dfrac{2 - x}{x + 3}$ $(x \neq -3)$ and $g(x) = \dfrac{2}{x - 1}$ $(x \neq 1)$

find $(f \circ g)^{-1}$ in the form $\dfrac{ax + b}{cx + d}$.

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6 *[Maximum mark: 6]*

Find the possible values of x such that $45e^{x}$, $7e^{2x}$ and e^{3x} are consecutive terms of an arithmetic sequence.

7 *[Maximum mark: 6]*

Use L'Hôpital's rule to find

$$\lim_{x \to \pi} \frac{x \sin x}{\ln\left(\frac{x}{\pi}\right)}.$$

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8 *[Maximum mark: 7]*

Sketch the graph of

$$y = \frac{2x^2 + 5x - 12}{x + 3}$$

State the coordinates of all axis intercepts and the equations of all asymptotes.

9 *[Maximum mark: 6]*

a Prove that $\log_2 5$ is an irrational number. *[4]*

b Aron says that $\log_2 n$ is an irrational number for every integer $n \geqslant 10$. Give a counterexample to disprove this statement. *[2]*

Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

10 *[Maximum mark: 20]*

a Sketch the graph of $y = x^2 + 3x - 10$, showing clearly the axes intercepts and the coordinates of the vertex. *[4]*

b **i** Show that the line $y = 2x - 20$ does not intersect the graph of $y = x^2 + 3x - 10$.

ii Find the set of values of k for which the line $y = 2x - k$ intersects the graph of $y = x^2 + 3x - 10$ at two distinct points. *[7]*

c Describe fully a sequence of transformations which transforms the graph of $y = x^2 + 3x - 10$ to the graph of $y = \left(2x + \frac{3}{2}\right)^2 + 2$. *[4]*

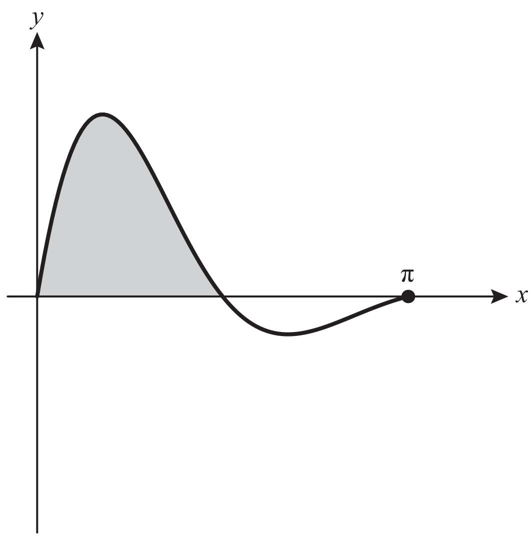
d Sketch the following graphs, indicating clearly all axes intercepts, asymptotes and turning points:

i $y = |x^2 + 3x - 10|$

ii $y = \dfrac{1}{x^2 + 3x - 10}$ *[5]*

11 *[Maximum mark: 16]*

The graph of $y = e^{-x} \sin 2x$ for $0 \leqslant x \leqslant \pi$ is shown below.



The graph has a maximum point at P, a minimum point at Q and points of inflection at R and S.

a Show that the x-coordinates of point P and point Q satisfy $\tan 2x = 2$. *[4]*

b Show that the x-coordinates of points R and S satisfy $\tan 2x = -\frac{4}{3}$. *[4]*

c Show that the area of the shaded region enclosed below the curve and above the x-axis is given by $a + be^c$, where a, b and c are constants to be found. *[8]*

12 *[Maximum mark: 19]*

a State and prove de Moivre's theorem. *[5]*

b Use de Moivre's theorem to prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. *[4]*

c Solve the equation $\cos 5\theta = 0$ for $0 \leqslant \theta \leqslant \pi$. *[2]*

d By considering the equation $16c^5 - 20c^3 + 5c = 0$, where $c = \cos\theta$, find the exact value of $\cos\left(\frac{\pi}{10}\right)$.

Justify your choice. *[6]*

e Find the exact value of $\cos\left(\frac{\pi}{10}\right)\cos\left(\frac{7\pi}{10}\right)$. *[2]*