Mathematics: analysis and approaches
Higher level
Paper 2 Practice Set A

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 *[Maximum mark: 6]*

A child makes a caterpillar out of modelling clay. The density of the clay is $1.45\,\text{g}\,\text{cm}^{-3}$. She starts by making a sphere of radius 3 cm.

a Find the mass of this sphere. *[2]*

She then adds more spheres, each with radius half the previous one.

b Show that the mass of the caterpillar can never reach 200 g. *[4]*

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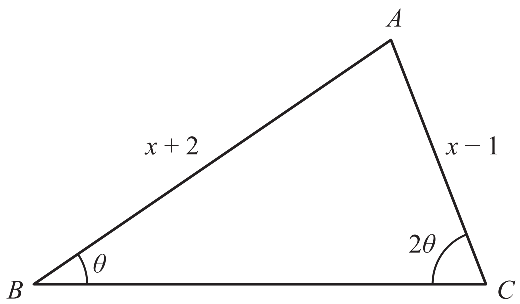
2 *[Maximum mark: 6]*

Find the standard deviation of a continuous random variable with the probability density function

$$f(x) = \begin{cases} 0.4106 \sin x \sqrt{x - 2\pi} & \text{for } 2\pi \leqslant x \leqslant 3\pi \\ 0 & \text{otherwise} \end{cases}.$$

3 *[Maximum mark: 7]*

A triangle has sides of length $(x - 1)$ and $(x + 2)$, and angles θ and 2θ, as shown in the diagram.



Show that $x = \dfrac{4}{4 - 3\sec^2\left(\frac{\theta}{2}\right)}$.

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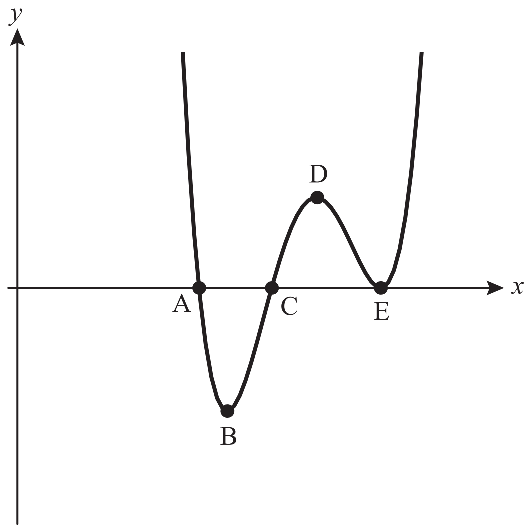
4 *[Maximum mark: 5]*

The graph of $y = f'(x)$ is shown in the diagram.



Write down the labels of the following points, justifying your choice in each case:

a local maximum point(s) of $f(x)$ *[2]*

b point(s) of inflection of $f(x)$. *[3]*

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5 *[Maximum mark: 6]*

Find the first four terms in the binomial expansion of $\frac{1}{\sqrt{4-x}}$, and state the set of values of x for which the expansion is valid.

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6 *[Maximum mark: 6]*

A particle moves in a straight line, with the velocity at time t seconds given by $v = \dfrac{\sin t}{\sqrt{t+1}}\,\text{m}\,\text{s}^{-1}$.

a Find the distance travelled by the particle in the first five seconds of motion, giving your answer to one decimal place. *[3]*

b Find the first two times when the magnitude of acceleration is $0.3\,\text{m}\,\text{s}^{-2}$. *[3]*

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7 *[Maximum mark: 6]*

Find the values of b and c such that the function

$$f(x) = \begin{cases} x^2 - 1 \text{ for } x \leqslant 1 \\ -x^2 + bx + c \text{ for } x > 1 \end{cases}$$

is both continuous and differentiable.

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8 *[Maximum mark: 5]*

Vectors **a** and **b** satisfy

$\mathbf{a} \cdot \mathbf{b} = 17$ and $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$.

Find, in degrees, the size of the angle between the two vectors.

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9 *[Maximum mark: 7]*

Ten children, including two pairs of (non-identical) twins, line up in a straight line for a photo.

a Find the number of arrangements in which each pair of twins stands together. *[3]*

b The photographer arranges the children at random. Find the probability that neither pair of twins stands together. *[4]*

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Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

10 *[Maximum mark: 21]*

The marks of Miss Rahman's class of twelve students on Mathematics Paper 1 and Paper 2 are given in the table.

Student	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Paper 1	72	105	98	106	63	58	52	87	75	72	91	68
Paper 2	72	87	91	98	68	56	61	72	73	61	97	52

a Find the mean and standard deviation of each set of marks. Hence write two comments comparing the marks on the two papers. *[4]*

b The critical value of the Pearson's correlation coefficient for 12 pieces of data is 0.532. Determine whether there is significant positive correlation between the two sets of marks. *[3]*

c Two students did not sit Paper 1.

i Student 13 scored 86 marks on Paper 2. Use an appropriate regression line to estimate what mark he would have got on Paper 1.

ii Student 14 scored 45 marks on Paper 2. Can your regression line be used to estimate her mark for Paper 1? Justify your answer. *[5]*

d It is known that, in the population of all the students in the world who took Paper 1, the marks followed the distribution $N(68, 11^2)$. It is also known that 12% of all students achieved Grade 7 in this paper.

i How many of the 12 students in Miss Rahman's class achieved Grade 7 in Paper 1?

ii Find the probability that, in a randomly selected group of 12 students, there are more Grade 7s than in Miss Rahman's class. *[6]*

e Paper 1 is marked out of 110. In order to compare the results to another paper, Miss Rahman rescales the marks so that the maximum mark is 80. Find the mean and standard deviation of the rescaled Paper 1 marks for the 12 students in the class. *[3]*

11 *[Maximum mark: 16]*

a Find the general solution of the differential equation $\frac{dy}{dx} - y \tan x = 0$, expressing y in terms of x. *[5]*

b Consider the differential equation $\frac{dy}{dx} - y \tan x = \cos x$.

i Show that the integrating factor is $\cos x$.

ii Hence find the general solution of the differential equation. *[7]*

c Consider instead the differential equation $\frac{dy}{dx} - y^2 \tan x = \cos x$ with the initial condition $y = 2$ when $x = 0$.

Use Euler's method with step length 0.1 to estimate the value of y when $x = 0.5$. *[4]*

12 *[Maximum mark: 19]*

The lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ -8 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

a **i** Show that the lines l_1 and l_2 intersect.

ii Find the coordinates of the point of intersection, P. *[7]*

b Find a vector perpendicular to l_1 and l_2. *[2]*

c Hence find the equation of the plane Π containing l_1 and l_2. Give your answer in the form $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$. *[2]*

The line l_3 passes through the point $Q(-11, 0, 1)$ and intersects Π at the point P.

d Find the exact value of the sine of the acute angle between l_3 and Π. *[6]*

e Hence find the shortest distance from the point Q to Π. *[2]*