Mathematics: analysis and approaches
Higher level
Paper 1 Practice Set B

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 *[Maximum mark: 7]*

Find the value of $a > 0$ such that $\int_0^a \frac{4x}{x^2+3}\,\mathrm{d}x = \ln 16$.

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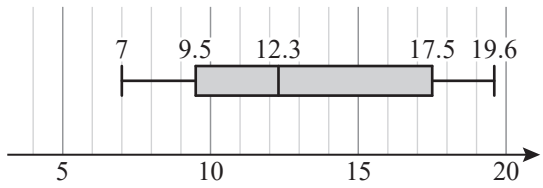
2 *[Maximum mark: 6]*

The box plot summarizes the times taken by a group of 40 children to complete an obstacle course.



Two of the 40 children are selected at random.

a Find the probability that both children completed the course in less than 9.5 minutes. *[3]*

b Find the probability that one child completed the course in less than 9.5 minutes and the other in between 9.5 and 17.5 minutes. *[3]*

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3 *[Maximum mark: 5]*

Find the equation of the normal to the graph of $y = \frac{\sin x}{x}$ at the point where $x = \pi$.

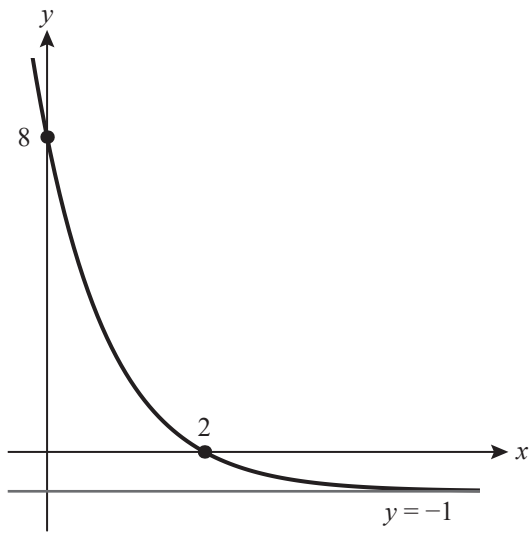
4 *[Maximum mark: 5]*
Solve the inequality $|x - 3| \leqslant |2x + 1|$.

5 *[Maximum mark: 6]*

Given that $P(A) = 0.3$, $P(B|A) = 0.6$ and $P(A \cup B) = 0.8$, find $P(A|B)$. Give your answer as a simplified fraction.

6 *[Maximum mark: 6]*

The graph in the diagram has equation $y = A + Be^{-kx}$.



Find the values of A, B and k.

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7 *[Maximum mark: 6]*

Use mathematical induction to prove that $7^n + 3^{n-1}$ is divisible by 4 for all integers $n \geqslant 1$.

8 *[Maximum mark: 7]*

Find, in the form $z = re^{i\theta}$, the roots of the equation $z^3 = 4 - (4\sqrt{3})i$.

9 *[Maximum mark: 7]*

Find the first two non-zero terms in the Maclaurin series for $\frac{\cos x}{\sqrt{1-x^2}}$.

Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

10 *[Maximum mark: 18]*

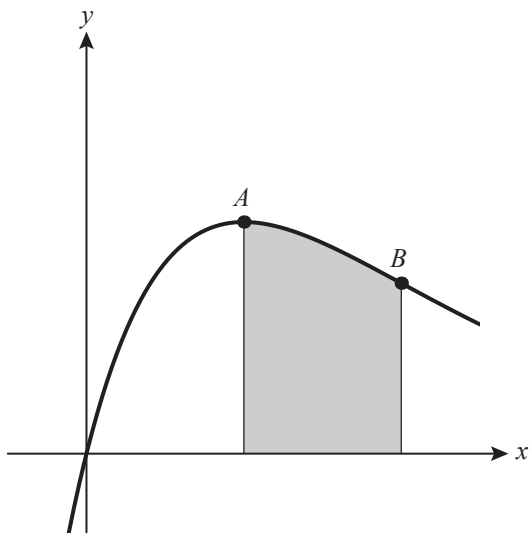
Let $f(x) = xe^{-kx}$ where $x \in \mathbb{R}$ and $k > 0$.

a Show that $f'(x) = (1 - kx)e^{-kx}$ and find $f''(x)$ in the form $(a + bx)e^{-kx}$. *[5]*

b Find the x-coordinate of the stationary point of $f(x)$ and show that it is a maximum. *[5]*

c Find the coordinates of the point of inflection of $f(x)$. *[3]*

d The graph of $y = f(x)$ is shown below. A is the maximum point and B is the point of inflection. Show that the shaded area equals $\frac{2e - 3}{k^2e^2}$. *[5]*



11 *[Maximum mark: 15]*

The following system of equations does not have a unique solution.

$$\begin{cases} 6x + ky + 2z = a \\ 6x - y - z = 7 \\ 2x - 3y + z = 1 \end{cases}$$

a Find the value of k. *[6]*

Each equation represents a plane.

b Find

i the value of a for which the three planes intersect in a line

ii the equation of the line. *[7]*

c If the value of a is such that the three planes do not intersect in a line, describe their geometric configuration, justifying your answer. *[2]*

12 *[Maximum mark: 22]*

Let $f(x) = x^2 - 2x - 3$, $x \in \mathbb{R}$.

a Sketch the graph of $y = |f(x)|$. *[3]*

b Hence or otherwise, solve the inequality $|f(x)| > -\frac{1}{2}x + 4$. *[6]*

Let $g(x) = \frac{2x - 7}{f(x)}$.

c State the largest possible domain of g. *[1]*

d Find the coordinates of the turning points of g. *[5]*

e Sketch the graph of $y = g(x)$, labelling all axis intercepts and asymptotes. *[5]*

f Hence find the range of g for the domain found in part **c**. *[2]*