Practice exam papers 135

Mathematics: analysis and approaches Higher level Paper 2 Practice Set B

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is [110 marks].

Practice exam papers

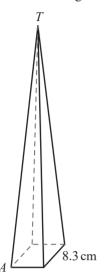
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2 [Maximum mark: 4]

A flag pole has the shape of a square-based pyramid shown in the diagram. The side length of the base is $8.3 \,\mathrm{cm}$. The edge AT makes an angle of 89.8° with the base.



Find the height of the flag pole. Give your answer in centimetres, in standard form, correct to two significant figures.

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122	124	127	131	134	134	136	147	
Determine w	hether any	of the height	ts are outlier	·s.				

4 [Maximum mark: 5]
Prove that

$$\frac{\sin\left(x + \frac{\pi}{3}\right) - \sin\left(x - \frac{\pi}{3}\right)}{\cos\left(x + \frac{\pi}{3}\right) - \cos\left(x - \frac{\pi}{3}\right)} \equiv -\cot x$$

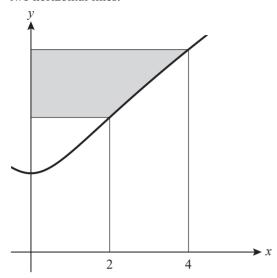
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	$\int \frac{2}{x(x-1)} \mathrm{d}x$
W	Write your answer in the form $\ln(f(x)) + c$.
•	
•	

	$3x^2 + 2xy - y^2 + 24 = 0$
7	where the tangent is parallel to the <i>x</i> -axis.
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8 [Maximum mark: 7]

The curve in the diagram has equation $y = \sqrt[3]{x^2 + 1}$. The shaded region is bounded by the curve, the *y*-axis and two horizontal lines.



a Find the area of the shaded region.

[4]

b Find the volume generated when the shaded region is rotated 2π radians about the y-axis.

[3]

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Do **not** write solutions on this page

Section B

Answer all questions in an answer booklet. Please start each question on a new page.

11 [Maximum mark: 18]

Battery life of a certain brand of smartphone can be modelled by a normal distribution with mean μ hours and standard deviation σ hours. It is known that 5% of the batteries last less than 24 hours, while 20% last more than 72 hours.

- a i Show that $\mu + 0.8416\sigma = 72$ and find another similar equation connecting μ and σ .
 - ii Show that approximately 65.7% of the batteries last longer than 48 hours. [7]
- **b** Find the interquartile range of battery life, giving your answer to the nearest hour. [2]
- c Find the probability that, out of 20 randomly selected batteries, at least 10 last more than 48 hours. [3]
- d Given that a battery has lasted for 48 hours, what is the probability that it will last for another 24 hours?
- **e** A customer buys a new smartphone and tests the battery. If the battery lasts less than 24 hours they return the phone with probability 0.9. If it lasts between 24 and 72 hours they return the phone with probability 0.2. Otherwise they do not return the phone.

Given that a customer keeps the phone, what is the probability that its battery lasts more than 72 hours? [4]

12 [Maximum mark: 18]

Two of the sides of a triangle have length x cm and 2x cm, and the angle between them is θ° .

The perimeter of the triangle is 10 cm.

- a In the case x = 2, find the area of the triangle. [4]
- **b** Explain why x must be less than $\frac{10}{3}$. [2]
- **c** i Show that $\cos \theta = \frac{15x x^2 25}{x^2}$.
 - ii Sketch the graph of $y = \frac{15x x^2 25}{x^2}$ for x > 0.
 - iii Hence find the range of possible values of x. [7]
- **d** Find the value of x for which the triangle has the largest possible area, and state the value of that area. [5]

13 [Maximum mark: 19]

Consider the differential equation $\frac{dy}{dx} = \frac{y}{x+y}$.

- a Find and simplify an expression for $\frac{d^2y}{dx^2}$ in terms of x and y. [7]
- **b** Show that the substitution y = xv transforms this equation into $x \frac{dv}{dx} = f(v)$, where f(v) is a function to be found. [4]
- c Hence find the particular solution of the equation $\frac{dy}{dx} = \frac{y}{x+y}$ for which y = 1 when x = 1. Give your answer in the form x = g(y).