Mathematics: analysis and approaches Higher level Paper 1 Practice Set C

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is [110 marks].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer all questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

- 1 [Maximum mark: 6] Let $f(x) = 2x^2 + 10x + 7, x \in \mathbb{R}$.
 - a Find the largest possible domain of the form $x \le k$ for which the inverse function, f^{-1} , exists.
 - [2] **b** For the value of k from part **a**, find the inverse function $f^{-1}(x)$, stating its domain. [4]

	tet $z = 3 - 2i$ and $w = -1 + i$. Represent z and w on an Argand diagram.
	Find $\frac{w}{z}$ in the form $a + bi$.
	Find the real numbers p and q such that $pz + qw = 6$.
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	set of values of k for which the		
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	$\int_{1}^{6} \frac{3x - 16}{3x^2 + 10x - 8} \mathrm{d}x$
(Give your answer in the form $\ln k$.

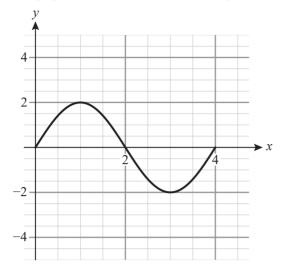
b Hence find the approximate solutions of the equation $\frac{1}{10} \sin 3x = x^2$.	a	Use Maclaurin series to find constant a such that $\frac{1}{10} \sin 3x \approx ax$ when $x \approx 0$.	
	b	Hence find the approximate solutions of the equation $\frac{1}{10} \sin 3x = x^2$.	
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all terms of the series are positive, find the common ratio of the series.

$\log_4($	$(3 - 2x) = \log_{16}(6x^2)$	-5x + 12).	

9 [Maximum mark: 5]

The graph of y = f(x) is shown in the diagram. The domain of f is $0 \le x \le 4$.



[3]

b Find the domain and range of the function g(x) = 2f(x - 1).

[2]

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a	Let $y = \arcsin x$. Express $\arccos x$ in terms of y . Hence show that $\arcsin x + \arccos x \equiv k$, where k is a constant to be found.	
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Do **not** write solutions on this page

Section B

Answer all questions in an answer booklet. Please start each question on a new page.

- **11** [Maximum mark: 18]
 - a Points A, B and D have coordinates A(1, -4, 3), B(2, 1, -1) and D(-1, 3, 3).
 - i Find the equation of the line l_1 through A and B.
 - ii Write down the equation of the line l_2 which passes through D and is parallel to AB. [5]
 - **b** i Find the exact distance AB.
 - ii Find the coordinates of two possible points C on the line l_2 such that CD = 2AB.
 - iii Denote the two possible points C by C_1 and C_2 . Determine whether angle C_1AC_2 is acute, right or obtuse. [8]
 - **c** i Find $\overrightarrow{AB} \times \overrightarrow{AD}$.
 - ii Hence find the equation of the plane containing the points A, B and D. [5]
- **12** [Maximum mark: 16]
 - a Use compound angle identities to express $\cos 3\theta$ in terms of $\cos \theta$. [4]
 - **b** Consider the equation $8x^3 6x + 1 = 0$.
 - i Given that $x = \cos \theta$, for $0 \le \theta \le \pi$, find the value of $\cos 3\theta$.
 - ii Hence find the possible values of x and show that they are all distinct. [7]
 - c Show that $8\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right) = -\sec\left(\frac{8\pi}{9}\right)$. [3]
 - **d** State, with a reason, the value of $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right)$. [2]
- **13** [Maximum mark: 21]

Let $f(x) = \frac{x}{1 + x^2}$ for $x \in \mathbb{R}$.

a Determine algebraically whether f is an even function, an odd function or neither. [3]

The continuous random variable X has probability density function given by

$$g(x) = \begin{cases} \frac{kx}{1+x^2} & \text{for } 0 \le x \le \sqrt{3} \\ 0 & \text{otherwise} \end{cases}.$$

- **b** Show that $k = \frac{1}{\ln 2}$. [4]
- \mathbf{c} Find the median of X.
- **d** Find the mode of X. [5]
- e Find the mean of X. [5]