Mathematics: analysis and approaches Higher level Paper 2 Practice Set C

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is [110 marks].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 [Maximum mark: 5]

Sacha is investigating the relationship between time spent doing homework and time spent on social media. In her year at school, at the time of the survey, 60% of students are aged 17 and the rest are aged 18. Sacha wants to represent both age groups fairly, so she takes a random sample of six 17-year-olds and four 18-year-olds.

a State the name of this sampling technique. [1]
The results are shown in the table, showing the number of hours per day spent on each activity.

Student	1	2	3	4	5	6	7	8	9	10
Time spent on social media (x)	1.7	3.5	2.6	1.7	2.1	3.2	3.8	2.5	3.1	3.6
Time spent on homework (y)	4.2	2.1	3.2	3.5	4.2	2.5	0.6	2.5	2.7	1.5

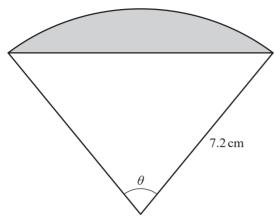
Sacha finds that there is a strong negative correlation between the two variables, and decides to use a linear regression line to model the relationship between them.

b Find the equation of the regression line in the form $y = ax + b$.	[2]
\mathbf{c} Interpret, in the context of this question, the meaning of the coefficients a and b .	[2]

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2 [Maximum mark: 5]

The diagram shows a sector of a circle with radius 7.2 cm. The angle at the centre is θ radians.



The area of the shaded region is $9.7\,\mathrm{cm^2}$. Find the value of θ .

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3	[Maximum	mark:	81

The discrete random variable X has the following probability distribution:

x	1	2	4	7
P(X=x)	k	2k	3 <i>k</i>	4k

a Find the value of k.

[2]

b Find the variance of X.

[4]

c Find the variance of $20 - 5X$.	

4	[Maximum mark: 5] Prove the identity

$\frac{\sec\theta\sin\theta}{\tan\theta + \cot\theta}$	$\equiv \sin^2 \theta$.	

1 /	0x + 200 minutes. work at most 12 hours	in a day, what is th	e largest number of	cakes he can
Ilya can choose	e to decorate between 2 time he can take per ca		day, inclusive. Find	he minimum

fiction and	0 books, 12 are f at least two non- ny different sele-	fiction books.		nts to take at	east three	
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	$\int \frac{\mathrm{e}^x}{\mathrm{e}^{2x} + \mathrm{e}^x - 2} \mathrm{d}x.$
(Give your answer in the form $ln(f(x))$.
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such	that $(2x + 3)$) is a facto	or of f(x)	, and wh	en f(x) is	divided	by $x - 2$ t	he remair	ider is 5.	

1	increasing when the radius equals 10 cm.

Do **not** write solutions on this page

Section B

Answer all questions in an answer booklet. Please start each question on a new page.

- **10** [Maximum mark: 20]
 - **a** Stella is planning to start a small business selling cosmetics gift boxes. She plans to start by selling 30 boxes in the first month. In each subsequent month she plans to sell 10 more boxes than in the previous month.
 - i According to Stella's plan, how many boxes will she sell in the 12th month?
 - ii How many boxes will she sell in the first year?
 - iii In which month will she sell her 2000th box?
 - **b** Giulio also sells cosmetics gift boxes. He also sells 30 boxes in the first month, but expects to increase his sales by 10% each month.
 - i How many boxes will Giulio sell in the first year?
 - ii In which month will Giulio first sell more than 100 boxes? [6]
 - c Stella makes a profit of £2.20 per box and Guilio makes a profit of £3.10 per box.
 - i Find the profit each person makes in the first year.
 - ii In which month will Giulio's total profit first overtake Stella's? [6]
- **11** [Maximum mark: 17]

The velocity (in $m s^{-1}$) of an object at t seconds is given by

$$v(t) = \frac{8 - 3t}{t^2 - 6t + 10}, \ 0 \le t \le 10.$$

Find

- a the initial speed [1]
- **b** the maximum speed [2]
- \mathbf{c} the length of time for which the speed is greater than $1 \,\mathrm{m\,s^{-1}}$
- d the time at which the object changes direction [2]
- e the length of time for which the object is decelerating [2]
- the length of time for which the object is decelerating
- f the acceleration after 5 seconds [2]
- g the distance travelled after 10 seconds [2]
- **h** the time when the object returns to its starting position. [3]
- **12** [Maximum mark: 18]
 - a Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln|\sec x + \tan x|\right) = \sec x.$$
 [3]

b Find the general solution to the differential equation

$$\cos x \, \frac{\mathrm{d}y}{\mathrm{d}x} + y = 1, -\frac{\pi}{2} < x < \frac{\pi}{2}.$$
 [7]

[8]

Consider now the differential equation

$$\frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} + y = 1, -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

c i Show that

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = (\sin x - 1) \frac{\mathrm{d}y}{\mathrm{d}x} - \cos x \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}.$$

ii Given that y = 2 and $\frac{dy}{dx} = 1$ when x = 0, find the Maclaurin series solution up to and including the term in x^3 . [8]