Cumulative frequency [53 marks]

University students were surveyed and asked how many hours, h , they worked each month. The results are shown in the following table.

| Hours per month, h | Frequency | Cumulative frequency |
|--------------------|-----------|----------------------|
| $0 < h \le 10$ | 3 | 3 |
| $10 < h \le 20$ | 7 | 10 |
| $20 < h \le 30$ | 10 | 20 |
| $30 < h \le 40$ | 14 | 34 |
| $40 < h \le 50$ | р | 44 |
| $50 < h \le 60$ | 6 | 50 |
| $60 < h \le 70$ | 4 | 54 |
| $70 < h \le 80$ | 2 | q |

Use the table to find the following values.

1a. p.

Markscheme p = 10 (A1) (C1)

Note: Award (A1) for each correct value. [1 mark]

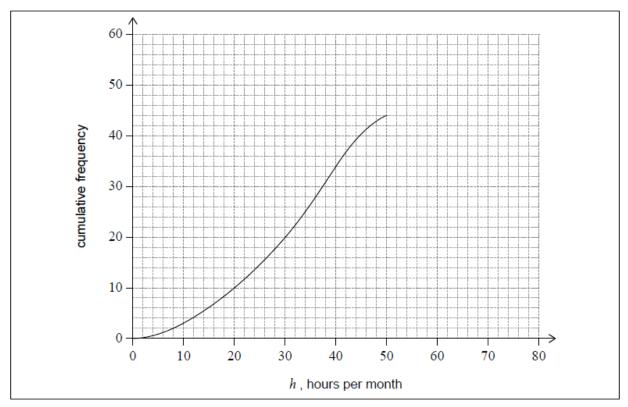
1b. q.

[1 mark]

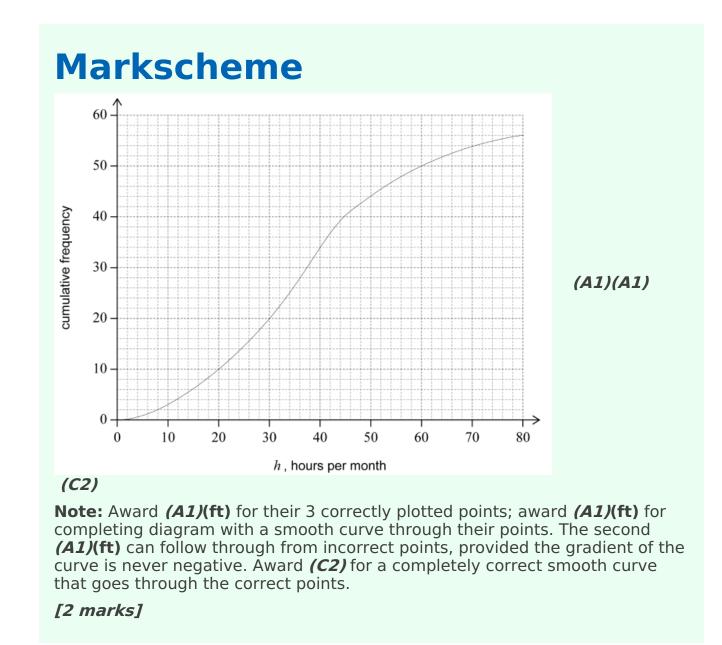
Markscheme

q = 56 (A1) (C1) Note: Award (A1) for each correct value. [1 mark]

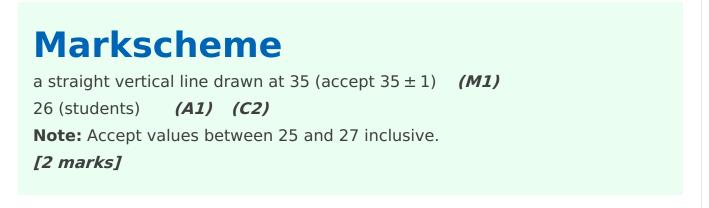
The first five class intervals, indicated in the table, have been used to draw part of a cumulative frequency curve as shown.



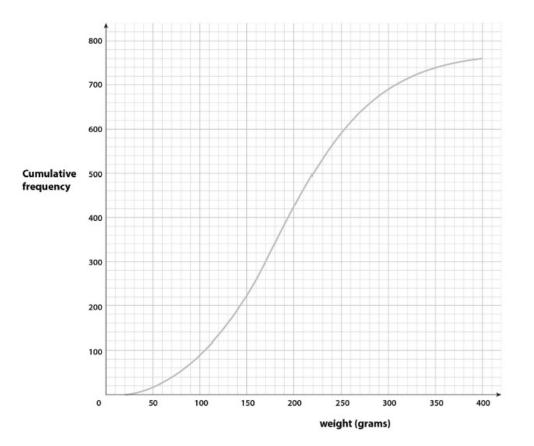
1c. On the same grid, complete the cumulative frequency curve for these [2 marks] data.



1d. Use the cumulative frequency curve to find an estimate for the number [2 marks] of students who worked at most 35 hours per month.



A food scientist measures the weights of 760 potatoes taken from a single field and the distribution of the weights is shown by the cumulative frequency curve below.



2a. Find the number of potatoes in the sample with a weight of more than [2 marks] 200 grams.

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

 $760 - 420 = 340 \; (g) \;$ (M1)A1

[2 marks]

2b. Find the median weight.

$$\label{eq:Markscheme} \begin{split} & \textbf{Markscheme} \\ & \textbf{Median} = 190 \ (g) \ \textbf{A1} \end{split}$$

[1 mark]

2c. Find the lower quartile.

| Markscheme | |
|-----------------------------------------|---|
| Lower quartile $= 135 - 140~{ m (g)}$ A | 1 |
| [1 mark] | |

2d. Find the upper quartile.

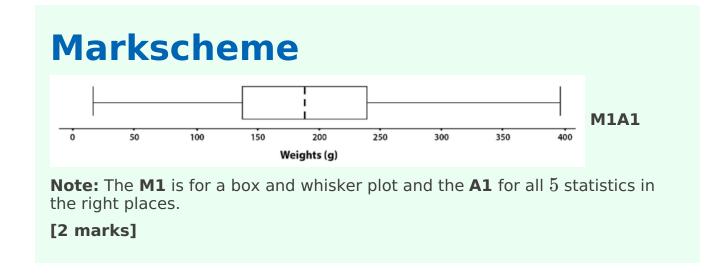
| Upper quartile $=242-247~{ m (g)}$ A1 [1 mark] | Markscheme |
|------------------------------------------------|---------------------------------------|
| [1 mark] | Upper quartile $=242-247~{ m (g)}$ A1 |
| | [1 mark] |

2e. The weight of the smallest potato in the sample is 20 grams and the *[2 marks]* weight of the largest is 400 grams.

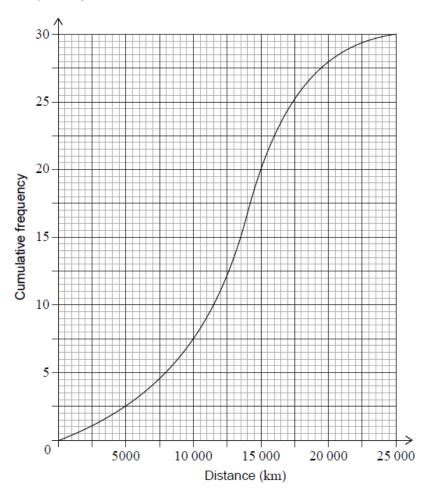
Use the scale shown below to draw a box and whisker diagram showing the distribution of the weights of the potatoes. You may assume there are no outliers.

| ò | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
|---|----|-----|-----|-------------|-----|-----|-----|-----|
| | | | w | /eights (g) | | | | |

[1 mark]



A transportation company owns 30 buses. The distance that each bus has travelled since being purchased by the company is recorded. The cumulative frequency curve for these data is shown.



3a. Find the number of buses that travelled a distance between 15000 and *[2 marks]* 20000 kilometres.

| Markscheme | |
|---------------------------------------------------------------------------------------------------------------------|--|
| * This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. | |
| 28 – 20 (A1) | |
| <i>Note:</i> Award <i>(A1)</i> for 28 and 20 seen. | |
| 8 (A1)(G2) | |
| [2 marks] | |
| | |

3b. Use the cumulative frequency curve to find the median distance. [2 marks]

| Markscheme |
|-----------------------------------------------------|
| 13500 (G2) |
| Note: Accept an answer in the range 13500 to 13750. |
| [2 marks] |
| |

3c. Use the cumulative frequency curve to find the lower quartile.

| Markscheme |
|-----------------------------------------------------|
| 10000 (G1) |
| Note: Accept an answer in the range 10000 to 10250. |
| [1 mark] |
| |

[1 mark]

3d. Use the cumulative frequency curve to find the upper quartile.[1 mark]

| Markscheme |
|-----------------------------------------------------|
| 16000 (G1) |
| Note: Accept an answer in the range 16000 to 16250. |
| [1 mark] |
| |
| |

| | irkscheme |
|-------|---------------------------------------------------|
| 6000 | <i>(A1)</i> (ft) |
| Note: | Follow through from their part (b)(ii) and (iii). |
| [1 ma | rk] |

3f. Write down the percentage of buses that travelled a distance greater [1 mark] than the upper quartile.

| Markscheme | |
|-----------------|--|
| 25% (A1) | |
| [1 mark] | |
| | |

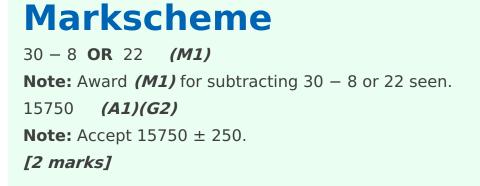
3g. Find the number of buses that travelled a distance less than or equal to [1 mark] 12 000 km.

Markscheme

It is known that 8 buses travelled more than m kilometres.

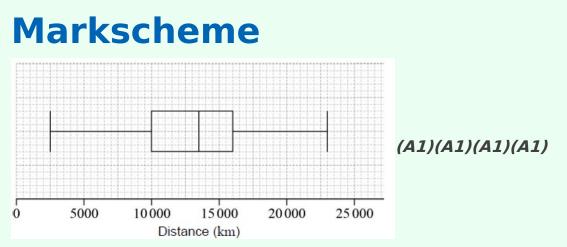
3h. Find the value of *m*.

[2 marks]



3i. The smallest distance travelled by one of the buses was 2500 km. The longest distance travelled by one of the buses was 23 000 km. [4 marks]

On graph paper, draw a box-and-whisker diagram for these data. Use a scale of 2 cm to represent 5000 km.



Note: Award **(A1)** for correct label and scale; accept "distance" or "km" for label.

(A1)(ft) for correct median,

(A1)(ft) for correct quartiles and box,

(A1) for endpoints at 2500 and 23 000 joined to box by straight lines.

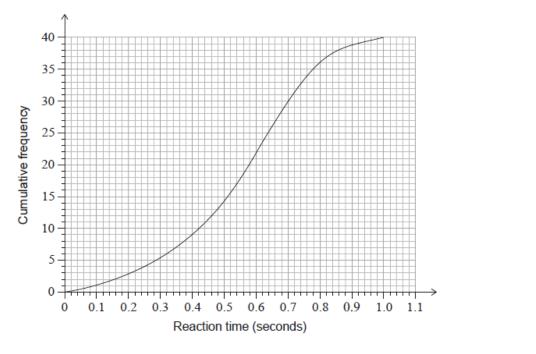
Accept ± 250 for the median, quartiles and endpoints.

Follow through from their part (b).

The final **(A1)** is not awarded if the line goes through the box.

[4 marks]

Mackenzie conducted an experiment on the reaction times of teenagers. The results of the experiment are displayed in the following cumulative frequency graph.



Use the graph to estimate the

4a. median reaction time.

Markscheme 0.58(s) A1 [1 mark]

4b. interquartile range of the reaction times.

[3 marks]

| Markscheme 0.7-0.42 (A1)(M1) | |
|--------------------------------------------------------------------------------------------------------|--|
| Note: Award A1 for correct quartiles seen, M1 for subtraction of their quartiles. | |
| 0.28(s) A1 | |
| [3 marks] | |
| | |
| Find the estimated number of teenagers who have a reaction time [2 marks] greater than 0.4 seconds. | |

| Markscheme | |
|---------------------------------------------|------|
| 9 (people have reaction time ≤ 0.4) | (A1) |
| 31 (people have reaction time > 0.4) | AI |
| [2 marks] | |

4d. Determine the $90 \mathrm{th}$ percentile of the reaction times from the cumulative [2 marks] frequency graph.

Markscheme $(90\% \times 40 =) 36$ OR 4 (A1) 0.8 s A1 [2 marks]

Mackenzie created the cumulative frequency graph using the following grouped frequency table.

| Reaction time, t (s) | Frequency |
|-----------------------------------|-----------|
| 0 < <i>t</i> ≤ 0 .2 | 3 |
| $0.2 < t \le 0.4$ | а |
| $0.4 < t \le 0.6$ | 13 |
| $0.6 < t \le 0.8$ | 14 |
| 0.8 < <i>t</i> ≤ 1.0 | Ь |

4e. Write down the value of a.

[1 mark]



4f. Write down the value of b.

Markscheme(b =) 4A1[1 mark]

4g. Write down the modal class from the table.

[1 mark]



4h. Use your graphic display calculator to find an estimate of the mean reaction time.

[2 marks]

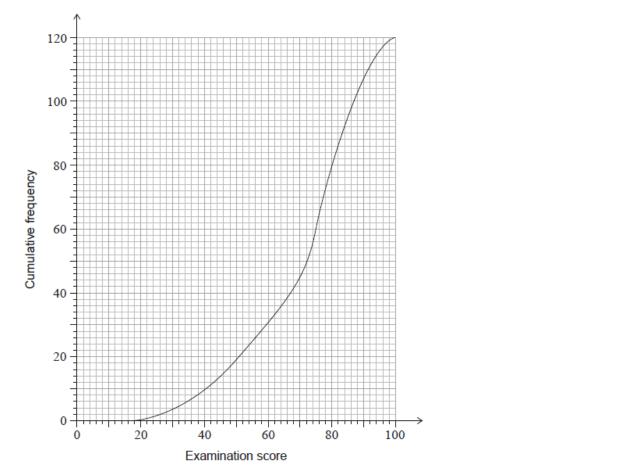
| Markscheme |
|------------------|
| 0.55 s A2 |
| [2 marks] |
| |

Upon completion of the experiment, Mackenzie realized that some values were grouped incorrectly in the frequency table. Some reaction times recorded in the interval $0 < t \le 0.2$ should have been recorded in the interval $0.2 < t \le 0.4$.

4i. Suggest how, if at all, the estimated mean and estimated median [4 marks] reaction times will change if the errors are corrected. Justify your response.

Markscheme the mean will increase A1 because the incorrect reaction times are moving from a lower interval to a higher interval which will increase the numerator of the mean calculation *R1* the median will stay the same A1 because the median or middle of the data is greater than both intervals being changed R1 Inter: Do not award A1R0. Interval

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



5a. Find the median of the scores obtained.

[1 mark]

| Markscheme | | | |
|------------|----|--|--|
| 75 | A1 | | |
| [1 mark | k] | | |

The students were awarded a grade from $1\ {\rm to}\ 5$, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

| Grade | 1 | 2 | 3 | 4 | 5 |
|--------------------|---|----|----|---|---|
| Number of students | 6 | 13 | 26 | а | b |

5b. Find an expression for a in terms of b.

[2 marks]

| Markschemerecognition that all entries add up to 120 (M1) $a = 120 - 6 - 13 - 26 - b$ OR $a = 75 - b$ A1 | |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| [2 marks] | |
| The mean grade for these students is $3.65.$ | |
| 5c. Find the number of students who obtained a grade $5.$ | [3 marks] |
| $\frac{\text{Markscheme}}{\frac{6 \times 1 + 13 \times 2 + 26 \times 3 + (75 - b) \times 4 + b \times 5}{120}} = 3.65 (M1)(A1)$ | |
| Note: Award (M1) for attempt to substitute into mean formula, L expression is sufficient for the M mark. Award (A1) for correct su one variable OR in two variables, followed by evidence of solving simultaneously with $a + b = 75$. | bstitutions in |
| $(b=)\ 28$ AI | |
| [3 marks] | |
| 5d. Find the minimum score needed to obtain a grade 5. | [2 marks] |

| Markscheme |
|----------------------------------------------------------------|
| 120-their part (c)(i) seen (e.g. 92 indicated on graph) (M1) |
| 84 A1 |
| [2 marks] |



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