

Do **not** write solutions on this page.

12. [Maximum mark: 18]

On the day of her birth, 1st January 1998, Mary’s grandparents invested $\$x$ in a savings account. They continued to deposit $\$x$ on the first day of each month thereafter.

The account paid a fixed rate of 0.4% interest per month. The interest was calculated on the last day of each month and added to the account.

Let $\$A_n$ be the amount in Mary’s account on the last day of the n th month, immediately after the interest had been added.

- (a) Find an expression for A_1 and show that $A_2 = 1.004^2x + 1.004x$. [2]
- (b) (i) Write down a similar expression for A_3 and A_4 .
- (ii) Hence show that the amount in Mary’s account the day before she turned 10 years old is given by $251(1.004^{120} - 1)x$. [6]
- (c) Write down an expression for A_n in terms of x on the day before Mary turned 18 years old showing clearly the value of n . [1]
- (d) Mary’s grandparents wished for the amount in her account to be at least \$20 000 the day before she was 18. Determine the minimum value of the monthly deposit $\$x$ required to achieve this. Give your answer correct to the nearest dollar. [4]
- (e) As soon as Mary was 18 she decided to invest \$15 000 of this money in an account of the same type earning 0.4% interest per month. She withdraws \$1000 every year on her birthday to buy herself a present. Determine how long it will take until there is no money in the account. [5]

