Sequences, FM and BT revision *[118 marks]*

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of $5.5\,\%$, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

1a. Find the value of r, giving your answer to four significant figures. *[3 marks]*

Markscheme

$\left(1+\frac{5.5}{4\times 100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

1b. Laurie makes no further deposits to or withdrawals from the account. *[3 marks]*

Find the year in which the amount of money in Laurie's account will become double the amount she invested.

Markscheme

EITHER

$2P = P \times \left(1 + \frac{5.5}{100 \times 4}\right)^{4n}$ **OR** $2P = P \times (\text{their } (a))^{m}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

PV = ±1
FV = ∓1
I% = 5.5
P/Y = 4
C/Y = 4
n = 50.756… ***(M1)(A1)***

OR

PV = ±1
FV = ∓2
I% = 100(their (*a*) − 1)
P/Y = 1
C/Y = 1 ***(M1)(A1)***

THEN

⇒ 12.7 years

Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

2. geometric sequence has a first term of 50 and a fourth term of 86.4. *[5 marks]*

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

Markscheme

$86.4 = 50r^3$ *(A1)*

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere *(A1)*

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ *(A1)*

attempt to solve their geometric S_n inequality or equation *(M1)*

sketch OR $n > 26.9045,\ n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ ***A1***

[5 marks]

3. Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$. *[6 marks]*

Find the value of a.

Markscheme

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^9C_r + (ax)^{9-r} + (1)^r$.

valid approach for expansion (must be the product of a binomial coefficient with $n = 9$ and a power of ax) *(M1)*

${}^9C_r(ax)^{9-r}(1)^r$ OR ${}^9C_{9-r}(ax)^r(1)^{9-r}$ OR ${}^9C_0(ax)^0(1)^9 + {}^9C_1(ax)^1(1)^8 + \ldots$

recognizing that the term in x^6 is needed *(M1)*

$\frac{\text{Term in } x^6}{21x^2} = kx^4$ OR $r = 6$ OR $r = 3$ OR $9 - r = 6$

correct term or coefficient in binomial expansion (seen anywhere) *(A1)*

${}^9C_6(ax)^6(1)^3$ OR ${}^9C_3a^6x^6$ OR $84\left(a^6x^6\right)(1)$ OR $84a^6$

EITHER

correct term in x^4 or coefficient (may be seen in equation) ***(A1)***

$\frac{^9C_6}{21}a^6x^4$ OR $4a^6x^4$ OR $4a^6$

Set their term in x^4 or coefficient of x^4 equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of x) ***(M1)***

$\frac{^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4$ OR $4a^6 = \frac{8}{7}a^5$

OR

correct term in x^6 or coefficient of x^6 (may be seen in equation) ***(A1)***

$84a^6x^6$ OR $84a^6$

set their term in x^6 or coefficient of x^6 equal to $24a^5x^6$ or $24a^5$ (do not accept other powers of x) ***(M1)***

$84a^6x^6 = 24a^5x^6$ OR $84a = 24$

THEN

$a = \frac{2}{7} \approx 0.286\,(0.285714\ldots)$ ***A1***

Note: Award **A0** for the final mark for $a = \frac{2}{7}$ and $a = 0$.

[6 marks]

4. Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all possible values of n for which the expansion has a non-zero constant term. *[5 marks]*

Markscheme

EITHER

attempt to obtain the general term of the expansion

$T_{r+1} = {}_nC_r\left(8x^3\right)^{n-r}\left(-\frac{1}{2x}\right)^r$ OR $T_{r+1} = {}_nC_{n-r}\left(8x^3\right)^r\left(-\frac{1}{2x}\right)^{n-r}$ ***(M1)***

OR

recognize power of x starts at $3n$ and goes down by 4 each time ***(M1)***

THEN

recognizing the constant term when the power of x is zero (or equivalent) ***(M1)***

$r = \frac{3n}{4}$ or $n = \frac{4}{3}r$ or $3n - 4r = 0$ OR $3r - (n - r) = 0$ (or equivalent) ***A1***

r is a multiple of 3 $(r = 3, 6, 9, \ldots)$ or one correct value of n (seen anywhere) ***(A1)***

$n = 4k,\ k \in \mathbb{Z}^+$ ***A1***

Note: Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \ldots$
Do not accept $n = 4, 8, 12$

Note: Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \ldots$ and for recognizing that this pattern continues.

[5 marks]

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

5a. Find the amount that Sam will have in his account after 10 years. *[3 marks]*

Markscheme

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

EITHER

$N = 10$ OR $N = 20$

$I\% = 2.74$ $I\% = 2.74$

$PV = (\mp)1700$ $PV = (\mp)1700$

$P/Y = 1$ $P/Y = 2$

$C/Y = 2$ $C/Y = 2$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. Accept a positive or negative value for PV.

OR

$1700\left(1 + \frac{0.0274}{2}\right)^{2\times 10}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula.
Award ***(A1)*** for correct substitution.

THEN

$\$2231.71$ ***A1***

[3 marks]

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

5b. Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. *[2 marks]*

Markscheme

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

EITHER

$N = 10$

$PV = \mp 1700$

$FV = \pm 2231.71\ldots$

$P/Y = 1$

$C/Y = 1$ ***(M1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen.

OR

$1700\left(1 + \frac{r}{100}\right)^{10} = 2231.71\ldots$ ***(M1)***

THEN

$r = 2.75876\ldots$

$r = 2.76$ ***A1***

Note: Ignore omission of opposite signs for PV and FV if $r = 2.76$ is obtained.

[2 marks]

5c. Find the interest David will earn over the 10 years. *[1 mark]*

Markscheme

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

$\$531.71$ ***A1***

[1 mark]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

6a. Find the first term of the sequence, u_1. *[2 marks]*

Markscheme

$u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24}\left(= \frac{7}{12} = 0.583333\ldots\right)$ ***A1***

[2 marks]

6b. Find S_∞. *[3 marks]*

Markscheme

$r = \frac{7}{8}(= 0.875)$ ***(A1)***

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3}(= 4.66666\ldots)$ ***A1***

[3 marks]

6c. Find the least value of n such that $S_\infty - S_n < 0.001$. *[4 marks]*

Markscheme

attempt to substitute their values into the inequality or formula for S_n ***(M1)***

$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r < 0.001$ OR $S_n = \frac{\frac{7}{12}\left(1-\left(\frac{7}{8}\right)^n\right)}{\left(1-\frac{7}{8}\right)}$

attempt to solve their inequality using a table, graph or logarithms

(must be exponential) ***(M1)***

Note: Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036\ldots$ OR $S_\infty - S_{64} = 0.000906\ldots$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683\ldots$ OR

$S_\infty - S_{64} - 0.001 = 0.0000931777\ldots$

least value is $n = 64$ ***A1***

[4 marks]

7. Consider the expansion of $\left(3 + x^2\right)^{n+1}$, where $n \in \mathbb{Z}^+$. *[5 marks]*

Given that the coefficient of x^4 is $20\ 412$, find the value of n.

Markscheme

METHOD 1

product of a binomial coefficient, a power of 3 (and a power of x^2) seen ***(M1)***

evidence of correct term chosen ***(A1)***

$^{n+1}C_2 \times 3^{n+1-2} \times \left(x^2\right)^2 \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4\right)$ **OR** $n - r = 1$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

EITHER

$^{n+1}C_2 \times 3^{n-1} = 20412$ ***(A1)***

OR

$^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$ ***(A1)***

THEN

$n = 7$ ***A1***

METHOD 2

$3^{n+1}\left(1 + \frac{x^2}{3}\right)^{n+1}$

product of a binomial coefficient, and a power of $\frac{x^2}{3}$ **OR** $\frac{1}{3}$ seen ***(M1)***

evidence of correct term chosen ***(A1)***

$3^{n+1} \times \frac{n(n+1)}{2!} \times \left(\frac{x^2}{3}\right)^2 \left(= \frac{3^{n-1}}{2} n(n+1)x^4\right)$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

$3^{n-1} \times \frac{n(n+1)}{2} = 20412$ ***(A1)***

$n = 7$ ***A1***

[5 marks]

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

8a. Find the value of Amelia’s investment after 5 years to the nearest hundred dollars. *[3 marks]*

Markscheme

EITHER

$9000 \times \left(1 + \frac{7}{100}\right)^5$ ***(A1)***

$12622.965\ldots$ ***(A1)***

OR

$n = 5$
$\mathrm{I\%} = 7$
$\mathrm{PV} = \mp 9000$
$\mathrm{P/Y} = 1$
$\mathrm{C/Y} = 1$ ***(A1)***
$\pm 12622.965\ldots$ ***(A1)***

THEN

$(\$)\ 12600$ ***A1***

[3 marks]

8b. Determine the number of years required for Amelia's investment to reach the target. *[2 marks]*

Markscheme

EITHER

$9000\left(1 + \frac{7}{100}\right)^x = 20000$ ***(A1)***

OR

$\mathrm{I\%} = 7$
$\mathrm{PV} = \mp 9000$
$\mathrm{FV} = \pm 20000$
$\mathrm{P/Y} = 1$
$\mathrm{C/Y} = 1$ ***(A1)***

THEN

$= 12$ (years) ***A1***

[2 marks]

8c. Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places. *[3 marks]*

Find the minimum value of r needed for Bill to reach the target after 10 years.

Markscheme

METHOD 1

attempt to substitute into compound interest formula (condone absence of compounding periods) ***(M1)***

$9000\left(1+\frac{r}{100\times12}\right)^{12\times10}=20000$

$8.01170\ldots$ ***(A1)***

$r=8.02\,(\%)$ ***A1***

METHOD 2

$n=10$
$\text{PV}=\pm9000$
$\text{FV}=\mp20000$
$\text{P/Y}=1$
$\text{C/Y}=12$
$r=8.01170\ldots$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for $(r=)8.01170\ldots$

$r=8.02\,(\%)$ ***A1***

[3 marks]

A third friend Chris also wants to reach the $\$20\ 000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

8d. Show that Chris will never reach the target if his initial deposit is $\$9000$. *[5 marks]*

Markscheme

recognising geometric series (seen anywhere) ***(M1)***

$r = \frac{4500}{9000}\left(= \frac{1}{2}\right)$ ***(A1)***

EITHER

considering S_∞ ***(M1)***

$\frac{9000}{1-0.5}(= 18000)$ ***A1***

correct reasoning that $18000 < 20000$ ***R1***

Note: Accept $S_\infty < 20000$ only if S_∞ has been calculated.

OR

considering S_n for a large value of $n, n \geq 80$ ***(M1)***

Note: Award ***M1*** only if the candidate gives a valid reason for choosing a value of n, where $50 \leq n < 80$.

correct value of S_n for their n ***A1***

valid reason why Chris will not reach the target, which involves their choice of n, their value of S_n and Chris' age OR using two large values of n to recognize asymptotic behaviour of S_n as $n \to \infty$. ***R1***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

THEN

Therefore, Chris will never reach the target. ***AG***

[5 marks]

8e. Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. *[3 marks]*

Markscheme

recognising geometric sum ***M1***

$\frac{u_1\left(1-0.5^5\right)}{0.5} = 20000$ ***(A1)***

$10322.58\ldots$

$(\$)\ 10323$ ***A1***

[3 marks]

9. Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$. *[6 marks]*

The coefficient of the term in x^6 is 6048. Find the value of k.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for r). ***(M1)***

eg
$$\binom{9}{r}\left(3x^2\right)^{9-r}\left(-\frac{k}{x}\right)^r,\ \left(3x^2\right)^9 + \binom{9}{1}\left(3x^2\right)^8\left(-\frac{k}{x}\right)^1 + \binom{9}{2}\left(3x^2\right)^7\left(-\frac{k}{x}\right)^2 + \ldots$$

valid attempt to identify correct term ***(M1)***

eg $2(9-r) - r = 6,\ \left(x^2\right)^r\left(x^{-1}\right)^{9-r} = x^6$

identifying correct term (may be indicated in expansion) ***(A1)***

eg $r = 4,\ r = 5$

correct term or coefficient in binominal expansion ***(A1)***

eg $\binom{9}{4}\left(3x^2\right)^5\left(-\frac{k}{x}\right)^4,\ 126\left(243x^{10}\right)\left(\frac{k^4}{x^4}\right),\ 30618k^4$

correct equation in k ***(A1)***

eg $\binom{9}{4}(243)\left(k^4\right)x^6 = 6048x^6,\ 30618k^4 = 6048$

$k = \frac{2}{3}$ (exact) 0.667 ***A1 N3***

Note: Do not award ***A1*** if additional answers given.

[6 marks]

10. Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2} - \frac{x}{2}\right)^9$. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of Binomial expansion to find a term in either

$\left(\frac{1}{3x^2} - \frac{x}{2}\right)^9$, $\left(\frac{1}{3x^{7/3}} - \frac{x^{2/3}}{2}\right)^9$, $\left(\frac{1}{3} - \frac{x^3}{2}\right)^9$, $\left(\frac{1}{3x^3} - \frac{1}{2}\right)^9$ or $\left(2 - 3x^3\right)^9$

(M1)(A1)

Note: Award ***M1*** for a product of three terms including a binomial coefficient and powers of the two terms, and ***A1*** for a correct expression of a term in the expansion.

finding the powers required to be 2 and 7 ***(M1)(A1)***

constant term is ${}_9C_2 \times \left(\frac{1}{3}\right)^2 \times \left(-\frac{1}{2}\right)^7$ ***(M1)***

Note: Ignore all x's in student's expression.

therefore term independent of x is $-\frac{1}{32}$ $(= -0.03125)$ ***A1***

[6 marks]

11. Consider the expansion of $(2 + x)^n$, where $n \geqslant 3$ and $n \in \mathbb{Z}$. *[6 marks]*

The coefficient of x^3 is four times the coefficient of x^2. Find the value of n.

Markscheme

attempt to find coefficients in binomial expansion ***(M1)***

coefficient of x^2 : $\binom{n}{2} \times 2^{n-2}$; coefficient of x^3 : $\binom{n}{3} \times 2^{n-3}$ ***A1A1***

Note: Condone terms given rather than coefficients. Terms may be seen in an equation such as that below.

$\binom{n}{3} \times 2^{n-3} = 4\binom{n}{2} \times 2^{n-2}$ ***(A1)***

attempt to solve equation using GDC or algebraically ***(M1)***

$\binom{n}{3} = 8\binom{n}{2}$

$\frac{n!}{3!(n-3)!} = \frac{8n!}{2!(n-2)!}$

$\frac{1}{3} = \frac{8}{n-2}$

$n = 26$ ***A1***

[6 marks]

In an arithmetic sequence, $u_1 = 1.3$, $u_2 = 1.4$ and $u_k = 31.2$.

12a. Find the exact value of S_k. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg $\frac{300}{2}(1.3 + 31.2)$, $\frac{300}{2}[2(1.3) + (300-1)(0.1)]$, $\frac{300}{2}[2.6 + 299(0.1)]$

$S_k = 4875$ ***A1 N2***

[2 marks]

Consider the terms, u_n, of this sequence such that $n \leq k$.

Let F be the sum of the terms for which n is not a multiple of 3.

12b. Show that $F = 3240$. *[5 marks]*

Markscheme

recognizing need to find the sequence of multiples of 3 (seen anywhere) ***(M1)***

eg first term is u_3 (= 1.5) (accept notation $u_1 = 1.5$) ,

$d = 0.1 \times 3$ (= 0.3) , 100 terms (accept $n = 100$), last term is 31.2

(accept notation $u_{100} = 31.2$) , $u_3 + u_6 + u_9 + \ldots$ (accept $F = u_3 + u_6 + u_9 + \ldots$)

correct working for sum of sequence where n is a multiple of 3 ***A2***

$\frac{100}{2}(1.5 + 31.2)$, $50\left(2 \times 1.5 + 99 \times 0.3\right)$, 1635

valid approach (seen anywhere) ***(M1)***

eg $S_k - (u_3 + u_6 + \ldots)$, $S_k - \frac{100}{2}(1.5 + 31.2)$, $S_k -$ (their sum for $(u_3 + u_6 + \ldots)$)

correct working (seen anywhere) ***A1***

eg $S_k - 1635$, 4875 − 1635

$F = 3240$ ***AG N0***

[5 marks]

12c. An infinite geometric series is given as $S_\infty = a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \ldots, a \in \mathbb{Z}^+$ *[5 marks]*

.

Find the largest value of a such that $S_\infty < F$.

Markscheme

attempt to find r ***(M1)***

eg dividing consecutive terms

correct value of r (seen anywhere, including in formula)

eg $\frac{1}{\sqrt{2}}$, 0.707106... , $\frac{a}{0.293...}$

correct working (accept equation) ***(A1)***

eg $\frac{a}{1-\frac{1}{\sqrt{2}}} < 3240$

correct working ***A1***

METHOD 1 (analytical)

eg $3240 \times \left(1 - \frac{1}{\sqrt{2}}\right)$, $a < 948.974$, 948.974

METHOD 2 (using table, must find both S_∞ values)

eg when $a = 948$, $S_\infty = 3236.67...$ **AND** when $a = 949$, $S_\infty = 3240.08...$

$a = 948$ ***A1 N2***

[5 marks]

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

13a. Find the common ratio. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct substitution into infinite sum ***(A1)***

eg $200 = \frac{4}{1-r}$

r = 0.98 (exact) ***A1 N2***

[2 marks]

13b. Find the sum of the first 8 terms. *[2 marks]*

Markscheme

correct substitution *(A1)*

$\frac{4(1-0.98^8)}{1-0.98}$

29.8473

29.8 ***A1 N2***

[2 marks]

13c. Find the least value of n for which $S_n > 163$. *[3 marks]*

Markscheme

attempt to set up inequality (accept equation) ***(M1)***

eg $\frac{4(1-0.98^n)}{1-0.98} > 163, \ \frac{4(1-0.98^n)}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values ***(A1)***

eg $n > 83.5234$, $n = 83.5234$, $S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ ***A1 N1***

[3 marks]

It is known that the number of fish in a given lake will decrease by 7% each year unless some new fish are added. At the end of each year, 250 new fish are added to the lake.

At the start of 2018, there are 2500 fish in the lake.

14a. Show that there will be approximately 2645 fish in the lake at the start of 2020. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

EITHER

2019: $2500 \times 0.93 + 250 = 2575$ ***(M1)A1***

2020: $2575 \times 0.93 + 250$ ***M1***

OR

2020: $2500 \times 0.93^2 + 250(0.93 + 1)$ ***M1M1A1***

Note: Award ***M1*** for starting with 2500, ***M1*** for multiplying by 0.93 and adding 250 twice. ***A1*** for correct expression. Can be shown in recursive form.

THEN

$(= 2644.75) = 2645$ ***AG***

[3 marks]

14b. Find the approximate number of fish in the lake at the start of 2042. *[5 marks]*

Markscheme

2020: $2500 \times 0.93^2 + 250(0.93 + 1)$
2042: $2500 \times 0.93^{24} + 250(0.93^{23} + 0.93^{22} + \ldots + 1)$ ***(M1)(A1)***

$= 2500 \times 0.93^{24} + 250\frac{(0.93^{24}-1)}{(0.93-1)}$ ***(M1)(A1)***

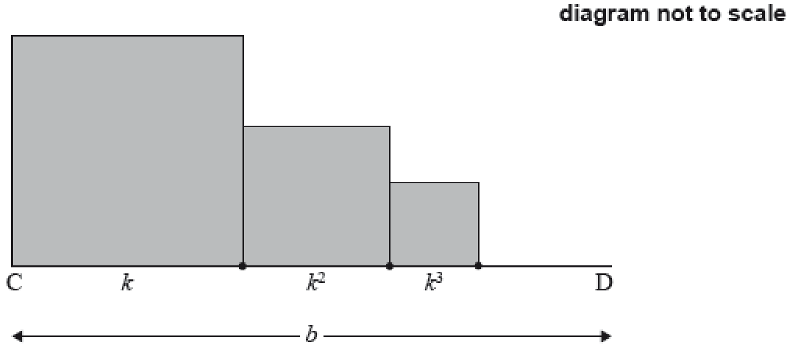
$=3384$ ***A1***

Note: If recursive formula used, award ***M1*** for $u_n = 0.93\ u_{n-1}$ **and** u_0 or u_1 seen (can be awarded if seen in part (a)). Then award ***M1A1*** for attempt to find u_{24} or u_{25} respectively (different term if other than 2500 used) (***M1A0*** if incorrect term is being found) and ***A2*** for correct answer.

Note: Accept all answers that round to 3380.

[5 marks]

15. The following diagram shows [CD], with length $b\,\text{cm}$, where $b > 1$. *[9 marks]*
Squares with side lengths $k\,\text{cm}, k^2\,\text{cm}, k^3\,\text{cm}, \ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

Markscheme

recognizing infinite geometric series with squares ***(M1)***

eg $k^2 + k^4 + k^6 + \ldots, \frac{k^2}{1-k^2}$

correct substitution into $S_\infty = \frac{9}{16}$ (must substitute into formula) ***(A2)***

eg $\frac{k^2}{1-k^2} = \frac{9}{16}$

correct working ***(A1)***

eg $16k^2 = 9 - 9k^2, 25k^2 = 9, k^2 = \frac{9}{25}$

$k = \frac{3}{5}$ (seen anywhere) ***A1***

valid approach with segments and CD (may be seen earlier) ***(M1)***

eg $r = k, S_\infty = b$

correct expression for b in terms of k (may be seen earlier) ***(A1)***

eg $b = \frac{k}{1-k}, b = \sum_{n=1}^{\infty} k^n, b = k + k^2 + k^3 + \ldots$

substituting **their** value of k into **their** formula for b ***(M1)***

eg $\frac{\frac{3}{5}}{1-\frac{3}{5}}, \frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \frac{3}{2}$ ***A1 N3***

[9 marks]

16. The coefficient of x^2 in the expansion of $\left(\frac{1}{x}+5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a+5x)^7,\ a\in\mathbb{R}$. Find the value of a. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

$^8C_r\left(\frac{1}{x}\right)^{8-r}(5x)^r = {}^8Cr(5)^r x^{2r-8}$ ***(M1)***

$r=5$ ***(A1)***

$^8C_5\times 5^5 = ^7C_4 a^3\times 5^4$ ***M1A1***

$56\times 5 = 35a^3$

$a^3=8$ ***(A1)***

$a=2$ ***A1***

METHOD 2

attempt to expand both binomials ***M1***

$175000x^2$ ***A1***

$21875a^3x^4$ ***A1***

$175000 = 21875a^3$ ***M1***

$a^3=8$ ***(A1)***

$a=2$ ***A1***

[6 marks]