Sequences, FM and BT revision *[118 marks]*

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, *r*.

1a. Find the value of *r*, giving your answer to four significant figures. *[3 marks]*

1b. Laurie makes no further deposits to or withdrawals from the account. *[3 marks]*

Find the year in which the amount of money in Laurie's account will become double the amount she invested.

2. geometric sequence has a first term of 50 and a fourth term of 86.4. *[5 marks]*

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

3. Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$. *[6 marks]*

Find the value of a.

4. Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all possible values of n for which the expansion has a non-zero constant term. *[5 marks]*

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

5a. Find the amount that Sam will have in his account after 10 years. *[3 marks]*

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

5b. Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. *[2 marks]*

5c. Find the interest David will earn over the 10 years. *[1 mark]*

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

6a. Find the first term of the sequence, u_1. *[2 marks]*

6b. Find S_∞. *[3 marks]*

6c. Find the least value of n such that $S_\infty - S_n < 0.001$. *[4 marks]*

7. Consider the expansion of $\left(3 + x^2\right)^{n+1}$, where $n \in \mathbb{Z}^+$. *[5 marks]*

Given that the coefficient of x^4 is $20\ 412$, find the value of n.

Two friends Amelia and Bill, each set themselves a target of saving $\$20\ 000$. They each have $\$9000$ to invest.

Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

8a. Find the value of Amelia's investment after 5 years to the nearest hundred dollars. *[3 marks]*

8b. Determine the number of years required for Amelia's investment to reach the target. *[2 marks]*

8c. Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places. *[3 marks]*

Find the minimum value of r needed for Bill to reach the target after 10 years.

A third friend Chris also wants to reach the $\$20\ 000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

8d. Show that Chris will never reach the target if his initial deposit is $\$9000$. *[5 marks]*

8e. Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. *[3 marks]*

9. Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$. *[6 marks]*

The coefficient of the term in x^6 is 6048. Find the value of k.

10. Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2} - \frac{x}{2}\right)^9$. *[6 marks]*

11. Consider the expansion of $(2+x)^n$, where $n \geqslant 3$ and $n \in \mathbb{Z}$. *[6 marks]*

The coefficient of x^3 is four times the coefficient of x^2. Find the value of n.

In an arithmetic sequence, $u_1 = 1.3$, $u_2 = 1.4$ and $u_k = 31.2$.

12a. Find the exact value of S_k. *[2 marks]*

Consider the terms, u_n, of this sequence such that $n \leq k$.

Let F be the sum of the terms for which n is not a multiple of 3.

12b. Show that $F = 3240$. *[5 marks]*

12c. An infinite geometric series is given as $S_\infty = a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \ldots$, $a \in \mathbb{Z}^+$ *[5 marks]*

.

Find the largest value of a such that $S_\infty < F$.

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

13a. Find the common ratio. *[2 marks]*

13b. Find the sum of the first 8 terms. *[2 marks]*

13c. Find the least value of n for which $S_n > 163$. *[3 marks]*

It is known that the number of fish in a given lake will decrease by 7% each year unless some new fish are added. At the end of each year, 250 new fish are added to the lake.

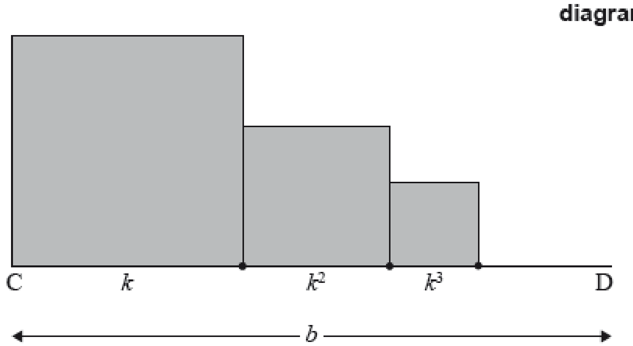
At the start of 2018, there are 2500 fish in the lake.

14a. Show that there will be approximately 2645 fish in the lake at the start of 2020. *[3 marks]*

14b. Find the approximate number of fish in the lake at the start of 2042. *[5 marks]*

15. The following diagram shows [CD], with length $b\,\mathrm{cm}$, where $b > 1$. Squares with side lengths $k\,\mathrm{cm}, k^2\,\mathrm{cm}, k^3\,\mathrm{cm}, \ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares. *[9 marks]*

diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

16. The coefficient of x^2 in the expansion of $\left(\frac{1}{x} + 5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a + 5x)^7,\ a \in \mathbb{R}$. Find the value of a. *[6 marks]*