**1.** Find the term in $x^3$ in the expansion of $\left(\frac{2}{3}x-3\right)^8$.

**(Total 5 marks)**

evidence of using binomial expansion (M1)

*e.g.* selecting correct term, $a^8b^0+\binom{8}{1}a^7b+\binom{8}{2}a^6b^2+\ldots$

evidence of calculating the factors, in any order A1A1A1

*e.g.* 56, $\frac{2^3}{3^3}$, $-3^5$, $\binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $=-4030x^3$ to 3 s.f.) A1 N2

**[5]**

**2.** One of the terms of the expansion of $(x + 2y)^{10}$ is $ax^8y^2$. Find the value of $a$.

**(Total 6 marks)**

Identifying the required term (seen anywhere) M1

*eg* $\binom{10}{8}\times 2^2$

$\binom{10}{8} = 45$ (A1)

$4y^2$, $2 \times 2$, 4 (A2)

$a = 180$ A2 N4

**[6]**

**3.** Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in $x^{12}$.

**(Total 6 marks)**

(a) 7 terms A1 N1

(b) A valid approach (M1)

Correct term chosen $\binom{6}{3}(x^3)^3(-3x)^3$ A1

Calculating $\binom{6}{3}=20$, $(-3)^3=-27$ (A1)(A1)

Term is $-540x^{12}$ A1 N3

**[6]**

**4.** Find the term in $x^4$ in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

**(Total 6 marks)**

evidence of substituting into binomial expansion (M1)

*e.g.* $a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \ldots$

identifying correct term for $x^4$ (M1)

evidence of calculating the factors, in any order A1A1A1

*e.g.* $\binom{5}{2}$, $27x^6, \frac{4}{x^2}$; $10(3x^2)^3\left(\frac{-2}{x}\right)^2$

***Note:*** *Award A1 for each correct factor.*

term = $1080x^4$ A1 N2

***Note:*** *Award M1M1A1A1A1A0 for 1080 with working shown.*

**[6]**

**5.** (a) Expand $(2 + x)^4$ and simplify your result.

**(3)**

(b) Hence, find the term in $x^2$ in $(2 + x)^4\left(1 + \frac{1}{x^2}\right)$.

**(3)**

**(Total 6 marks)**

(a) evidence of expanding M1

*e.g.* $2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4$, $(4 + 4x + x^2)(4 + 4x + x^2)$

$(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ A2 N2

(b) finding coefficients 24 and 1 (A1)(A1)

term is $25x^2$ A1 N3

**[6]**

**6.** (a) Expand $(x-2)^4$ and simplify your result.

**(3)**

(b) Find the term in $x^3$ in $(3x+4)(x-2)^4$.

**(3)**
**(Total 6 marks)**

(a) evidence of expanding M1

*e.g.* $(x-2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$

$(x-2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$ A2 N2

(b) finding coefficients, $3 \times 24\ (= 72), 4 \times (-8)(= -32)$ (A1)(A1)

term is $40x^3$ A1 N3

**[6]**

**7.** Determine the constant term in the expansion of $\left(x - \frac{2}{x^2}\right)^9$.

**(Total 4 marks)**

The constant term will be the term independent of the variable $x$. (R1)

$\left(x - \frac{2}{x^2}\right)^9 = x^9 + 9x^8\left(\frac{-2}{x^2}\right) + \ldots + \binom{9}{3}x^6\left(\frac{-2}{x^2}\right)^3 + \ldots + \left(\frac{-2}{x^2}\right)^9$ (M1)

$\binom{9}{3}x^6\left(\frac{-2}{x^2}\right)^3 = 84x^6\left(\frac{-8}{x^6}\right)$ (A1)

$= -672$ (A1)

**[4]**

**8.** (a) Expand $\left(e+\frac{1}{e}\right)^4$ in terms of e.

**(4)**

(b) Express $\left(e+\frac{1}{e}\right)^4 + \left(e-\frac{1}{e}\right)^4$ as the sum of three terms.

**(2)**
**(Total 6 marks)**

(a) For finding second, third and fourth terms correctly (A1)(A1)(A1)

Second term $\binom{4}{1}e^3\left(\frac{1}{e}\right)$, third term $\binom{4}{1}e^2\left(\frac{1}{e}\right)^2$,

fourth term $\binom{4}{1}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1)

$$\left(e+\frac{1}{e}\right)^4=\binom{4}{0}e^4+\binom{4}{1}e^3\left(\frac{1}{e}\right)+\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+\binom{4}{3}e\left(\frac{1}{e}\right)^3+\binom{4}{4}\left(\frac{1}{e}\right)^4$$

$$\left(e+\frac{1}{e}\right)^4=e^4+4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2+4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4+4e^2+6+\frac{4}{e^2}+\frac{1}{e^4}\right)$$ N4

(b) $$\left(e-\frac{1}{e}\right)^4=e^4-4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2-4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4-4e^2+6-\frac{4}{e^2}+\frac{1}{e^4}\right)$$ (A1)

Adding gives $2e^4+12+\frac{2}{e^4}$

$$\left(\text{accept}\ 2\binom{4}{0}e^4+2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$$ A1 N2

**[6]**

**9.** Given that $\left(3+\sqrt{7}\right)^3 = p + q\sqrt{7}$ where $p$ and $q$ are integers, find

(a) $p$;

(b) $q$.

**(Total 6 marks)**

Using binomial expansion (M1)

$$\left(3+\sqrt{7}\right)^3 = 3^3 + \binom{3}{1}3^2\left(\sqrt{7}\right) + \binom{3}{2}3\left(\sqrt{7}\right)^2 + \left(\sqrt{7}\right)^3 \quad \text{(A1)}$$

$$= 27 + 27\sqrt{7} + 63 + 7\sqrt{7} \quad \text{(A2)}$$

$$\left(3+\sqrt{7}\right)^3 = 90 + 34\sqrt{7} \quad (\text{so } p = 90,\ q = 34) \quad \text{(A1)(A1)(C3)(C3)}$$

**10.** Find the coefficient of $x^3$ in the expansion of $(2 - x)^5$.

**(Total 6 marks)**

Term involving $x^3$ is $\binom{5}{3}(2)^2(-x)^3$ (A1)(A1)(A1)

$\binom{5}{3} = 10$ (A1)

Therefore, term $= -40x^3$ (A1)

$\Rightarrow$ The coefficient is $-40$ (A1) (C6)

**[6]**

**11.** Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.

(a) How many terms are there in this expansion?

(b) Find the constant term in this expansion.

**(Total 6 marks)**

(a) 10 (A2) (C2)

(b) $(3x^2)^3\left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6}3^3x^6\frac{1}{x^6}\left(\text{or } 84\times 3^3x^6\frac{1}{x^6}\right)$ (A1)

constant = 2268 (A1) (C4)

**[6]**

**12.** When the expression $(2 + ax)^{10}$ is expanded, the coefficient of the term in $x^3$ is 414 720. Find the value of $a$.

**(Total 6 marks)**

$\binom{10}{3}2^7(ax)^3$ $\left(\text{accept}\binom{10}{7}\right)$ (A1)(A1)(A1)

$\binom{10}{3} = 120$ (A1)

$120 \times 2^7 a^3 = 414\ 720$ (M1)

$a^3 = 27$

$a = 3$ (A1) (C6)

***Note:*** *Award (A1)(A1)(A0) for* $\binom{10}{3}2^7ax^3$. *If this leads to the answer* $a = 27$, *do not award the final (A1).*

**[6]**