

# chi squared *[54 marks]*

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

|                          | Salad meals per week |     |     |    |
|--------------------------|----------------------|-----|-----|----|
|                          | 0                    | 1–2 | 3–4 | >4 |
| Students                 | 45                   | 26  | 18  | 6  |
| Professors               | 15                   | 8   | 5   | 12 |
| Staff and Administration | 16                   | 13  | 10  | 6  |

Jing conducted a  $\chi^2$  test for independence at a 5 % level of significance.

1a. State the null hypothesis.

*[1 mark]*

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1b. Calculate the  $p$ -value for this test.

*[2 marks]*

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1c. State, giving a reason, whether the null hypothesis should be accepted. [2 marks]

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Manny and Annabelle, mathematics teachers at Burnham High School, give their students the same examination. A random sample of the examination scores were collected from each of their classes.

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|--|----|----|----|----|----|----|----|----|
| <b>Examination scores from Manny's class</b>     | 76 | 77 | 82 | 84 | 88 | 90 | 91 | 98 |
| <b>Examination scores from Annabelle's class</b> | 68 | 79 | 81 | 89 | 91 | 92 | 92 | 95 |

Annabelle uses these scores to conduct a two-tailed  $t$ -test to compare the means of the two classes, at the 5% level of significance. It is assumed the examination scores for both classes have the same variance and are normally distributed.

The null hypothesis is  $\mu_1 = \mu_2$ , where  $\mu_1$  is the mean examination score from Manny's class and  $\mu_2$  is the mean examination score from Annabelle's class.

2a. Write down the alternative hypothesis.

[1 mark]

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2b. Find the  $p$ -value for this test. Give your answer correct to five decimal places. [2 marks]

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Annabelle concludes there is insufficient evidence to reject the null hypothesis.

2c. State whether Annabelle’s conclusion is correct. Give a reason for your answer. [2 marks]

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A group of 1280 students were asked which electronic device they preferred. The results per age group are given in the following table.

| Preferred device | Age   |       |       | Total |
|------------------|-------|-------|-------|-------|
|                  | 11–13 | 14–16 | 17–18 |       |
| Laptop           | 143   | 160   | 153   | 456   |
| Tablet           | 205   | 224   | 131   | 560   |
| Mobile phone     | 72    | 128   | 64    | 264   |
| <b>Total</b>     | 420   | 512   | 348   | 1280  |

A student from the group is chosen at random. Calculate the probability that the student

3a. prefers a tablet.

*[2 marks]*

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3b. is 11–13 years old and prefers a mobile phone.

*[2 marks]*

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3c. prefers a laptop **given that** they are 17–18 years old.

[2 marks]

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3d. prefers a tablet or is 14–16 years old.

[3 marks]

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A  $\chi^2$  test for independence was performed on the collected data at the 1% significance level. The critical value for the test is 13.277.

3e. State the null and alternative hypotheses.

[1 mark]

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3f. Write down the number of degrees of freedom.

[1 mark]

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3g. Write down the  $\chi^2$  test statistic.

[2 marks]

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3h. Write down the  $p$ -value.

[1 mark]

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3i. State the conclusion for the test in context. Give a reason for your answer.

[2 marks]

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Casanova restaurant offers a set menu where a customer chooses **one** of the following meals: pasta, fish or shrimp.

The manager surveyed 150 customers and recorded the customer's age and chosen meal. The data is shown in the following table.

|          | Pasta | Fish | Shrimp | Total |
|----------|-------|------|--------|-------|
| Adults   | 24    | 25   | 32     | 81    |
| Children | 20    | 14   | 35     | 69    |
| Total    | 44    | 39   | 67     | 150   |

A  $\chi^2$  test was performed at the 10% significance level. The critical value for this test is 4.605.

4a. State  $H_0$ , the null hypothesis for this test. [1 mark]

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4b. Write down the number of degrees of freedom. [1 mark]

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4c. Show that the expected number of children who chose shrimp is 31, correct to two significant figures. [2 marks]

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Write down

4d. the  $\chi^2$  statistic.

[2 marks]

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4e. the  $p$ -value.

[1 mark]

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4f. State the conclusion for this test. Give a reason for your answer.

[2 marks]

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A customer is selected at random.

4g. Calculate the probability that the customer is an adult.

[2 marks]

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4h. Calculate the probability that the customer is an adult or that the customer chose shrimp.

[2 marks]

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4i. Given that the customer is a child, calculate the probability that they chose pasta or fish. [2 marks]

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A survey was conducted on a group of people. The first question asked how many pets they each own. The results are summarized in the following table.

|                             |    |    |    |    |    |   |
|-----------------------------|----|----|----|----|----|---|
| <b>Number of pets owned</b> | 0  | 1  | 2  | 3  | 4  | 5 |
| <b>Number of people</b>     | 20 | 45 | 40 | 30 | 20 | 5 |

5a. Write down the total number of people, from this group, who are **pet owners**. [1 mark]

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5b. Write down the modal number of pets. [1 mark]

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5c. For these data, write down the median number of pets.

[1 mark]

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5d. For these data, write down the lower quartile.

[1 mark]

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5e. For these data, write down the upper quartile.

[1 mark]

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The second question asked each member of the group to state their age and preferred pet. The data obtained is organized in the following table.

| Preferred pet | Age      |              |
|---------------|----------|--------------|
|               | Teenager | Non-teenager |
| cat           | 23       | 32           |
| dog           | 35       | 23           |
| bird          | 16       | 13           |
| other         | 11       | 7            |

5f. Write down the ratio of teenagers to non-teenagers in its simplest form.

[1 mark]

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A  $\chi^2$  test is carried out at the 10 % significance level.

5g. State the null hypothesis.

[1 mark]

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5h. State the alternative hypothesis.

[1 mark]

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5i. Write down the number of degrees of freedom for this test.

[1 mark]

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5j. Calculate the expected number of teenagers that prefer cats.

[2 marks]

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5k. State the conclusion for this test. Give a reason for your answer.

[2 marks]

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