Practice Set A: Paper 1 Mark scheme

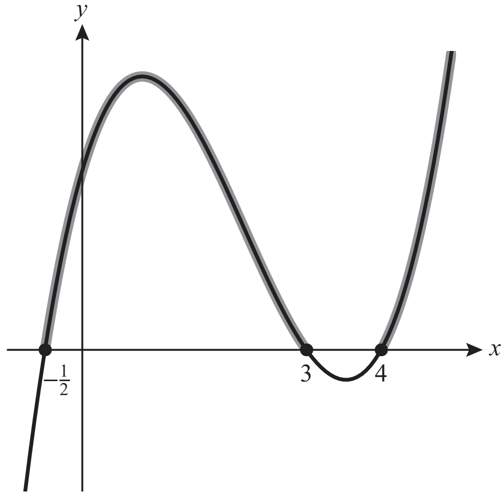
SECTION A

1	P(late) = 0.8 × 0.4 + 0.2 × 0.1 (= 0.34)	(M1)
	P(late and not coffee) = 0.2 × 0.1 (= 0.02)	(M1)
	P(not coffee|late)	M1
	$= \frac{0.02}{0.34}$	A1
	$= \frac{1}{17}$	A1
		[5 marks]
2	Substitute dx = du, 5x = 5(u + 3)	M1
	Change limits	M1
	Obtain $\int_0^4 5(u+3)\sqrt{u}\,\mathrm{d}u$	A1
	Expand the brackets before integrating: $\int_0^4 5u^{\frac{3}{2}} + 15u^{\frac{1}{2}}\,\mathrm{d}u$	M1
	$= \left[2u^{\frac{5}{2}} + 10u^{\frac{3}{2}}\right]_0^4$	A1
	$= 2 \times 2^5 + 10 \times 2^3$	(M1)
	= 144	A1
		[7 marks]
3	Write $z = x + \mathrm{i}y$	(M1)
	Then $3x + 3\mathrm{i}y - 4x + 4\mathrm{i}y = 18 + 21\mathrm{i}$	A1
	Compare real and imaginary parts	M1
	$z = -18 + 3\mathrm{i}$	A1
	$\left\lvert\frac{z}{3}\right\rvert = \sqrt{6^2 + 1^2}$	M1
	$= \sqrt{37}$	A1
		[6 marks]
4 a	EITHER	
	Use factor theorem:	
	$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 13\left(-\frac{1}{2}\right)^2 + 17\left(-\frac{1}{2}\right) + 12$	M1
	$= -\frac{1}{4} - \frac{13}{4} - \frac{34}{4} + \frac{48}{4}$	
	$= 0$	
	So $(2x + 1)$ is a factor	A1
	OR	
	Compare coefficients or long division:	
	$2x^3 - 13x^2 + 17x + 12 = (2x + 1)(x^2 - 7x + 12)$	M1A1

b $(2x+1)(x-3)(x-4)=0$

$x=-\frac{1}{2}, 3, 4$ (M1)

Sketch graph or consider sign of factors (M1)



$-\frac{1}{2}<x<3$ or $x>4$

Note: Award M1A0 for correct region from their roots M1A1

[6 marks]

5 $f \circ g(x)=\dfrac{2-\frac{2}{x-1}}{\frac{2}{x-1}+3}$ M1

$=\dfrac{2(x-1)-2}{2+3(x-1)}$ (M1)

$=\dfrac{2x-4}{3x-1}$ A1

$x=\dfrac{2y-4}{3y-1}$

$3xy-x=2y-4$ (M1)

$3xy-2y=x-4$ M1

$y=\dfrac{x-4}{3x-2}$ A1

[6 marks]

6 $7e^{2x}-45e^{x}=e^{3x}-7e^{2x}$ M1

$e^{3x}-14e^{2x}+45e^{3x}=0$ A1

$e^{x}(e^{x}-9)(e^{x}-5)=0$ M1A1

Reject $e^{x}=0$ R1

$x=\ln 5$ or $\ln 9$ A1

[6 marks]

7 Attempt to differentiate both top and bottom. M1

Top: $\sin x + x\cos x$ M1A1

Bottom: $\frac{1}{x}$ A1

$\lim_{x\to\pi}(x\sin x + x^2\cos x)$ M1

$=-\pi^2$ A1

[6 marks]

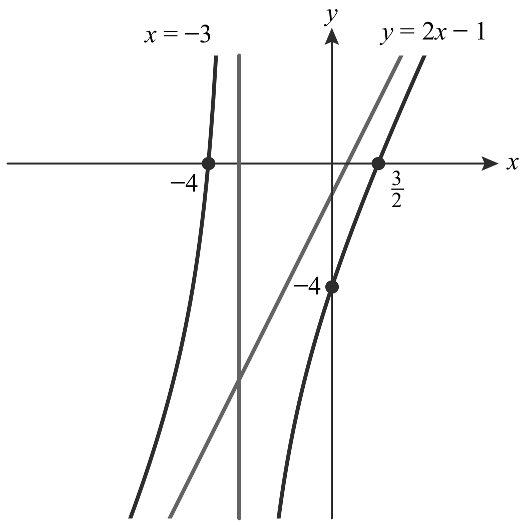
8 Factorize denominator to find x-intercepts: $(2x-3)(x+4)$ (M1)

Long division or compare coefficients:

$$\frac{2x^2+5x-12}{x+3} = \frac{(x+3)(2x-1)-9}{x+3}$$ M1

$$= 2x - 1 - \frac{9}{x+3}$$ A1

Correct shape A1



Axis intercepts: $\left(\frac{3}{2}, 0\right)$, $(-4, 0)$, $(0, -4)$ A1

Vertical asymptote: $x = -3$ A1

Oblique asymptote: $y = 2x - 1$ A1

[7 marks]

9 **a** Suppose that $\log_2 5$ is rational, and write $\log_2 5 = \frac{p}{q}$. M1

Then $2^{\frac{p}{q}} = 5$, so $2^p = 5^q$. M1

e.g. LHS is even and RHS is odd. A1

This is a contradiction, so $\log_2 5$ is irrational A1

b Any suitable example, e.g. $n = 16$ M1

Complete argument, e.g. $\log_2 16 = 4$, which is rational A1

[6 marks]

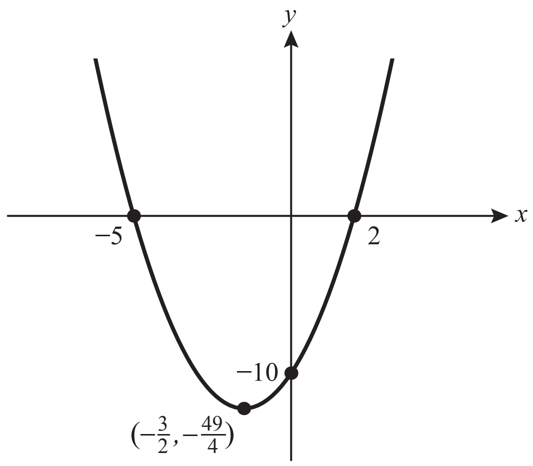
SECTION B

10 a Factorize to find x-intercepts: $(x + 5)(x - 2)$ (M1)

Complete the square for vertex (or half-ways between intercepts):

$\left(x+\frac{3}{2}\right)^2 - \frac{49}{4}$ (M1)

Correct shape and all intercepts A1



Correct vertex $\left(-\frac{3}{2}, -\frac{49}{4}\right)$ A1

[4 marks]

b i $x^2 + 3x - 10 = 2x - 20$

$\Leftrightarrow x^2 + x + 10 = 0$ M1

discriminant $= 1 - 40$ $(= -39)$ M1

< 0 so no intersections A1

ii $x^2 + x + (k - 10) = 0$ M1A1

$1 - 4(k - 10) > 0$ M1

$k < \frac{41}{4}$ A1

[7 marks]

c Compare to $\left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$ (M1)

Vertical translation $\frac{57}{4}$ units up A1

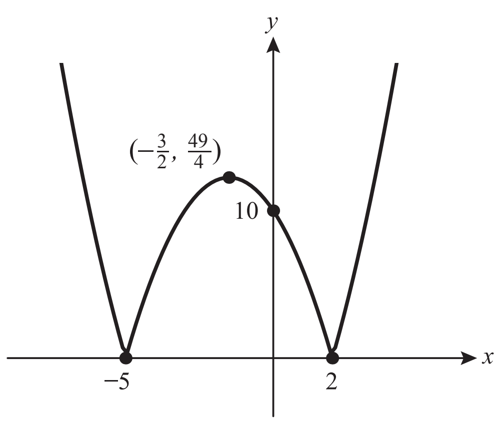
Horizontal stretch A1

Scale factor $\frac{1}{2}$ A1

[4 marks]

d i Correct shape A1

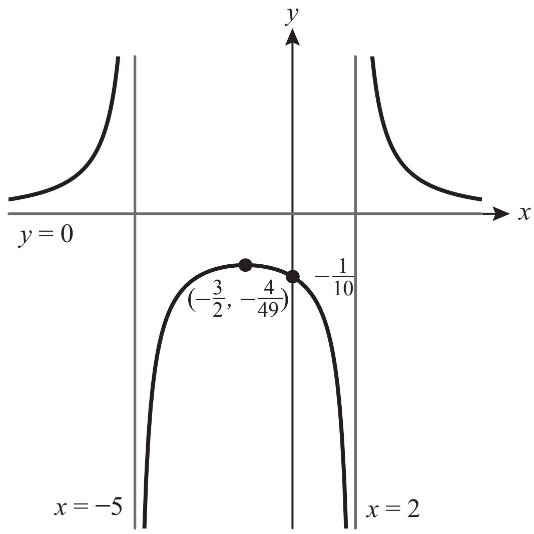
Correct intercepts and turning point labelled A1



ii Vertical asymptotes at x = −5, 2, y-int −0.1 — A1

Parts of curve in correct quadrants — M1

Turning point $\left(-\frac{3}{2}, -\frac{4}{49}\right)$ — A1



[5 marks]
Total [20 marks]

11 a $\frac{dy}{dx} = -e^{-x}\sin 2x + 2e^{-x}\cos 2x$ — M1A1

M1 for attempt at product rule or quotient rule

$-e^{-x}\sin 2x + 2e^{-x}\cos 2x = 0$

$-\sin 2x + 2\cos 2x = 0$ — M1

$\frac{\sin 2x}{\cos 2x} = 2$ — A1

$\tan 2x = 2$ — AG

[4 marks]

b $\frac{d^2y}{dx^2} = e^{-x}\sin 2x - 2e^{-x}\cos 2x - 2e^{-x}\cos 2x - 4e^{-x}\sin 2x$ — (M1)

M1 for attempt at product rule or quotient rule on their $\frac{dy}{dx}$

$= -3e^{-x}\sin 2x - 4e^{-x}\cos 2x$ — A1

$-3e^{-x}\sin 2x - 4e^{-x}\cos 2x = 0$ — M1

$\frac{\sin 2x}{\cos 2x} = -\frac{4}{3}$ — A1

$\tan 2x = -\frac{4}{3}$ — AG

[4 marks]

c x-intercept $= \frac{\pi}{2}$ — A1

$\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x - \int -2e^{-x}\cos 2x\,dx$ — (M1)

$= -e^{-x}\sin 2x + 2\int e^{-x}\cos 2x\,dx$ — A1

$= -e^{-x}\sin 2x + 2(-e^{-x}\cos 2x - \int 2e^{-x}\sin 2x\,dx)$

$= -e^{-x}\sin 2x - 2e^{-x}\cos 2x - 4\int e^{-x}\sin 2x\,dx$ — A1

$5\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x - 2e^{-x}\cos 2x$ — (M1)

$\int_0^{\frac{\pi}{2}} e^{-x}\sin 2x\,dx = \frac{1}{5}[-e^{-x}\sin 2x - 2e^{-x}\cos 2x]_0^{\frac{\pi}{2}}$

$= \frac{1}{5}\left(2e^{-\frac{\pi}{2}} + 2\right)$

$= \frac{2}{5} + \frac{2}{5}e^{-\frac{\pi}{2}}$ — A1A1A1

[8 marks]
Total [16 marks]

12	**a**	For any positive integer n, $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$	A1
		True when $n = 1$: $(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta$	A1
		Assume it is true for $n = k$:	
		$(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$	M1
		Then	
		$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	
		$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1
		The statement is true for $n = 1$ and if it is true for some $n = k$ then it is also true for $n = k + 1$; it is therefore true for all integers $n > 1$ [by the principle of mathematical induction].	R1
			[5 marks]
	b	[Writing $c = \cos\theta$, $s = \sin\theta$:]	
		$(\cos\theta + \mathrm{i}\sin\theta)^5 = c^5 + 5\mathrm{i}c^4s - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + \mathrm{i}s^5$	A1
		Equating real parts of $\cos 5\theta + \mathrm{i}\sin 5\theta$ and $(\cos\theta + \mathrm{i}\sin\theta)^5$:	
		$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$	M1
		Using $s^2 = 1 - c^2$	
		$\cos 5\theta = c^5 - 10c^3(1 - c^2) + 5c(1 - c^2)^2$	(M1)
		$= c^5 - 10c^3 + 10c^5 + 5c(1 - 2c^2 + c^4)$	A1
		$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	AG
			[4 marks]
	c	$5\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$	M1
		Obtain at least $\theta = \frac{\pi}{10}$	A1
			[2 marks]
	d	The roots of the equation are cos(values above)	(M1)
		Either $c = 0$, in which case $\theta = \frac{\pi}{2}\ldots$	A1
		… or $16c^4 - 20c^2 + 5 = 0$	A1
		$c^2 = \frac{5 \pm \sqrt{5}}{8}$	A1
		$\cos\left(\frac{\pi}{10}\right)$ is positive and the largest of the roots	R1
		So $\cos\left(\frac{\pi}{10}\right) = \sqrt{\frac{5+\sqrt{5}}{8}}$	A1
			[6 marks]
	e	[$\cos\left(\frac{7\pi}{10}\right)$ is negative and not equal to $-\cos\left(\frac{\pi}{10}\right)$]	
		$\cos\left(\frac{\pi}{10}\right)\cos\left(\frac{7\pi}{10}\right) = \left(\sqrt{\frac{5+\sqrt{5}}{8}}\right)\left(-\sqrt{\frac{5-\sqrt{5}}{8}}\right)$	M1
		$\left[= -\sqrt{\frac{25-5}{64}}\right] = -\frac{\sqrt{5}}{4}$	A1
			[2 marks]
			Total [19 marks]

Practice Set A: Paper 2 Mark scheme

SECTION A

1 **a** $= \frac{4}{3}\pi(3^3) \times 1.45$ (M1)

$= 164\,\text{g}$ A1

b Each volume [mass] is $\frac{1}{8}$ the previous one. A1

Sum to infinity $= \frac{164}{1-\frac{1}{8}} = 187\,\text{g}$ M1A1

Hence the mass is always smaller than 200 g. A1

[6 marks]

2 $\text{E}(X) = \int_{2\pi}^{3\pi} 0.4106x \sin x\sqrt{x-2\pi}\,\text{d}x[= 8.018]$ M1

A1 for correct limits A1

$\text{E}(X^2) = \int_{2\pi}^{3\pi} 0.4106x^2 \sin x\sqrt{x-2\pi}\,\text{d}x[= 64.71]$ M1

$\text{Var}(X) = 64.71 - 8.018^2$ M1

$= 0.425$ (A1)

$\sqrt{0.425} = 0.652$ (A1)

[6 marks]

3 Attempt sine rule:

$\frac{\sin\theta}{x-1} = \frac{\sin 2\theta}{x+2}$ A1

Use double angle formula:

$= \frac{2\sin\theta\cos\theta}{x+2}$ M1

$2\cos\theta(x-1) = x+2$ A1

Rearrange for x:

$x(2\text{ Cos}\theta - 1) = 2 + 2\cos x$ M1

$x = \frac{2(1+\cos\theta)}{2\cos\theta - 1}$ A1

Use $\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$:

$= \frac{4\cos^2\left(\frac{\theta}{2}\right)}{4\cos^2\left(\frac{\theta}{2}\right) - 3}$ M1

Divide by $\cos^2\left(\frac{\theta}{2}\right)$, clearly using $= \frac{1}{\cos\left(\frac{\theta}{2}\right)} = \sec\left(\frac{\theta}{2}\right)$:

$= \frac{4}{4 - 3\sec^2\left(\frac{\theta}{2}\right)}$ A1AG

[7 marks]

4 **a** A A1

Gradient is zero and changing from positive to negative R1

b B, D A1

and E A1

Second derivative is zero and changes sign R1

[5 marks]

5 $(4-x)^{-\frac{1}{2}}$ M1

$= 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1

$\approx \frac{1}{2}(1+\frac{x}{8}+\ldots$ A1

$\ldots+\frac{3}{8}\left(-\frac{x}{4}\right)^2-\frac{5}{16}\left(-\frac{x}{4}\right)^3)$ M1

$=\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\frac{5x^3}{2048}$ A1

Valid for $|x|<4$ A1

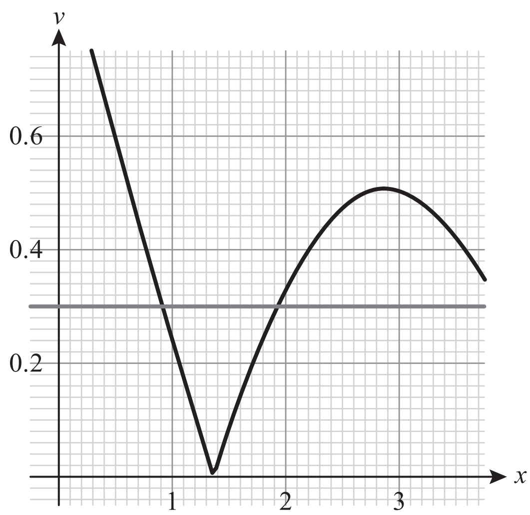
[6 marks]

6 a integrate $|v|$ (M1)

With limits 0 and 5 (M1)

Distance = 1.8 m A1

b Sketch $\left|\frac{dv}{dt}\right|$ [or $\frac{dv}{dt}$] (M1)



Intersect with $y = 0.3$ [or with both 0.3 and -0.3] (M1)

$t = 0.902$ and 1.93 seconds A1

[6 marks]

7 Consider f(1):

$1^2 - 1 = -(1)^2 + b(1) + c$ M1

$\Rightarrow b + c = +1$ A1

Consider f′ (1):

$+2(1) = -2(1) + b$ M1A1

$\Rightarrow b = 4$ A1

$c = -3$ A1

[6 marks]

8 $|\mathbf{a}||\mathbf{b}|\cos\theta = 17$ M1

$|\mathbf{a}||\mathbf{b}|\sin\theta = \sqrt{4+1+25}$ [$=\sqrt{30}$] M1

$\tan\theta = \frac{\sqrt{30}}{17}$ M1A1

$\theta = 17.9°$ A1

[5 marks]

9	**a**		8! seen	A1
			8!2!2! = 161280	(M1)A1
	b		Pair 1 stands together: 9!2! [= 725760]	M1
			2 × 725760 –(their **a**) [=1 290 240]	M1
			$\frac{10! - 1\,290\,240}{10!}$	M1
			= 0.644 (3 s.f.)	A1
				[7 marks]

SECTION B

10	**a**		Paper 1: mean = 78.9, SD = 17.4	A1
			Paper 2: mean = 74.0, SD = 15.1	A1
			Paper 1 has higher marks on average.	A1
			Paper 2 has more consistent marks.	A1
				[4 marks]
	b		$r = 0.868$	A1
			> 0.532	M1
			There is evidence of positive correlation between the two sets of marks.	A1
				[3 marks]
	c	**i**	Find regression line x on y	M1
			$x = 0.997y + 5.16$	A1
			$0.997 \times 86 + 5.16 \approx 91$ marks	A1
		ii	Can't be used.	A1
			Mark is outside of the range of available data (interpolation)	R1
				[5 marks]
	d	**i**	Boundary for 7: inverse normal of 0.88	M1
			Boundary = 81	A1
			5 students	A1
		ii	Use $B(12, 0.12)$	(M1)
			$P(>5) = 1 - P(\le 5)$	(M1)
			= 0.001 44	A1
				[6 marks]
	e		Scaled mark = $\frac{80}{110}$ × original mark	(M1)
			Mean = 57.4	A1
			SD = 12.7	A1
				[3 marks]
				Total [21 marks]

11	**a**		Separate variables and attempt integration	M1
			$\int \frac{dy}{y} = \int \tan x \, dx$	A1
			$\ln y = -\ln \lvert\cos x\rvert + c$	A1
			$y = Ae^{-\ln(\cos x)}$	M1
			$= \frac{A}{\cos x}$	A1
				[5 marks]
	b	**i**	$\int -\tan x \, dx = \int \frac{-\sin x}{\cos x} dx = \ln(\cos x)$	M1A1
			$I = e^{\ln(\cos x)} = \cos x$	M1(AG)
		ii	$y \cos x = \int \cos^2 x \, dx$	M1
			$= \int \frac{\cos 2x + 1}{2} dx$	M1
			$= \frac{1}{4}\sin 2x + \frac{1}{2}x + c$	A1
			$y = \frac{\sin 2x}{4\cos x} + \frac{x}{2\cos x} + \frac{c}{\cos x} \quad \left(= \frac{\sin x}{2} + \frac{x \sec x}{2} + c \sec x\right)$	A1
				[7 marks]

c Use $y_{n+1} = y_n + 0.1(y_n^2 \tan x_n + \cos x_n)$ M1A1

Table of values – at least the first two rows correct M1

x	y'	y
0	1.000	2.000
0.1	1.437	2.100
0.2	2.001	2.244
0.3	2.803	2.444
0.4	4.058	2.724
0.5	6.229	3.130

$y(0.5) = 3.13$ A1

[4 marks]

Total [16 marks]

12 a i Equate x, y, z components:

$$\begin{cases} 5+7\lambda = 1-\mu \,(1) \\ 3+2\lambda = -8+3\mu \,(2) \\ 1-3\lambda = -2+2\mu \,(3) \end{cases}$$ M1A1

From, e.g. (1) and (2): $\lambda = -1,\ \mu = 3$ A1A1

Check in (3):

$1 - 3(-1) = 4$

$-2 + 2(3) = 4$

So lines intersect. M1AG

ii Substitute their values of λ and μ into either equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$ (M1)

So coordinates $(-2,\ 1,\ 4)$ A1

[7 marks]

b Attempt to find cross product of direction vectors:

$$\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$ (M1)

$$= \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix}$$ A1

[2 marks]

c $\mathbf{r} \cdot$ their $\mathbf{n}$ = their $\mathbf{p} \cdot$ their $\mathbf{n}$

$$\mathbf{r} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix}$$ (M1)

$$\mathbf{r} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix} = 55$$ A1

[2 marks]

d $\overrightarrow{QP} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} -11 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 3 \end{pmatrix}$ (M1)

$\cos\phi = \dfrac{\begin{pmatrix} 9 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix}}{\sqrt{9^2+1^2+3^2}\sqrt{13^2+11^2+23^2}}$ M1A1

$= \dfrac{25}{39}$ A1

$\sin\theta = \sin(90-\phi) = \cos\phi$ M1

So, $\sin\theta = \dfrac{25}{39}$ A1

[6 marks]

e $d = \left|\overrightarrow{QP}\right| \sin\theta$ (M1)

$= \dfrac{25\sqrt{91}}{39}$ A1

[2 marks]

Total [19 marks]

Practice Set A: Paper 3 Mark scheme

1 **a** $F_3 = 2$ A1
$F_4 = 3$ A1
$F_5 = 5$ A1
[3 marks]

b $F_{12} = 144$ A1
This is another Fibonacci number which is a perfect square R1
[2 marks]

c Check that the statement is true for $n = 1$: M1
$LHS = 1^2 = 1$ $RHS = 1 \times 1 = 1$ A1
Assume true for $n = k$
$\sum_{i=1}^{i=k} (F_i)^2 = F_k F_{k+1}$ A1
Then
$\sum_{i=1}^{i=k+1} (F_i)^2 = \sum_{i=1}^{i=k} (F_i)^2 + (F_{k+1})^2$ M1
$= F_k F_{k+1} + (F_{k+1})^2$
$= F_{k+1}(F_k + F_{k+1})$
$= F_{k+1} F_{k+2}$ A1
So if the statement works for $n = k$ then it works for $n = k + 1$ and it works for $n = 1$ therefore it works for all $n \in \mathbb{Z}^+$. R1
[6 marks]

d Smallest such k is 5 A1
Check that the statement is true for $n = 5$ and $n = 6$: M1
$F_5 = 5, F_6 = 8$ A1
Assume true for $n = k$ and $n = k + 1$ M1
$F_k \geqslant k, F_{k+1} \geqslant k + 1$ A1
Then
$F_{k+2} = F_k + F_{k+1} \geqslant 2k + 1 > k + 2$ since $k > 1$ A1
So if the statement works for $n = k$ and $n = k + 1$ then it works for $n = k + 2$ and it works for $n = 5$ and $n = 6$ therefore it works for all integers $n \geqslant 5$ R1
[7 marks]

e $\alpha^{n+2} = \alpha^{n+1} + \alpha^n$ M1
Dividing by α^n since $\alpha \neq 0$: $\alpha^2 = \alpha + 1$ or $\alpha^2 - \alpha - 1 = 0$ A1
Using the quadratic formula $\alpha = \dfrac{1 \pm \sqrt{5}}{2}$ A1A1
[4 marks]

f $F_n + F_{n+1} = A\alpha_1^n + B\alpha_2^n + A\alpha_1^{n+1} + B\alpha_2^{n+1}$ M1
$A(\alpha_1^n + \alpha_1^{n+1}) + B(\alpha_2^n + \alpha_2^{n+1})$
$A\alpha_1^{n+2} + B\alpha_2^{n+2} = F_{n+2}$ A1
[2 marks]

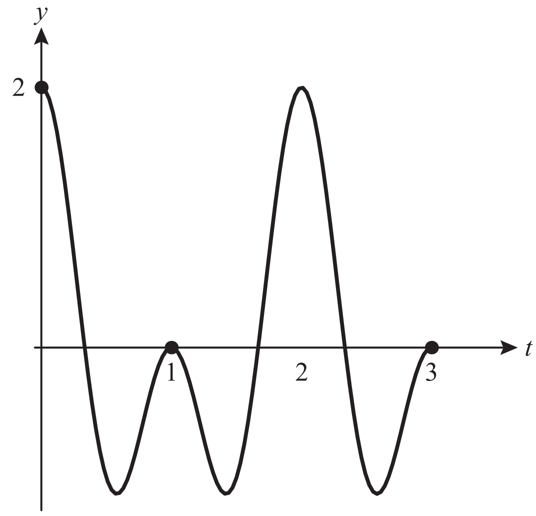
g $F_1 = A\alpha_1 + B\alpha_2 = 1$ A1
$F_2 = A\alpha_1^2 + B\alpha_2^2 = 1$ A1
Since $\alpha^2 = \alpha + 1$:
$A(\alpha_1 + 1) + B(\alpha_2 + 1) = 1$ M1
$A\alpha_1 + B\alpha_2 + A + B = 1$
$A + B = 0$
$A = -B$
Substituting into first equation: $A(\alpha_1 - \alpha_2) = 1$
$A = \dfrac{1}{\alpha_1 - \alpha_2} = \dfrac{1}{\sqrt{5}}$
$F_n = \dfrac{1}{\sqrt{5}}\left(\left(\dfrac{1+\sqrt{5}}{2}\right)^n - \left(\dfrac{1-\sqrt{5}}{2}\right)^n\right)$ A1
[4 marks]

h As n gets large, $\left(\dfrac{1-\sqrt{5}}{2}\right)^n \to 0$ M1
$\dfrac{F_{n+1}}{F_n} \approx \dfrac{\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}\right)}{\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n}\right)} = \dfrac{1+\sqrt{5}}{2}$ A1
[2 marks]
Total [30 marks]

2 **a** 2 — A1

[1 mark]

b **i**



A1

ii 2 — A1

[2 marks]

c **i** $A = 4$ — A1

$B = 8$ — A1

$C = 20$ — A1

ii $T = 2n$ — A1

[4 marks]

d $f(t + 2n) = \cos\left(\pi\left(t + 2n\right)\right) + \cos\left(\pi\left(1 + \frac{1}{n}\right)\left(t + 2n\right)\right)$ — M1

$= \cos\left(\pi t + 2n\pi\right) + \cos\left(\left(1 + \frac{1}{n}\right)\pi t + 2\pi\left(n + 1\right)\right)$ — A1

$= \cos\left(\pi t\right) + \cos\left(\left(1 + \frac{1}{n}\right)\pi t\right) = f\left(t\right)$

Since $\cos(x + 2\pi k) = \cos x$ if k is an integer — R1

[3 marks]

e **i** $\cos(A + B) + \cos(A - B)$

$= \cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B$

$= 2 \cos A \cos B$ — A1

ii If $P = A + B$ and $Q = A - B$ then

$A = \frac{P + Q}{2}, B = \frac{P - Q}{2}$ — M1

$\cos P + \cos Q = 2 \cos\left(\frac{P + Q}{2}\right) \cos\left(\frac{P - Q}{2}\right)$ — A1

[3 marks]

f $f(t) = 2 \cos\left(\pi\left(1 + \frac{1}{2n}\right)t\right) \cos\left(\frac{\pi}{2n} t\right)$ — A1

The graph of $\cos\left(\pi\left(1 + \frac{1}{2n}\right)t\right)$ provides the high frequency oscillations. — R1

Their amplitude is determined/enveloped by the lower frequency curve $\cos\left(\frac{\pi}{2n} t\right)$ — R1

[3 marks]

g $\frac{d^2 x}{dt^2} = -\omega^2 \cos \omega t$ — M1A1

The DE becomes:

$-\omega^2 \cos \omega t + 4 \cos \omega t = 0$ — M1

This is solved when $\omega^2 = 4$ so $\omega = 2$ — A1

[4 marks]

h $\frac{d^2x}{dt^2} = -4\cos 2t - k^2 g(k)\cos kt$ M1

The DE becomes:

$-4\cos 2t - k^2 g(k)\cos kt + 4\cos 2t + 4g(k)\cos kt = \cos kt$ M1

$(4g(k) - k^2 g(k))\cos kt = \cos kt$

This is true for all t when $g(k)(4 - k^2) = 1$

$g(k) = \frac{1}{4-k^2}$ A1

[3 marks]

i When $k = 2$ A1

Since $\frac{1}{4-k^2} \to \infty$ as $k \to 2$ R1

[2 marks]

Total [25 marks]

Practice Set B: Paper 1 Mark scheme

SECTION A

1	$k\ln(x^2+3)$	M1
	$2\ln(x^2+3)$	A1
	Limits: $2\ln(a^2+3) - 2\ln 3$	M1
	$2\ln\left(\frac{a^2+3}{3}\right) = \ln 16$ or $2\ln(a^2+3) = \ln(16 \times 9)$	A1
	$\left(\frac{a^2+3}{3}\right)^2 = 16$ or $(a^2+3)^2 = 16 \times 9$	M1
	$\frac{a^2+3}{3} = 4$ or $a^2+3 = 12$ only	A1
	$a = 3$	A1
		[7 marks]
2	**a** $\frac{1}{4}$ or 10 seen	A1
	$\frac{10}{40} \times \frac{9}{39}$	(M1)
	$= \frac{3}{52}$	A1
	b $\frac{10}{40} \times \frac{20}{39}$	(M1)
	$\times 2$	(M1)
	$= \frac{10}{39}$	A1
		[6 marks]
3	Attempt quotient rule:	
	$\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$	M1A1
	$= \frac{-\pi - 0}{\pi^2} = -\frac{1}{\pi}$	A1
	$y = 0$	A1
	$y = \pi(x - \pi)$	A1
		[5 marks]
4	Sketch both graphs or consider four cases	M1
	Attempts to find intersection points:	M1
	$x - 3 = 2x + 1 \Rightarrow x = -4$	A1
	$-x + 3 = 2x + 1 \Rightarrow x = \frac{2}{3}$	A1
	$x \leqslant -4$ or $x \geqslant \frac{2}{3}$	A1
		[5 marks]
5	$P(B\|A) = \frac{P(A \cap B)}{P(A)} : 0.6 = \frac{P(A \cap B)}{0.3}$	(M1)
	$P(A \cap B) = 0.18$	A1
	$P(A \cup B) = P(A) + P(B) - P(A \cap B) : 0.8 = 0.3 + P(B) - 0.18$	M1
	$P(B) = 0.68$	A1
	$P(B\|A) = \frac{P(A \cap B)}{P(B)}\left[= \frac{0.18}{0.68}\right]$	M1ft
	$= \frac{9}{34}$	A1
		[6 marks]

6 $A = -1$ A1
$x = 0$: $A + B = 8$ M1
$\Rightarrow B = 9$ A1
$-1 + 9e^{-2k} = 0 \Rightarrow e^{-2k} = \frac{1}{9}$ M1
Attempt taking logarithm of both sides, e.g. $2k = -\ln\left(\frac{1}{9}\right)$ M1
$k = \ln 3$ A1
[6 marks]

7 $7^1 + 3^0 = 8$, so true for $n = 1$ A1
Assume that, for some k, $7^k + 3^{k-1} = 4A$ M1
Then
$7^{k+1} + 3^k = 7 \times 7^k + 3 \times (4A - 7^k)$ M1
$= 4 \times 7^k + 12A$ A1
So $7^{k+1} + 3^k$ is divisible by 4 A1
The statement is true for $n = 1$, and if it is true for some $n = k$ then it is also true for $n = k + 1$. Therefore it is true for all integers $n \geqslant 1$ [by the principle of mathematical induction]. A1
[6 marks]

8 $\left|4 - 4\sqrt{3}\,\mathrm{i}\right| = \sqrt{16 + 48} = 8$ M1
$\Rightarrow |z| = 2$ A1
$\arg(4 - 4\sqrt{3}\,\mathrm{i}) = \arctan\left(-\sqrt{3}\right)$ M1
$= -\frac{\pi}{3}\left[\text{or } \frac{5\pi}{3}\right]$ A1
$\Rightarrow \arg z = -\frac{\pi}{9}, \ldots$ M1
$\ldots \frac{5\pi}{9}$ or $\frac{11\pi}{9}$ A1
$\therefore z = 2e^{-\frac{\pi}{9}\mathrm{i}}, 2e^{\frac{5\pi}{9}\mathrm{i}}, 2e^{\frac{11\pi}{9}\mathrm{i}}, \left[\text{or } 2e^{\frac{5\pi}{9}\mathrm{i}}, 2e^{\frac{11\pi}{9}\mathrm{i}}, 2e^{\frac{17\pi}{9}\mathrm{i}}\right]$ A1
[7 marks]

9 $\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$ A1
$\left(1 - x^2\right)^{-\frac{1}{2}}$ M1
$\approx 1 + \frac{x^2}{2}$ A1
$+\frac{3x^4}{8}$ A1
Multiply the two expansions, using at least two terms in each one M1
Attempt to simplify, obtain at least $1 + 0x^2$ A1
$1 + \frac{x^4}{6}$ A1
[7 marks]

SECTION B

10 a Use product rule
$f'(x) = e^{-kx} + x(-k)e^{-kx}$ M1
$= (1 - kx)e^{-kx}$ A1AG
Use product rule again, $u' = -k$, $v' = -ke^{-kx}$ M1
$f''(x) = (-k)e^{-kx} + (1 - kx)(-k)e^{-kx}$ A1
$= (k^2x - 2k)e^{-kx}$ A1
[5 marks]

b $f'(x) = 0: (1 - kx)e^{-kx} = 0$ M1

$e^{-kx} \neq 0$ A1

$x = \frac{1}{k}$ A1

$f''\left(\frac{1}{k}\right) = \left(\frac{k^2}{k} - 2k\right)e^{-\frac{k}{k}}$ M1

$= -ke^{-1} < 0 \therefore$ local maximum A1

[5 marks]

c $f''(x) = 0: k^2x - 2k = 0$ M1

$x = \frac{2}{k}$ A1

The coordinates are

$\left(\frac{2}{k}, \frac{2}{k}e^{-2}\right)$ A1

[3 marks]

Integration by parts:

$\int_{\frac{1}{k}}^{\frac{2}{k}} x e^{-kx}\,dx = \left[-\frac{x}{k}e^{-kx}\right]_{\frac{1}{k}}^{\frac{2}{k}} + \int_{\frac{1}{k}}^{\frac{2}{k}} \frac{1}{k}e^{-kx}\,dx$ M1A1

$= \left[-\frac{2}{k^2}e^{-2} + \frac{1}{k^2}e^{-1}\right] - \left[\frac{1}{k^2}e^{-kx}\right]_{\frac{1}{k}}^{\frac{2}{k}}$ A1

$= \frac{2}{k^2}e^{-1} - \frac{3}{k^2}e^{-2}$ A1

$= \frac{2}{k^2e} - \frac{3}{k^2e^2}$ A1

$= \frac{2e - 3}{k^2e^2}$ AG

[5 marks]

Total [18 marks]

11 a Eliminate a variable between two equations, e.g. x between equations (2) and (3):

$$\begin{cases} 6x + ky + 2z = a \\ 6x - y - z = 7 \\ 2y - z = 1 \end{cases}$$ M1

Eliminate the same variable between another pair of equations, e.g. x between (1) and (2):

$$\begin{cases} 6x + ky + 2z = a \\ (k+1)y + 3z = a - 7 \\ 2y - z = 1 \end{cases}$$ M1

Eliminate a variable between the pair of equations in two variables, e.g. z between (2) and (3):

$$\begin{cases} 6x + ky + 2z = a \\ (k+1)y + 3z = a - 7 \\ (k+7)y = a - 4 \end{cases}$$ M1

Leading to:

$(2k + 14)x = a + 10 + 2k$

OR

$(k + 7)y = a - 4$

OR

$(k + 7)z = 2a - 15 - k$ A1

Their coefficient of $x/y/z = 0$ (M1)

$k = -7$ A1

[6 marks]

b **i** Their RHS = 0 (with their value of k) (M1)

$a = 4$ A1

ii Let $z = \lambda$ M1

$2y - z = 1$ (M1)

$6x - y - z = 7$ (M1)

At least one of

$y = \frac{1+\lambda}{2}$, $x = \frac{5+\lambda}{4}$ A1ft

$$\mathbf{r} = \begin{pmatrix} \frac{5}{4} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$ A1

[7 marks]

c Normal vectors to each plane are

$$\begin{pmatrix} 6 \\ -7 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

Since none of these are multiples of each other, no two planes are parallel M1

So the planes form a triangular prism A1

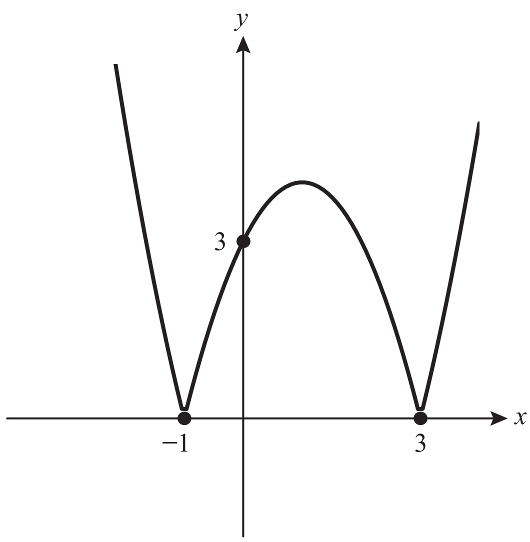
[2 marks]

Total [15 marks]

12 **a** Factorize to find x-intercepts: $(x - 3)(x + 1)$ M1

$(-1, 0)$ and $(3, 0)$ A1

Correct shape – reflected above x-axis A1



[3 marks]

b Solve $f(x) = -\frac{1}{2}x + 4$

$x^2 - 2x - 3 = -\frac{1}{2}x + 4$ (M1)

$2x^2 - 3x - 14 = 0$

$(2x - 7)(x + 2) = 0$

$x = \frac{7}{2}, -2$ A1

Solve $-f(x) = -\frac{1}{2}x + 4$

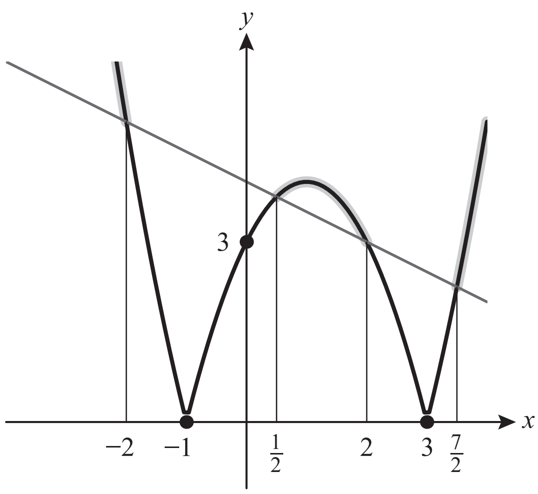
$-(x^2 - 2x - 3) = -\frac{1}{2}x + 4$ (M1)

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$ A1

Sketch of $y = |f(x)|$ and $y = -\frac{1}{2}x + 4$



$x < -2$ or $\frac{1}{2} < x < 2$ or $x > \frac{7}{2}$

Note: Award A1 for two correct regions A1A1

[6 marks]

c $x \in \mathbb{R}, x \neq -1, x \neq 3$ A1

[1 mark]

d $g'(x) = \dfrac{2(x^2 - 2x - 3) - (2x - 7)(2x - 2)}{(x^2 - 2x - 3)^2}$

Note: Award M1 for attempt at quotient rule M1A1

For turning points, $g'(x) = 0$:

$2(x^2 - 2x - 3) - (2x - 7)(2x - 2) = 0$ M1

$x^2 - 7x + 10 = 0$

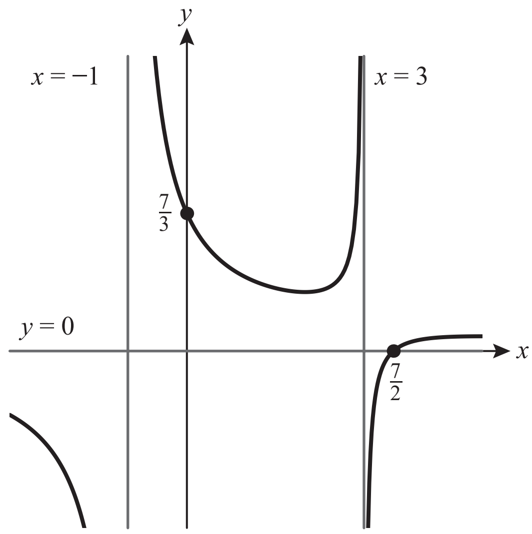
$(x - 2)(x - 5) = 0$

$x = 2, 5$ A1

So, coordinates (2, 1) and $\left(5, \frac{1}{4}\right)$ A1

[5 marks]

e



Correct shape between vertical asymptotes A1
Correct shape outside vertical asymptotes A1
Vertical asymptotes: $x = -1, x = 3$ A1
Horizontal asymptote: $y = 0$ A1
Axis intercepts at $\left(\frac{7}{2}, 0\right)$ and $\left(0, \frac{7}{3}\right)$ A1

[5 marks]

f $g(x) \in \left(-\infty, \frac{1}{4}\right] \cup [1, \infty)$ A2

[2 marks]

Total [22 marks]

Practice Set B: Paper 2 Mark scheme

SECTION A

1	$a + 4d = 7,\ a + 9d = 81$	M1A1
	Solving:	
	$a = -52.2,\ d = 14.8$	A1
	$S_{20} = \frac{20}{2}(-104.4 + 19 \times 14.8)$	(M1)
	$= 1768$	A1
		[5 marks]
2	Find the diagonal of the square base: $\sqrt{8.3^2 + 8.3^2}$	M1
	Height $= \frac{\sqrt{8.3^2 + 8.3^2}}{2}\tan(89.8°)$	M1
	$= 1681$	A1
	$= 1.7 \times 10^3$ cm	A1
		[4 marks]
3	mean = 131.9, SD = 7.41	A1
	Boundaries for outliers: mean ± SD	(M1)
	= 117.1, 146.7	A1A1ft
	147 is an outlier	A1
		[5 marks]
4	At least one correct use of compound angle formula	M1
	Correct values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ used	A1
	LHS $\equiv \dfrac{\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right) - \left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right)}{\left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right) - \left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)}$	A1
	$\equiv \dfrac{\sqrt{3}\cos x}{-\sqrt{3}\sin x}$	A1
	$\equiv -\cot x$	A1(AG)
		[5 marks]
5	$\frac{2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$	M1
	$2 = A(x-1) + Bx$	M1
	Using $x = 0$: $A = -2$	
	Using $x = 1$: $B = 2$ (both correct)	A1
	$\int -\frac{2}{x} + \frac{2}{x-1}\,dx = -2\ln x + 2\ln(x-1) + c$	M1A1
	$= \ln\left(\frac{x-1}{x}\right)^2 + c$	A1
		[6 marks]
6	gradient = 3.024	(M1)
	normal gradient $= -\frac{1}{\text{their gradient}}$ [−0.3307]	(M1)
	y-coordinate = 3.392	A1
	Equation of normal: $y - 3.392 = -0.3307(x - 1.5)$	A1
	A: $y = 0$, B: $x = 0$ [$x_A = 11.76$, $y_B = 3.888$]	(M1)
	Area = 22.9	A1
		[6 marks]
7	Differentiate implicitly: at least one term containing y correct	M1
	$6x + 2xy' + 2y - 2yy' = 0$	A1
	$y' = 0 \Rightarrow y = -3x$	M1
	Substitutes their expression for x or y back into curve:	
	$3x^2 + 2x(-3x) - (-3x)^2 + 24 = 0$	M1A1
	$12x^2 = 24 \Rightarrow x = \pm\sqrt{2}$	A1
	$(\sqrt{2}, -3\sqrt{2}), (-\sqrt{2}, 3\sqrt{2})$	A1
		[7 marks]

8	**a**	Limits $\sqrt[3]{5}, \sqrt[3]{17}$ (seen in either part)	A1
		$x = \sqrt{y^3 - 1}$	A1
		$\int \sqrt{y^3 - 1}\, dy$	M1
		$= 2.57$	A1
	b	Using x^2	M1
		$\int \pi (y^3 - 1)\, dy$	M1
		$= 24.9$	A1
			[7 marks]
9		Another root is $2 + i$	A1
		Consider sum of roots:	
		$(2 + i) + (2 - i) + x_3 = 7$ (allow -7)	M1
		$x_3 = 3$	A1
		Product of roots: $3(2 + i)(2 - i)$	M1
		$c = -15$	A1
			[5 marks]
10		The rth term is	
		$nC_r x^{2r} \left(\frac{1}{x}\right)^{n-r}$	(M1)
		For constant term: $2r - (n - r) = 0$	(M1)
		$n = 3r$	A1
		So need $(3r)C_r = 495$	(M1)
		Using GDC: $r = 4$ so $n = 12$	A1
			[5 marks]

SECTION B

11	**a**	**i** $\frac{72 - \mu}{\sigma} = 0.8416$	M1
		$72 - \mu = 0.8416\sigma$	A1
		$\mu + 0.8416\sigma = 72$	(AG)
		$\frac{24 - \mu}{\sigma} = \ldots$	M1
		$\ldots -1.645$	A1
		$\mu - 1.645\sigma = 24$	A1
		ii (From GDC) $\mu = 55.8$, $\sigma = 19.3$	A1
		P(>48) = 0.657	A1
			[7 marks]
	b	Use inverse normal with $p = 0.25$ or $p = 0.75$	
		($Q_1 = 42.8$ or $Q_3 = 68.8$)	M1
		IQR = 26 (hours)	A1
			[2 marks]
	c	Use $B(20, 0.656)$	(M1)
		$1 - P(\leqslant 9)$	(M1)
		$= 0.953$	A1
			[3 marks]
	d	$\frac{P(>72)}{P(>48)}$	(M1)
		$= 0.305$	A1
			[2 marks]
	e	P(keep phone) = $1 - (0.05 \times 0.9 + 0.75 \times 0.2)$	(M1M1)
		$\frac{0.2}{\text{P(keep phone)}}$	M1
		$= 0.248$	A1
			[4 marks]
			Total [18 marks]
12	**a**	$\cos\theta = \frac{2^2 + 4^2 - 4^2}{2 \times 2 \times 4}$ [= 0.25]	(M1)
		$\sin\theta = \sqrt{\frac{15}{16}}$ [= 0.968]	M1
		Area $= \frac{1}{2}(2 \times 4) \times$ their $\sin\theta$	M1
		$= 3.87$ [cm^2]	A1
			[4 marks]

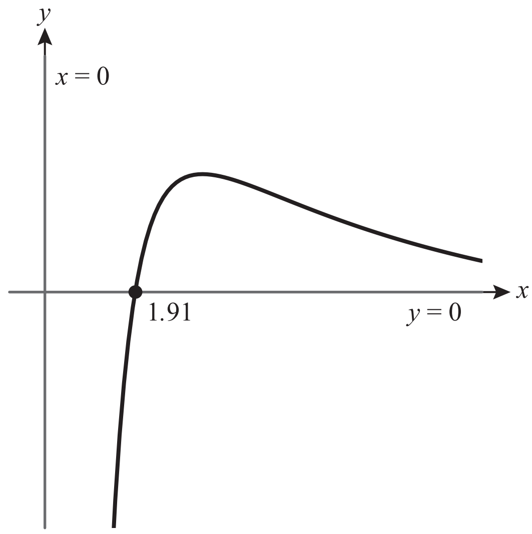
b The third side is $10 - 3x$ … M1
… which must be positive. A1
[2 marks]

c i $(10 - 3x)^2 = x^2 + (2x)^2 - 2x(2x)\cos\theta$ M1

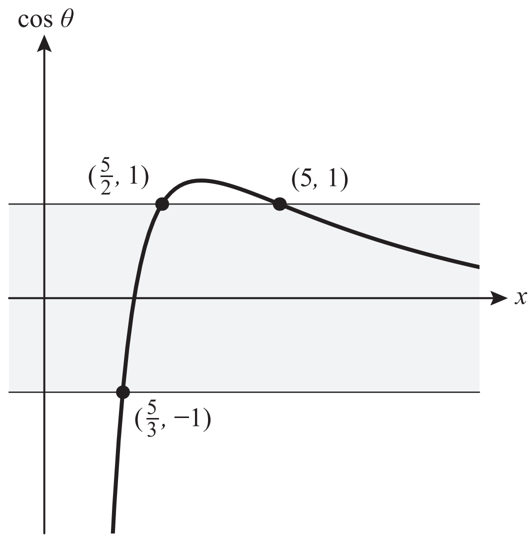
$\cos\theta = \dfrac{60x - 4x^2 - 100}{4x^2}$

$= \dfrac{15x - x^2 - 15}{x^2}$ A1(AG)

ii A2



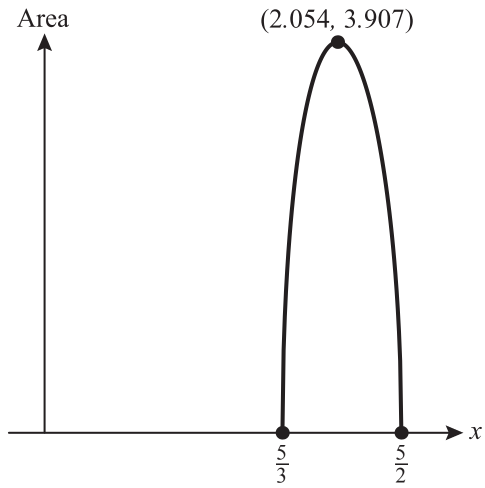
iii Need $-1 < \cos\theta < 1$ (allow ⩽ here) M1



Intersections at $x = \frac{5}{3}, \frac{5}{2}, 5$ A1

So $\frac{5}{3} < x < \frac{5}{2}$ A1
[7 marks]

d State or use $\sin\theta = \sqrt{1 - \cos^2\theta}$ M1
State or use Area $= \frac{1}{2}x(2x)\sin\theta$ M1
Sketch area as a function of x: M1



Max area for $x = 2.05$ A1

Max area = 3.91 [cm^2] A1

[5 marks]

Total [18 marks]

13 a Use quotient rule M1

Use implicit differentiation M1

$$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}(x+y) - y\left(1 + \frac{dy}{dx}\right)}{(x+y)^2}$$ A1

$$= \frac{x\frac{dy}{dx} - y}{(x+y)^2}$$ A1

Substitute $\frac{dy}{dx} = \frac{y}{x+y}$:

$$\frac{d^2y}{dx^2} = \frac{\frac{xy}{x+y} - y}{(x+y)^2}$$ M1

$$= \frac{xy - y(x+y)}{(x+y)^3}$$ M1

$$-\frac{y^2}{(x+y)^3}$$ A1

[7 marks]

b $\frac{dy}{dx} = v + x\frac{dv}{dx}$ M1

$v + x\frac{dv}{dx} = \frac{xv}{x+xv}$ M1

$x\frac{dv}{dx} = \frac{v}{1+v} - v$ M1

$= -\frac{v^2}{1+v}$ A1

[4 marks]

c Separate variables: $\frac{1+v}{v^2}\frac{dv}{dx} = -\frac{1}{x}$ or equivalent M1

$\int \frac{1+v}{v^2}\, dv = \int -\frac{1}{x}\, dx$ M1

$-\frac{1}{v} + \ln v = -\ln x + c$ A1

Using $x = 1, y = 1, v = 1$: $-1 + 0 = 0 + c$ M1

$c = -1$ A1

$-\frac{1}{v} + \ln(xv) = -1$ (M1)

$\frac{x}{y} = \ln y + 1$ (M1)

$x = y(\ln y + 1)$ A1

[8 marks]

Total [19 marks]

Practice Set B: Paper 3 Mark scheme

1 a Check that the statement is true for $n = 1$: M1

LHS = 1 RHS = $\frac{1 \times 2}{2} = 1$ A1

Assume true for $n = k$ M1

$\sum_{r=1}^{r=k} r = \frac{k(k+1)}{2}$ A1

Then

$\sum_{r=1}^{r=k+1} r = \sum_{r=1}^{r=k} r + (k+1) = \frac{k(k+1)}{2} + (k+1)$ M1

$= (k+1)\left(\frac{k}{2} + 1\right)$

$= \frac{(k+1)(k+2)}{2}$ A1

So if the statement works for $n = k$ then it works for $n = k + 1$ and it works for $n = 1$, therefore it works for all $n \in \mathbb{Z}^+$. A1

[7 marks]

b $3n^2 + 3n + 1$ M1A1

[2 marks]

c $\sum_{r=1}^{n} (r+1)^3 - r^3$

$= [(n+1)^3 - n^3] + [n^3 - (n-1)^3] \ldots + [3^3 - 2^3] + [2^3 - 1^3]$ M1

$= (n+1)^3 - 1 = n^3 + 3n^2 + 3n$ A1

Also:

$\sum_{r=1}^{n} (r+1)^3 - r^3 = \sum_{r=1}^{n} 3r^2 + 3r + 1$

$= 3\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$ M1

$= 3\sum_{r=1}^{n} r^2 + \frac{3n(n+1)}{2} + n$ A1A1

Therefore:

$3\sum_{r=1}^{n} r^2 = n^3 + 3n^2 + 3n - \frac{3n(n+1)}{2} - n$ M1

$= n^3 + \frac{3}{2}n^2 + \frac{1}{2}n$

$= \frac{1}{2}n(2n^2 + 3n + 1)$

$= \frac{1}{2}n(n+1)(2n+1)$ A1

Therefore $\sum_{r=1}^{n} r^2 + \frac{n(n+1)(2n+1)}{6}$ AG

[7 marks]

d The coordinate of the bottom right hand corner of the rth rectangle is $\frac{rx}{n}$. M1

The height of the rectangle is $\left(\frac{rx}{n}\right)^2$ A1

So the area of each rectangle is $\frac{x}{n}\left(\frac{rx}{n}\right)^2$ A1

The total area is $\sum_{r=1}^{n} \frac{x}{n}\left(\frac{rx}{n}\right)^2$

Each rectangle has a portion above the curve, so the total area is an overestimate of the true area under the curve. A1

Tip: A diagram would be a great way to form and illustrate this argument!

[4 marks]

e The coordinate of the bottom left hand corner of the rth rectangle is $\frac{(r-1)x}{n}$ M1

The height of the rectangles with top left corner on the curve is $\left(\frac{(r-1)x}{n}\right)^2$ A1

The total area is $\sum_{r=1}^{n} \frac{x}{n}\left(\frac{(r-1)x}{n}\right)^2$

This is less than the area under the curve, so M1

$\frac{x}{n}\sum_{r=1}^{n} \left(\frac{(r-1)x}{n}\right)^2 \leqslant \int_0^x t^2 \, dt$ A1

[4 marks]

f $\sum_{r=1}^{n} \frac{x}{n}\left(\frac{rx}{n}\right)^2 = \frac{x^3}{n^3}\sum_{r=1}^{n} r^2 = \frac{x^3}{n^3}\frac{n(n+1)(2n+1)}{6}$ M1

$= x^3 \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$ A1

$\frac{x}{n}\sum_{r=1}^{n}\left(\frac{(r-1)x}{n}\right)^2 = \frac{x}{n}\sum_{r=1}^{n-1}\left(\frac{rx}{n}\right)^2$ M1A1

$= \frac{x^3}{n^3}\frac{(n-1)(n)(2n-1)}{6}$

$= x^3 \frac{\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}$

Taking the limit

$\lim_{n\to\infty} x^3 \frac{\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6} \leqslant \int_0^x t^2\,dt \leqslant \lim_{n\to\infty} x^3 \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$ M1

$\frac{x^3}{3} \leqslant \int_0^x x^2 dx \leqslant \frac{x^3}{3}$

Since $\int_0^x t^2\,dt$ is sandwiched between two quantities tending towards $\frac{x^3}{3}$, it must also tend towards $\frac{x^3}{3}$. A1

[6 marks]

Total [30 marks]

2 a $\overline{X} = \frac{X_1 + X_2}{2}$ A1

[1 mark]

b $E(\overline{X}) = E\left(\frac{1}{2}X_1 + \frac{1}{2}X_2\right) = \frac{1}{2}E(X_1) + \frac{1}{2}E(X_2)$ M1

$= \frac{1}{2}\mu + \frac{1}{2}\mu$ A1

$= \mu$ AG

$\text{Var}(\overline{X}) = \text{Var}\left(\frac{1}{2}X_1 + \frac{1}{2}X_2\right) = \frac{1}{4}\text{Var}(X_1) + \frac{1}{4}\text{Var}(X_2)$ M1

$\frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2$

$= \frac{1}{2}\sigma^2$ A1

[4 marks]

c i $E(X^2) = \text{Var}(X) + E(X)^2$ A1

ii $E(S^2) = E\left(\frac{X_1^2 + X_2^2}{2} - \overline{X}^2\right) = \frac{1}{2}E(X_1^2) + \frac{1}{2}E(X_2^2) - E(\overline{X}^2)$ M1

$= \frac{1}{2}(\text{Var}(X_1) + E(X_1)^2) + \frac{1}{2}(\text{Var}(X_1) + E(X_1)^2)$ M1

$-(\text{Var}(\overline{X}) + E(\overline{X})^2)$

$= \frac{1}{2}(\sigma^2 + \mu^2) + \frac{1}{2}(\sigma^2 + \mu^2) - \left(\frac{1}{2}\sigma^2 + \mu^2\right)$ A1

$= \frac{1}{2}\sigma^2$ AG

[4 marks]

d i $E(M) = \frac{2}{5}E(X_1) + \frac{3}{5}E(X_2)$ M1

$= \frac{2}{5}\mu + \frac{3}{5}\mu$ A1

$= \mu$ AG

ii $\text{Var}(M) = \frac{4}{25}\text{Var}(X_1) + \frac{9}{25}\text{Var}(X_2)$ M1

$= \frac{13}{25}\sigma^2$ A1

$> \frac{1}{2}\sigma^2$ therefore $\overline{X}$ is a more efficient estimator A1

[5 marks]

e i $L = P(Y = a)P(Y = b)$ M1

$= p(1-p)^{a-1} \times p(1-p)^{b-1}$ A1

ii $L = p^2(1-p)^{a+b-2}$

$\frac{dL}{dp} = 2p(1-p)^{a+b-2} - (a+b-2)p^2(1-p)^{a+b-3}$ M1A1

At a max, $\frac{dL}{dp} = 0$ M1

$p(1-p)^{a+b-3}(2(1-p) - (a+b-2)p) = 0$ M1

Since $p \neq 0$ and $p \neq 1$ at the maximum value of L A1

$2 - 2p = ap + bp - 2p$

$2 = ap + bp$

$p = \frac{2}{a+b}$ A1

[8 marks]

f **i** $S^2 = \frac{4^2 + 8^2}{2} - 6^2 = 4$ M1

Unbiased estimate of $\sigma^2 = 2S^2 = 8$ A1

ii $p = \frac{2}{4+8} = \frac{1}{6}$ A1

[3 marks]

Total [25 marks]

Practice Set C: Paper 1 Mark scheme

SECTION A

1 **a** Attempt to find x-coordinate of turning point:

$\frac{dy}{dx} = 0 : 4x + 10 = 0$ M1

$x = -\frac{5}{2}$

So required domain: $x \leqslant -\frac{5}{2}$ A1

b $y = 2\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 7$ (M1)

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{11}{2}$ A1

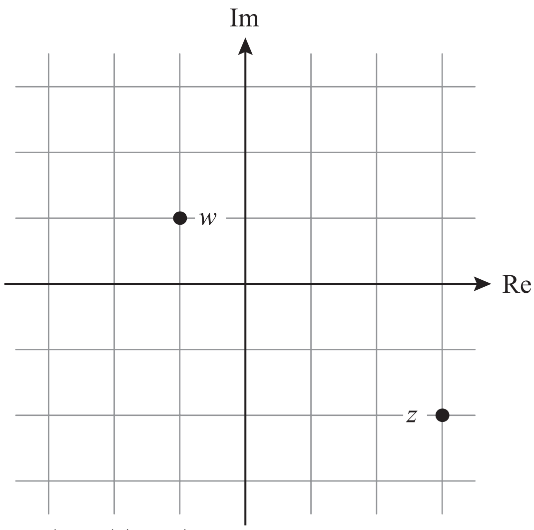
Since $x \leqslant 1$, $f^{-1}(x) = \frac{-5 - \sqrt{2x+11}}{2}$ M1

Domain of $f^{-1} : x \geqslant -\frac{11}{2}$ A1

[6 marks]

2 **a** z correct A1

w correct A1



b $\frac{(-1+i)(3+2i)}{9+4}$ M1

$= -\frac{5}{13} + \frac{1}{13}i$ A1

c Compare real and imaginary parts: M1

$3p - q = 6, -2p + q = 0$

$p = 6, q = 12$ A1

[6 marks]

3 Find the intersection points:

$2x + 1 = x - 3$ OR $2x + 1 = -x + 3$

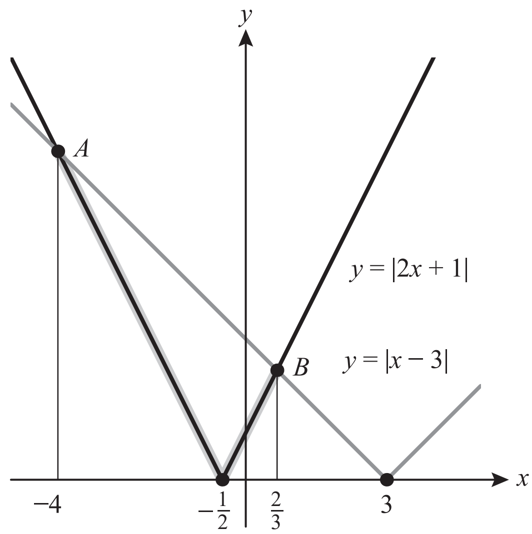
OR

square to get $4x^2 + 4x + 1 = x^2 - 6x + 9$ M1

$x = -4$ A1

$x = \frac{2}{3}$ A1

Graph sketch (or consider signs of factors) M1



$-4 < x < \frac{2}{3}$ A1

[5 marks]

4 To be strictly increasing for all x, f must have no stationary points M1

$f'(x) = 3x^2 + 2kx + k$ A1

$3x^2 + 2kx + k = 0$ has no solutions when $(2k)^2 - 4 \times 3k < 0$ M1

$k(k - 3) < 0$ A1

$0 < k < 3$ A1

[5 marks]

5 Attempt to use partial fractions

$$\frac{3x-16}{(3x-2)(x+4)} = \frac{A}{3x-2} + \frac{B}{x+4}$$

$3x - 16 = A(x + 4) + B(3x - 2)$ M1

$x = -4: -28 = B(-14)$

$B = 2$ A1

$x = \frac{2}{3}: -14 = A\left(\frac{14}{3}\right)$

$A = -3$ A1

$\int_1^6 \frac{2}{x+4} - \frac{3}{3x-2}\,dx = \left[2\ln|x+4| - \ln|3x-2|\right]_1^6$ A1ft

Substitute in limits

$= 2 \ln 10 - \ln 16 - 2 \ln 5 + \ln 1$ M1

$= \ln\frac{1}{4}$ A1

[6 marks]

6 a Use $\sin x \approx x$ M1

$\frac{1}{10}\sin 3x \approx \frac{3}{10}x$ A1

b $\frac{3}{10}x \approx x^2$ M1

$x = 0$ A1

$x \approx 0.3$ A1

[5 marks]

7 Use $\frac{u_1}{(1-r)}=5$ M1

Use $u_1 + u_1 r = 3$ M1

Express u_1 from both equations and equate:

$5(1-r)=\frac{3}{1+r}$ M1

$1-r^2=\frac{3}{5}$ A1

$r=\sqrt{\frac{2}{5}}$ A1

[5 marks]

8 EITHER

$\log_4(3-2x)=\frac{\log_{16}(3-2x)}{\log_{16}4}=\frac{\log_{16}(3-2x)}{\frac{1}{2}}$ M1A1

$2\log_{16}(3-2x)=\log_{16}(6x^2-5x+12)$

$\log_{16}(3-2x)^2=\log_{16}(6x^2-5x+12)$ A1

OR

$\log_{16}(6x^2-5x+12)=\frac{\log_4(6x^2-5x+12)}{\log_4 16}=\frac{\log_4(6x^2-5x+12)}{2}$ M1A1

$2\log_4(3-2x)=\log_4(6x^2-5x+12)$

$\log_4(3-2x)^2=\log_4(6x^2-5x+12)$ A1

$(3-2x)^2=6x^2-5x+12$ M1

$2x^2+7x+3=0$ A1

$(2x+1)(x+3)=0$

$x=-\frac{1}{2},-3$ A1

Checks their solutions in equation:

$x=-\frac{1}{2}$: $3-2x=4>0$ and $6x^2-5x+12=16>0$

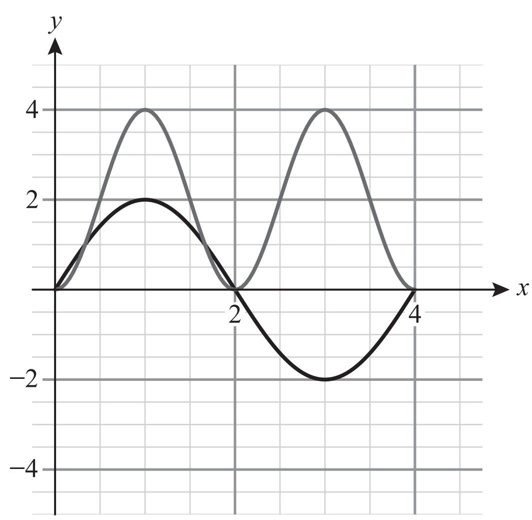
$x=-3$: $3-2x=9>0$ and $6x^2-5x+12=81>0$

So solutions are $x=-\frac{1}{2}, -3$

Note: Award A1 if conclusion consistent with working A1

[7 marks]

9 **a**



y in the range 0 to 4 A1

Intersections at $y = 0$ A1

Intersections at $y = 1$ A1

b Domain: $1 \leqslant x \leqslant 5$ A1

Range: $-4 \leqslant g(x) \leqslant 4$ A1

[5 marks]

10 a $\sin y = x$ (M1)

$\cos\left(\frac{\pi}{2} - y\right) = x$ (M1)

$\arccos x = \frac{\pi}{2} - y$ A1

b $\arcsin x + \arccos x = y + \frac{\pi}{2} - y$ M1

So $\arcsin x + \arccos x \equiv \frac{\pi}{2}$ A1

[5 marks]

SECTION B

11 a i Find $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ A1

$\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$

OR

$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ A1A1

ii $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ A1A1ft

[5 marks]

b i $AB = \sqrt{1^2 + 5^2 + (-4)^2}$ M1

$= \sqrt{42}$ A1

ii $\mathbf{c} = \mathbf{d} \pm 2\overrightarrow{AB}$ (M1)

So the coordinates of C are (1, 13, –5) A1

OR (–3, –7, 11) A1

iii Consider $\overrightarrow{AC_1} \cdot \overrightarrow{AC_2}$ M1

$= 0 - 51 - 64$ [$= -115$] A1

< 0 so obtuse A1

[8 marks]

c i Use $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 7 \\ 0 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix} \times \begin{pmatrix} -2 \\ 7 \\ 0 \end{pmatrix}$ M1

$= \begin{pmatrix} 28 \\ 8 \\ 17 \end{pmatrix}$ A1

ii Scalar product of $\begin{pmatrix} 28 \\ 8 \\ 17 \end{pmatrix}$ with **a**, **b** or **d** attempted $A(1, -4, 3)$ M1

$28x + 8y + 17z$ M1

$= 47$ A1

[5 marks]

Total [18 marks]

12	**a**	$\cos(2\theta+\theta)=\cos(2\theta)\cos\theta-\sin(2\theta)\sin\theta$	M1
		$=(2\cos^2\theta-1)\cos\theta-2\sin^2\theta\cos\theta$	A1
		$=2\cos^3\theta-\cos\theta-2(1-\cos^2\theta)\cos\theta$	(M1)
		$=4\cos^3\theta-3\cos\theta$	A1
			[4 marks]
	b	**i** $8\cos^3\theta-6\cos\theta+1=0$	M1
		$2(4\cos^3\theta-3\cos\theta)=-1$	(M1)
		$\cos 3\theta=-\frac{1}{2}$	A1
		ii $3\theta=\frac{2\pi}{3},$	A1
		$\frac{4\pi}{3},\frac{8\pi}{3}$	A1
		$\theta=\frac{2\pi}{9},\frac{4\pi}{9},\frac{8\pi}{9}$	A1
		Hence $x=\cos\left(\frac{2\pi}{9}\right),\cos\left(\frac{4\pi}{9}\right),\cos\left(\frac{8\pi}{9}\right)$	A1
			[7 marks]
	c	Product of the roots of the cubic equation is $-\frac{1}{8}$	M1
		$\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right)\cos\left(\frac{8\pi}{9}\right)=-\frac{1}{8}$	M1
		$8\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right)=-\frac{1}{\cos\left(\frac{8\pi}{9}\right)}$	
		$=-\sec\left(\frac{8\pi}{9}\right)$	A1(AG)
			[3 marks]
	d	State 0	A1
		It is the sum of the roots of the equation, the coefficient of x^2 is 0	A1
			[2 marks]
			Total [16 marks]
13	**a**	$f(-x)=\frac{-x}{1+(-x)^2}$	M1
		$=-\frac{x}{1+x^2}$	
		$=-f(x)$	A1
		So f is an odd function	A1
			[3 marks]
	b	$\int_0^{\sqrt{3}}\frac{kx}{1+x^2}\,dx=1$	M1
		$\left[\frac{k}{2}\ln(1+x^2)\right]_0^{\sqrt{3}}=1$	A1
		$\frac{k}{2}\ln(4)=1$	A1
		$k\ln 4^{\frac{1}{2}}=1$	M1
		$k=\frac{1}{\ln 2}$	AG
			[4 marks]

c $\frac{1}{\ln 2}\int_0^m \frac{x}{1+x^2}\,dx = \frac{1}{2}$ (M1)

$\frac{1}{\ln 2}\frac{1}{2}\ln(1+m^2) = \frac{1}{2}$ A1

$\ln(1+m^2) = \ln 2$ A1

$1+m^2 = 2$

$m = 1$ A1

[4 marks]

d $g'(x) = \frac{1}{\ln 2}\left(\frac{1(1+x^2)-x(2x)}{(1+x^2)}\right) = 0$ M1A1

$1 - x^2 = 0$

$x = 1$ A1

$g(0) = 0$ and $g(1) = \frac{1}{2\ln 2} > 0$ so $x = 1$ is local maximum (or alternative justification) M1

So $x = 1$ is the mode A1

[5 marks]

e $E(X) = \frac{1}{\ln 2}\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}\,dx$ (M1)

$\frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2} = 1 - \frac{1}{1+x^2}$ M1

$E(X) = \frac{1}{\ln 2}\left[x - \arctan x\right]_0^{\sqrt{3}}$ A1

$= \frac{1}{\ln 2}\left(\sqrt{3} - \arctan\sqrt{3}\right)$ (M1)

$= \frac{1}{\ln 2}\left(\sqrt{3} - \frac{\pi}{3}\right)$ A1

[5 marks]

Total [21 marks]

Practice Set C: Paper 2 Mark scheme

SECTION A

1 **a** Stratified sampling A1
b Correct regression line attempted M1
$y = -1.33x + 6.39$ A1
c For every extra hour spent on social media, 1.33 hours less spent on homework. A1
No social media gives around 6.39 hours for homework. A1
[5 marks]

2 Shaded area $\frac{1}{2}(7.2)^2\,\theta\ (= 25.92\,\theta)$ M1
Triangle area $\frac{1}{2}(7.2)^2 \sin\theta\ (= 25.92 \sin\theta)$ M1
$\frac{1}{2}(7.2)^2\,\theta - \frac{1}{2}(7.2)^2 \sin\theta = 9.7$ or equivalent (e.g. $\theta - \sin\theta = 0.3742$) A1
Solve their equation using GDC M1
$\theta = 1.35$ A1
[5 marks]

3 **a** $k + 2k + 3k + 4k = 1$ (M1)
$k = 0.1$ A1
b $\mathrm{E}(X) = k + 4k + 12k + 28k$ (M1)
$\mathrm{E}(X^2) = k + 8k + 48k + 196k\ (= 25.3)$ M1
$\mathrm{Var}(X) = 25.3 - [4.5]^2$ (M1)
$= 5.05$ A1
c $25 \times \mathrm{Var}(X)$ (M1)
$= 126(.25)$ A1
[8 marks]

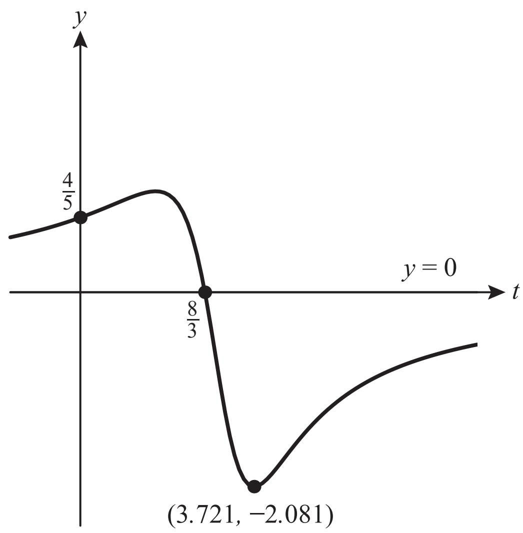
4 METHOD 1
Use of $\cot\theta = \frac{1}{\tan\theta}$
LHS $\equiv \dfrac{\sec\theta\sin\theta}{\tan\theta + \frac{1}{\tan\theta}}$ M1
$\equiv \dfrac{\sec\theta\sin\theta\tan\theta}{\tan^2\theta + 1}$ A1
Use of $\sec^2\theta \equiv \tan^2\theta + 1$
$\equiv \dfrac{\sec\theta\sin\theta\tan\theta}{\sec^2\theta}$ M1
$\equiv \dfrac{\sin\theta\tan\theta}{\sec\theta}$ A1
Express in terms of $\sin\theta$ and $\cos\theta$
$\equiv \sin\theta\dfrac{\sin\theta}{\cos\theta} \times \cos\theta$ M1
$\equiv \sin^2\theta$ AG
[5 marks]

METHOD 2
Use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
LHS $\equiv \dfrac{\sec\theta\sin\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$ M1
Add fractions in denominator (or multiply through by $\sin\theta\cos\theta$)
$\equiv \dfrac{\sec\theta\sin\theta}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$ M1
$\equiv \dfrac{\sin^2\theta\sec\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$ A1
$\equiv \dfrac{\sin^2\theta}{\sin^2\theta + \cos^2\theta}$ A1

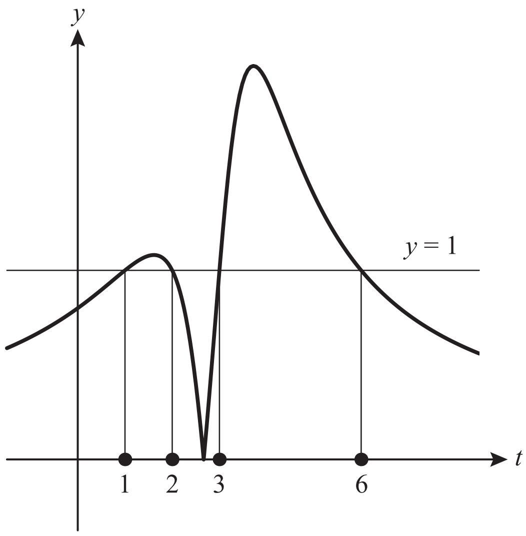
		Use of $\sin^2\theta + \cos^2\theta \equiv 1$	
		$\equiv \dfrac{\sin^2\theta}{1}$	M1
		$\equiv \sin^2\theta$	AG
			[5 marks]
5	**a**	Solve $0.003x^3 + 10x + 200 = 720$ using GDC	M1
		36 cakes	A1
	b	Sketch graph of $y = \dfrac{T(x)}{x}$	M1
		Minimum point marked at $x = 32.2$	M1
		Minimum = 19.3 minutes	A1
		Maximum = 21.2 minutes	A1
			[6 marks]
6	**a**	$20C6$	(M1)
		$= 38\,760$	A1
	b	Consider two cases: (3 F and 3 NF) or (4 F and 2 NF)	M1
		$12C3 \times 8C3$ (= 12 320) or $12C4 \times 8C2$ (= 13 860)	M1
		Both of the above terms seen (not necessarily added for this mark)	A1
		26 180 selections	A1
			[6 marks]
7		$\dfrac{du}{dx} = e^x \Rightarrow dx = \dfrac{1}{e^x}du$	M1
		$\displaystyle\int \frac{u}{u^2+u-2}\frac{1}{e^x}du = \int \frac{1}{u^2+u-2}du$	A1
		Attempt to use partial fractions	
		$\dfrac{1}{u^2+u-2} = \dfrac{A}{u-1} + \dfrac{B}{u+2}$	M1
		$1 = A(u+2) + B(u-1)$	
		$A = \dfrac{1}{3}, B = -\dfrac{1}{3}$	A1
		$\displaystyle\int \frac{\frac{1}{3}}{u-1} - \frac{\frac{1}{3}}{u+2}du = \frac{1}{3}(\ln\lvert u-1\rvert - \ln\lvert u+2\rvert)$	M1
		$\ln\left\lvert\dfrac{u-1}{u+2}\right\rvert^{\frac{1}{3}}$	A1
		$\displaystyle\int \frac{e^x}{e^{2x}+e^x-2}dx = \ln\left\lvert\frac{e^x-1}{e^x+2}\right\rvert^{\frac{1}{3}}(+c)$	A1
			[7 marks]
8		Assume there does exist such a function	M1
		By factor theorem $f\left(-\dfrac{3}{2}\right) = 0$:	
		$2\left(-\dfrac{3}{2}\right)^3 + b\left(-\dfrac{3}{2}\right)^2 + c\left(-\dfrac{3}{2}\right) + 3 = 0$	
		Note: award M1 for $f\left(\pm\dfrac{3}{k}\right) = 0$ where $k = 1$ or 2.	M1
		$3b - 2c - 5 = 0$	A1
		By remainder theorem $f(2) = 5$	
		$2(2)^3 + b(2)^2 + c(2) + 3 = 5$	
		Note: award M1 for $f(\pm 2) = 5$	M1
		$2b + c + 7 = 0$	A1
		Solving (1) and (2) simultaneously:	
		$b = -\dfrac{9}{7}, c = -\dfrac{31}{7}$	A1
		This is a contradiction as b, c were assumed to be integers.	
		So, there exists no such function.	A1
			[7 marks]
9		$\dfrac{dS}{dt} = 2\pi r\dfrac{dr}{dt} \ldots$	M1
		$\ldots + \pi\dfrac{dr}{dt}\sqrt{r^2+25}$	A1
		$\ldots + \pi r\dfrac{2r\frac{dr}{dt}}{2\sqrt{r^2+25}}$	M1A1
		Substitute $r = 10$, $\dfrac{dr}{dt} = 2$ into their expression	M1
		$\dfrac{dS}{dt} = 252\,\text{cm}^2\,\text{sec}^{-1}$	A1
			[6 marks]

SECTION B

10	**a**	**i**	Arithmetic sequence, $u_1 = 30$, $d = 10$	(M1)
			$u_{12} = 30 + 11 \times 10$	M1
			$= 140$	A1
		ii	$S_{12} = 6(60 + 11 \times 10)$ or $\frac{12(30 + 140)}{2}$	M1
			$= 1020$	A1
		iii	$\frac{N}{2}\left(60 + 10(N - 1)\right) = 2000$	
			OR Create table of values	M1
			$N = 17.7$	
			OR $S_{17} = 1870$, $S_{18} = 2070$	A1
			In the 18th month	A1
				[8 marks]
	b	**i**	Geometric sequence, $u_1 = 30$, $r = 1.1$	(M1)
			$S_{12} = \frac{30(1.1^{12} - 1)}{1.1 - 1}$	M1
			$= 642$	A1
		ii	$30 \times 1.1^{N-1} > 100$	M1
			$N = 13.6$	(M1)
			In the 14th month	A1
				[6 marks]
	c	**i**	Multiply answer to **a(ii)** or **b(i)** by the profit at least once	M1
			Stella: 1020 × 2.20 = £2244	A1
			Giulio: 642 × 3.10 = £1990	A1
		ii	$\frac{30(1.1^N - 1)}{0.1} \times 3.10 > \frac{N}{2}\left(60 + 10(N - 1)\right) \times 2.20$	M1
			$N = 22.9$	(M1)
			In the 22nd month	A1
				[6 marks]
				Total [20 marks]
11	**a**		$v(0) = \frac{8}{10} = 0.8\ \text{m s}^{-1}$	A1
				[1 mark]
	b		Sketch graph $y = v(t)$ and identify minimum point.	(M1)



		Maximum speed = $|-2.08| = 2.08\ \text{m s}^{-1}$	
		Note: Award M1A0 for $-2.08\ \text{m s}^{-1}$	A1
			[2 marks]
c		EITHER	
		$v > 1$ for $1 < t < 2$	M1
		$v < 1$ for $3 < t < 6$	M1
		OR	
		Graph $y = |v(t)|$	M1



$|v| > 1$ for $1 < t < 2$ or $3 < t < 6$ — M1

So speed > 1 for 4 seconds — A1

[3 marks]

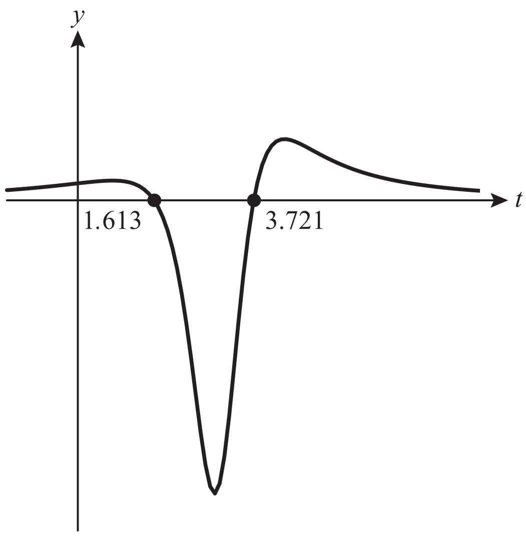
d Object changes direction when $v = 0$ — (M1)

$t = \frac{8}{3} = 2.67\,\text{s}$ — A1

[2 marks]

e EITHER

Sketch graph of $y = \frac{dv}{dt}$: $y < 0$ for $1.61 < t < 3.72$ — (M1)



OR

Use graph of $y = v(t)$: gradient negative for $1.61 < t < 3.72$ (between turning points) — (M1)

So $a < 0$ for 2.11 seconds — A1

[2 marks]

f From GDC, $\frac{dv}{dt}$ at $t = 5$ … — (M1)

… gives $a = 0.52\,\text{m}\,\text{s}^{-2}$ — A1

[2 marks]

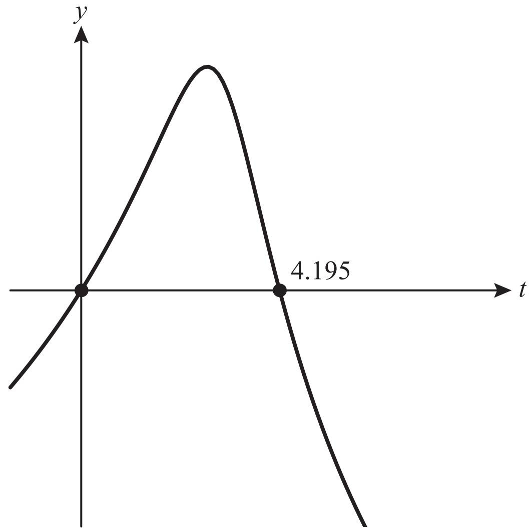
g From GDC:

distance $= \int_0^{10} \left| \frac{8 - 3t}{t^2 - 6t + 10} \right| dt$ — M1

$= 9.83\,\text{m}$ — A1

[2 marks]

h Sketch graph of $y = \int_0^x v\,\mathrm{d}t$ (M1)



Identify x-intercept as being point at which object back at start (M1)
$t = 4.20$ seconds A1

[3 marks]
Total [17 marks]

12 a $\frac{\mathrm{d}}{\mathrm{d}x}(\ln|\sec x + \tan x|) = \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$ M1A1

$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$ A1

$= \sec x$ AG

[3 marks]

b $\frac{\mathrm{d}y}{\mathrm{d}x} + \sec x y = \sec x$ M1

Integrating factor:
$e^{\int \sec x\,\mathrm{d}x} = e^{\ln|\sec x + \tan x|}$ M1
$= \sec x + \tan x$ A1

$\frac{\mathrm{d}}{\mathrm{d}x}\left(y(\sec x + \tan x)\right) = \sec^2 x + \sec x \tan x$ M1A1

$y(\sec x + \tan x) = \int \sec^2 x + \sec x \tan x\,\mathrm{d}x$

$y(\sec x + \tan x) = \tan x + \sec x + c$ A1

$y = 1 + \frac{c}{\sec x + \tan x}$ A1

[7 marks]

c i $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - \sin x\frac{\mathrm{d}y}{\mathrm{d}x} + \cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ M1A1

$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = (\sin x - 1)\frac{\mathrm{d}y}{\mathrm{d}x} - \cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ AG

ii Substitute given values into differential equation:
When $x = 0$

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \cos 0(1) + 2 = 1$ M1

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2$ A1

Substitute their value into expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$:
When $x = 0$

$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = (\sin 0 - 1)(1) - \cos 0(-2)$ M1
$= 1$ A1

Substitute their values into Maclaurin series

$y = 2 + x - \frac{2}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$ M1

$2 + x - x^2 + \frac{1}{6}x^3 + \cdots$ A1

[8 marks]
Total [18 marks]

Practice Set C: Paper 3 Mark scheme

1 a ( )( )   (( ))   (( )(A2

)( )(   ))((   ))( )

[2 marks]

b i 16   A1

ii 8   A1

iii 12 870   A1

$^{2n}C_n$ A1

[4 marks]

c i ( )( )   (( ))   A1

ii ((( ))) ( )(( ))   ( )( )( )   (( )) ( )   (( )( ))   M1

So $B_3 = 5$ A1

[3 marks]

d $\frac{B_1}{A_1} = \frac{1}{2}$ $\frac{B_2}{A_2} = \frac{1}{3}$ $\frac{B_3}{A_3} = \frac{5}{20} = \frac{1}{4}$ $\frac{B_8}{A_8} = \frac{1}{9}$ (M2)

This suggests $f(n) = \frac{1}{n+1}$ A1

$B_n = \frac{1}{n+1}\,{}^{2n}C_n$

[3 marks]

e When $n = 1$ M1

$B_1 = \frac{1}{2} \times {}^{2}C_1 = \frac{1}{2} \times 2 = 1$

So the conjecture is true when $n = 1$ A1

Assume that it is true when $n = k$ M1

$B_1 = \frac{1}{k+1}\,{}^{2k}C_k = \frac{1}{k+1}\,\frac{(2k)!}{k!k!}$ A1

Then using the given recursion relation:

$B_{k+1} = \frac{4k+2}{k+2} \times \frac{1}{k+1}\,\frac{(2k)!}{k!k!}$ M1

$= \frac{2(2k+1)}{(k+2)(k+1)} \times \frac{(2k)!}{k!k!}$

$= \frac{2(2k+1)}{(k+2)(k+1)} \times \frac{(2k)!}{k!k!} \times \frac{(2k+1)(2k+2)}{(2k+1)(2k+2)}$ M1

$= \frac{(2k+1)}{(k+2)(k+1)} \times \frac{(2k)!}{k!k!} \times \frac{(2k+1)(2k+2)}{(2k+1)(k+1)}$

$= \frac{1}{k+2} \times \frac{2(k+1)!}{(k+1)!(k+1)!}$ A1

$= \frac{1}{(k+1)+1}\,{}^{2(k+1)}C_{k+1}$

So if the statement works for $n = k$ then it works for $n = k + 1$ and it works for $n = 1$ therefore it works for all $n \in \mathbb{Z}^+$ A1

[8 marks]

Tip: You might wonder where the given recursion relation comes from. The most natural way is from the triangulation of a polygon interpretation of Catalan numbers.

f $\frac{B_{20}}{A_{20}} = f(20) = \frac{1}{21}$ M1A1

[2 marks]

g Let (be equivalent to a vote for Elsa and) be equivalent to a vote for Asher   M1

Then the total number of ways of ending in a draw is A_{50} and the number where Asher is never ahead is B_{50} M1

The probability is then $\frac{B_{50}}{A_{50}} = \frac{1}{51}$ A1

[3 marks]

Total [25 marks]

2 **a** $\left|e^{\frac{2\pi i}{3}}-1\right|=\left|\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}-1\right|$ M1

$=\left|-\frac{1}{2}+\frac{\sqrt{3}}{2}i-1\right|$ A1

$=\sqrt{\left(-\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$=\sqrt{3}$ A1

[3 marks]

b Bill: $e^{\frac{2\pi i}{3}}$ A1

Charlotte: $e^{\frac{4\pi i}{3}}$ A1

[2 marks]

c Using part **a**: $\sqrt{3}$ units in $\sqrt{3}$ seconds A1A1

[2 marks]

d The direction from z_A to z_B is z_B-z_A A1

The distance travelled per unit time is one, so this is $\frac{z_B-z_A}{|z_B-z_A|}$ A1

[2 marks]

e $z_B=e^{\frac{2\pi i}{3}}z_A$ A1

[1 mark]

f $\frac{dz_A}{dt}=\frac{dr}{dt}e^{i\theta}+ire^{i\theta}\frac{d\theta}{dt}$ M1A1

[2 marks]

g $\frac{dr}{dt}e^{i\theta}+ire^{i\theta}\frac{d\theta}{dt}=\frac{e^{\frac{2\pi i}{3}}z_A-z_A}{\left|e^{\frac{2\pi i}{3}}z_A-z_A\right|}=\frac{z_A\left(e^{\frac{2\pi i}{3}}-1\right)}{|z_A|\left|e^{\frac{2\pi i}{3}}-1\right|}$ M1A1

$=\frac{re^{i\theta}\left(e^{\frac{2\pi i}{3}}-1\right)}{r\left|e^{\frac{2\pi i}{3}}-1\right|}$ M1A1

$=\frac{e^{i\theta}}{\sqrt{3}}\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)$ A1

$=e^{i\theta}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

Dividing through by $e^{i\theta}$:

1. $\frac{dr}{dt}+ir\frac{d\theta}{dt}=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$

Comparing real and imaginary parts:

$\frac{dr}{dt}=-\frac{\sqrt{3}}{2}$ A1

$r\frac{d\theta}{dt}=\frac{1}{2}$ A1

[7 marks]

h $r=-\frac{\sqrt{3}}{2}t+c$ M1

When $t=0$, $r=1$ so $c=1$ M1

$r=1-\frac{\sqrt{3}}{2}t$ A1

$\frac{d\theta}{dt}=\frac{1}{2\left(1-\frac{\sqrt{3}}{2}t\right)}=\frac{1}{2-\sqrt{3}t}$ M1

$\theta=-\frac{1}{\sqrt{3}}\ln(2-\sqrt{3}t)+c$ A1

When $t=0$, $\theta=0$ so $c=\frac{1}{\sqrt{3}}\ln 2$ M1

$\theta=\frac{1}{\sqrt{3}}\ln\left(\frac{2}{2-\sqrt{3}t}\right)$ A1

[7 marks]

i Meet when $r=0$ M1

This happens when $1-\frac{\sqrt{3}}{2}t=0$

So $t=\frac{2}{\sqrt{3}}$ A1

Since $v=1$ the distance travelled is $\frac{2}{\sqrt{3}}$ units A1

As $t\to\frac{2}{\sqrt{3}}$, $\theta\to\infty$ so the snails make an infinite number of rotations A1

[4 marks]

Total [30 marks]