1. (a) Show that lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect and find the coordinates of P, the point of intersection.

(8)

(b) Find the Cartesian equation of the plane Π that contains the two lines.

(6)

(c) The point Q(3, 4, 3) lies on Π. The line L passes through the midpoint of [PQ]. Point S is on L such that $\left|\overrightarrow{PS}\right| = \left|\overrightarrow{QS}\right| = 3$, and the triangle PQS is normal to the plane Π. Given that there are two possible positions for S, find their coordinates.

(15)
(Total 29 marks)

2. Consider the points A(1, 2, 1), B(0, –1, 2), C(1, 0, 2) and D(2, –1, –6).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(2)

(b) Calculate $\overrightarrow{AB}\times\overrightarrow{BC}$.

(2)

(c) Hence, or otherwise find the area of triangle ABC.

(3)

(d) Find the Cartesian equation of the plane P containing the points A, B and C.

(3)

(e) Find a set of parametric equations for the line L through the point D and perpendicular to the plane P.

(3)

(f) Find the point of intersection E, of the line L and the plane P.

(4)

(g) Find the distance from the point D to the plane P.

(2)

(h) Find a unit vector that is perpendicular to the plane P.

(2)

(i) The point F is a reflection of D in the plane P. Find the coordinates of F.

(4)
(Total 25 marks)

3. The acceleration in m s^{-2} of a particle moving in a straight line at time t seconds, $t \geq 0$, is given by the formula $a = -\frac{1}{2}v$. When $t = 0$, the velocity is 40 m s^{-1}.

Find an expression for v in terms of t.

(Total 6 marks)

4. Calculate $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

(Total 6 marks)

5. (a) Show that the solution of the differential equation

$$\frac{dy}{dx} = \cos x \cos^2 y,$$

given that $y = \frac{\pi}{4}$ when $x = \pi$, is $y = \arctan(1 + \sin x)$.

(5)

(b) Determine the value of the constant a for which the following limit exists

$$\lim_{x \to \frac{\pi}{2}} \frac{\arctan(1+\sin x) - a}{\left(x - \frac{\pi}{2}\right)^2}$$

and evaluate that limit.

(12)
(Total 17 marks)

6. Consider the differential equation $\frac{dy}{dy} + \frac{xy}{4 - x^2} = 1$, where $|x| < 2$ and $y = 1$ when $x = 0$.

(a) Use Euler's method with $h = 0.25$, to find an approximate value of y when $x = 1$, giving your answer to two decimal places.

(10)

(b) (i) By first finding an integrating factor, solve this differential equation. Give your answer in the form $y = f(x)$.

(ii) Calculate, correct to two decimal places, the value of y when $x = 1$.

(10)

(c) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 1$. Use your sketch to explain why your approximate value of y is greater than the true value of y.

(4)
(Total 24 marks)