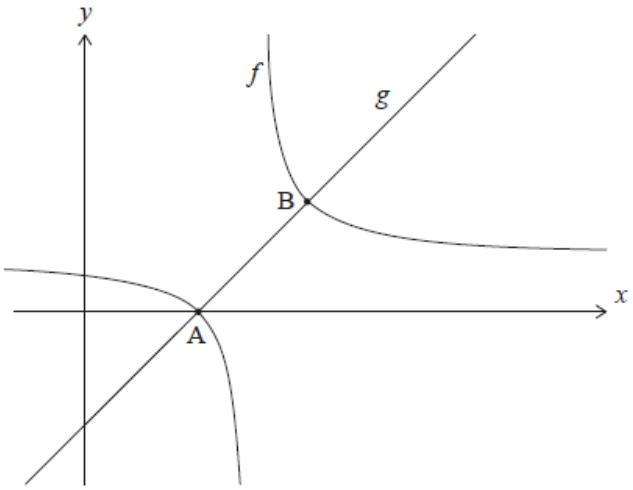
Definite Integrals *[113 marks]*

1. The derivative of the function f is given by $f'(x) = \frac{6x}{x^2+1}$. *[5 marks]*

   The graph of $y = f(x)$ passes through the point $(1,\ 5)$. Find an expression for $f(x)$.

Consider the functions $f(x) = \frac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

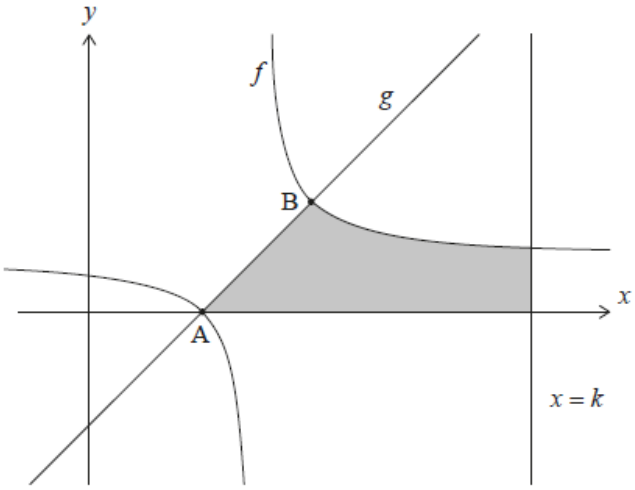
The following diagram shows the graphs of f and g.



The graphs of f and g intersect at points A and B. The coordinates of A are $(3,\ 0)$.

2a. Find the coordinates of B. *[5 marks]*

In the following diagram, the shaded region is enclosed by the graph of f, the graph of g, the x-axis, and the line $x = k$, where $k \in \mathbb{Z}$.



The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

2b. Find the value of k and the value of p. *[10 marks]*

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3. Find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}}\right) \mathrm{d}\, x$. *[5 marks]*

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Consider $f(x) = \frac{2x-4}{x^2-1}, -1 < x < 1$.

4a. Find $f'(x)$. *[2 marks]*

4b. Show that, if $f'(x) = 0$, then $x = 2 - \sqrt{3}$. *[3 marks]*

For the graph of $y = f(x)$,

4c. find the coordinates of the y-intercept. *[1 mark]*

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4d. show that there are no x-intercepts. *[2 marks]*

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4e. sketch the graph, showing clearly any asymptotic behaviour. *[2 marks]*

4f. Show that $\frac{3}{x+1} - \frac{1}{x-1} = \frac{2x-4}{x^2-1}$. *[2 marks]*

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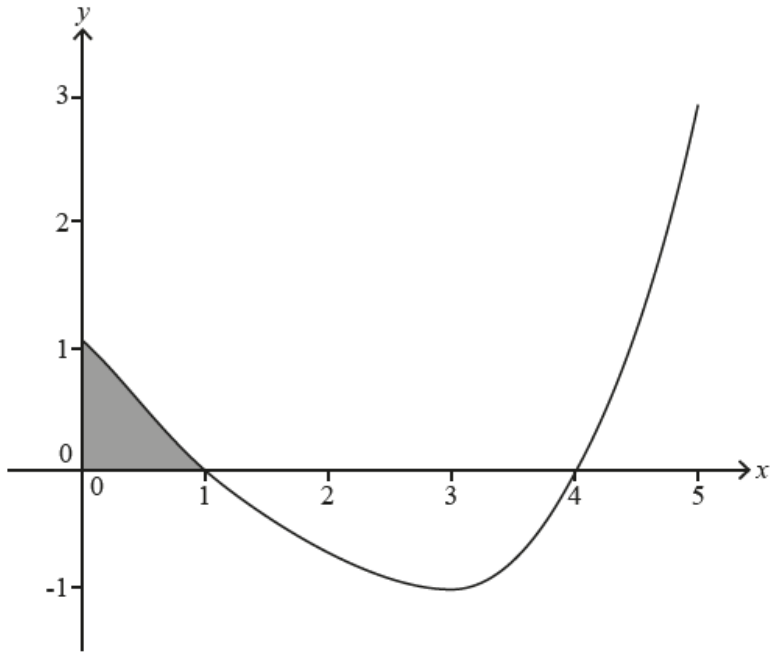
4g. The area enclosed by the graph of $y = f(x)$ and the line $y = 4$ can be expressed as $\ln v$. Find the value of v. *[7 marks]*

5. Given that $\int_0^{\ln k} \mathrm{e}^{2x} \mathrm{d}x = 12$, find the value of k. *[6 marks]*

The graph of $y = f'(x)$, $0 \leq x \leq 5$ is shown in the following diagram. The curve intercepts the x-axis at (1, 0) and (4, 0) and has a local minimum at (3, −1).



6a. Write down the x-coordinate of the point of inflexion on the graph of $y = f(x)$. *[1 mark]*

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The shaded area enclosed by the curve $y = f'(x)$, the x-axis and the y-axis is 0.5. Given that $f(0) = 3$,

6b. find the value of $f(1)$. *[3 marks]*

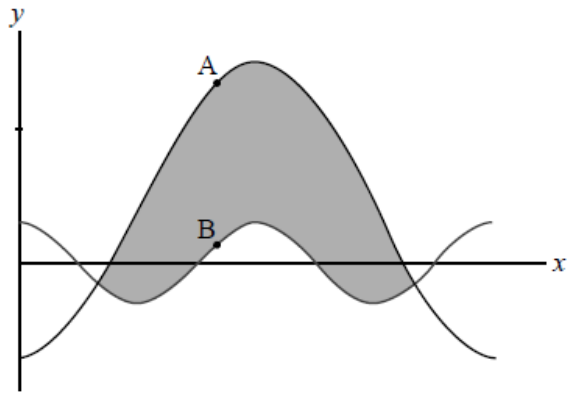
The area enclosed by the curve $y = f'(x)$ and the x-axis between $x = 1$ and $x = 4$ is 2.5 .

6c. find the value of $f(4)$. *[2 marks]*

6d. Sketch the curve $y = f(x)$, $0 \leq x \leq 5$ indicating clearly the coordinates of the maximum and minimum points and any intercepts with the coordinate axes. *[3 marks]*

Consider the functions f and g defined on the domain $0 < x < 2\pi$ by $f(x) = 3\cos 2x$ and $g(x) = 4 - 11\cos x$.

The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$



7a. Find the x-coordinates of the points of intersection of the two graphs. *[6 marks]*

7b. Find the exact area of the shaded region, giving your answer in the form $p\pi + q\sqrt{3}$, where p, $q \in \mathbb{Q}$. *[5 marks]*

7c. At the points A and B on the diagram, the gradients of the two graphs are equal. *[6 marks]*

Determine the y-coordinate of A on the graph of g.

Given that $\int_{-2}^{2} f(x)\,\mathrm{d}x = 10$ and $\int_{0}^{2} f(x)\,\mathrm{d}x = 12$, find

8a. $\int_{-2}^{0} (f(x) + 2)\,\mathrm{d}x$. *[4 marks]*

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8b. $\int_{-2}^{0} f\left(x+2\right) \mathrm{d}x.$ *[2 marks]*

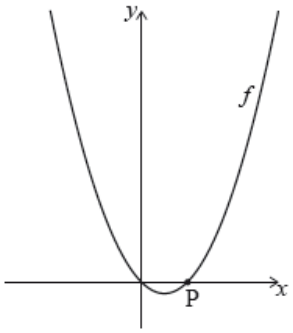
Let $y = \arccos\left(\frac{x}{2}\right)$

9a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[2 marks]*

9b. Find $\int_0^1 \arccos\left(\frac{x}{2}\right) \mathrm{d}x$. *[7 marks]*

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

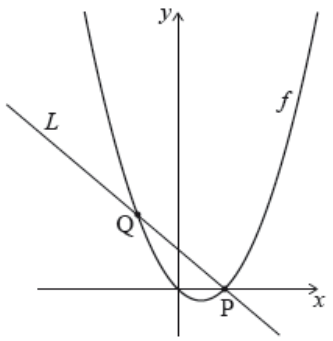
diagram not to scale



The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1, 0)$.

The line L intersects the graph of f at another point Q, as shown in the following diagram.

diagram not to scale



10. Find the area of the region enclosed by the graph of f and the line L. *[6 marks]*

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11a. Express $x^2 + 3x + 2$ in the form $(x + h)^2 + k$. *[1 mark]*

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11b. Factorize $x^2 + 3x + 2$. *[1 mark]*

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Consider the function $f(x) = \frac{1}{x^2+3x+2}, x \in \mathbb{R}, x \neq -2, x \neq -1$.

11c. Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y-intercept and the local maximum. *[5 marks]*

11d. Hence find the value of p if $\int_0^1 f(x)\mathrm{d}x = \ln(p)$. *[4 marks]*

11e. Sketch the graph of $y = f\left(|x|\right)$. *[2 marks]*

11f. Determine the area of the region enclosed between the graph of $y = f(|x|)$, the x-axis and the lines with equations $x = -1$ and $x = 1$. *[3 marks]*