Diff equations [84 marks]

Consider the differential equation $rac{\mathrm{d}y}{\mathrm{d}x}=rac{4x^2+y^2-xy}{x^2}$, with y=2 when x=1.

- 1a. Use Euler's method, with step length h = 0.1, to find an approximate [5 marks] value of y when x = 1.4.
- 1b. Express m^2-2m+4 in the form $\left(m-a
 ight)^2+b$, where $a,b\in\mathbb{Z}.$ [1 mark]
- 1c. Solve the differential equation, for x > 0, giving your answer in the [10 marks] form y = f(x).
- 1d. Sketch the graph of y = f(x) for $1 \le x \le 1.4$. [1 mark]
- 1e. With reference to the curvature of your sketch in part (c)(iii), and without[2 marks] further calculation, explain whether you conjecture f(1.4) will be less than, equal to, or greater than your answer in part (a).

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \frac{y}{x}$, where $x \neq 0$.

- 2a. Given that y(1) = 1, use Euler's method with step length h = 0.25 to [4 marks] find an approximation for y(2). Give your answer to two significant figures.
- ^{2b.} Solve the equation $\frac{dy}{dx} = 1 + \frac{y}{x}$ for y(1) = 1. [6 marks]
- 2c. Find the percentage error when y(2) is approximated by the final [3 marks] rounded value found in part (a). Give your answer to two significant figures.

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{x}{x^2+1}y = x$ where y = 1 when x = 0.

3a. Show that $\sqrt{x^2 + 1}$ is an integrating factor for this differential equation. [4 marks]

3b. Solve the differential equation giving your answer in the form y = f(x). [6 marks]

4a. Consider the differential equation

$$rac{\mathrm{d} y}{\mathrm{d} x} = f\left(rac{y}{x}
ight), \; x > 0.$$

Use the substitution y = vx to show that the general solution of this differential equation is

$$\int \frac{\mathrm{d}v}{f(v) - v} = \ln x + \text{Constant.}$$

4b. Hence, or otherwise, solve the differential equation

$$rac{\mathrm{d}y}{\mathrm{d}x}=rac{x^2+3xy+y^2}{x^2}, x>0,$$

given that y = 1 when x = 1. Give your answer in the form y = g(x).

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{2x}{1+x^2}\right)y = x^2$, given that y = 2 when x = 0.

- 5a. Show that $1 + x^2$ is an integrating factor for this differential equation. [5 marks]
- 5b. Hence solve this differential equation. Give the answer in the form [6 marks] y = f(x).

Consider the expression $\frac{1}{\sqrt{1+ax}} - \sqrt{1-x}$ where $a \in \mathbb{Q}, \ a \neq 0$. The binomial expansion of this expression, in ascending powers of x, as far as the term in x^2 is $4bx + bx^2$, where $b \in \mathbb{Q}$.

- 6a. Find the value of a and the value of b.
- 6b. State the restriction which must be placed on x for this expansion to be [1 mark] valid.

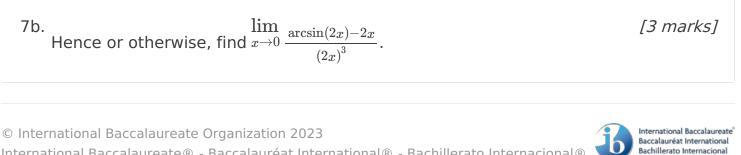
[3 marks]

[10 marks]

[6 marks]

The function f is defined by $f(x) = \arcsin{(2x)}$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.

7a. By finding a suitable number of derivatives of f, find the first two non- [8 marks] zero terms in the Maclaurin series for f.



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