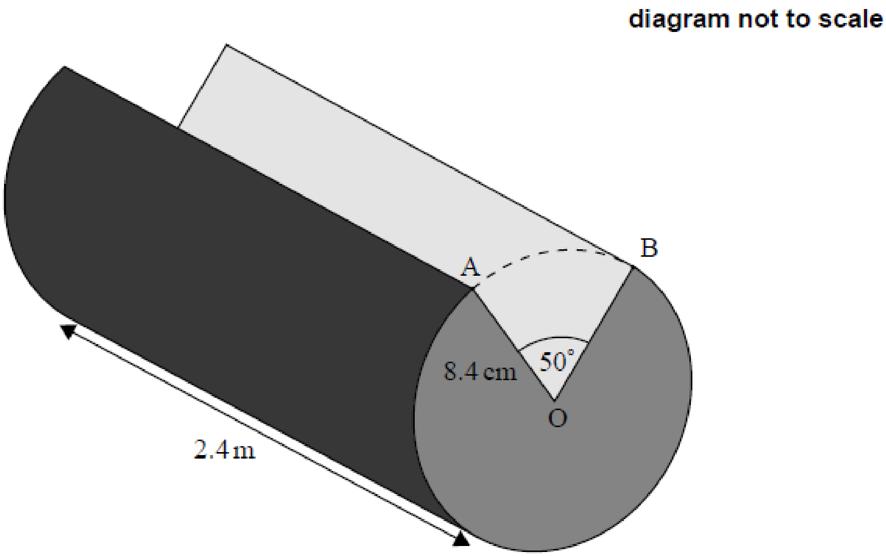
Mock exam review - geometry and trigonometry *[48 marks]*

1. Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram. *[4 marks]*



Find the volume of this log.

Markscheme

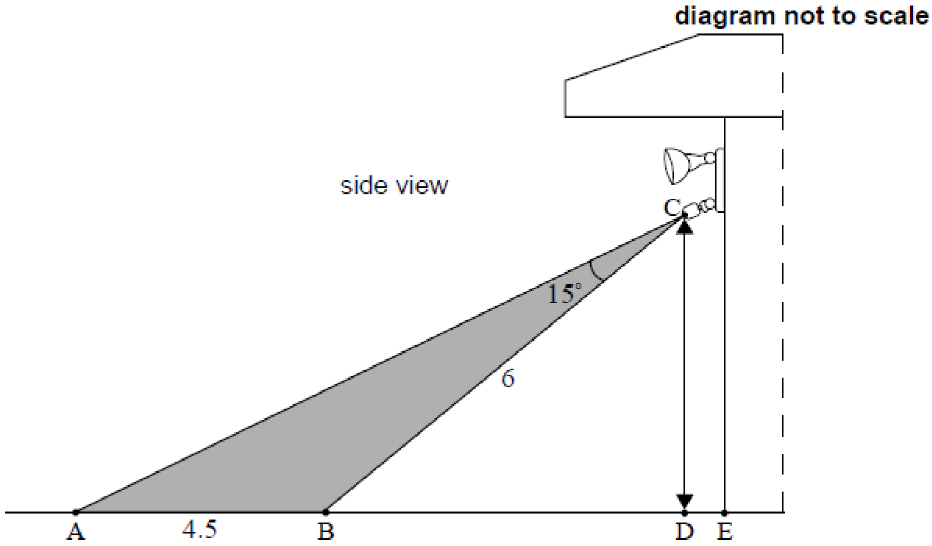
volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

Note: Award ***M1*** 240 × area, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

= 45800 (= 45811.96071) ***A1***

[4 marks]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $A\hat{C}B$ is 15°.



2a. Find $C\hat{A}B$. *[3 marks]*

Markscheme

$$\frac{\sin C\hat{A}B}{6} = \frac{\sin 15^\circ}{4.5}$$ ***(M1)(A1)***

$C\hat{A}B = 20.2°$ (20.187415…) ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

2b. Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B. *[5 marks]*

Find the distance Ollie is **from the entrance to his house** when he first activates the sensor.

Markscheme

$\text{C}\hat{\text{B}}\text{D} = 20.2 + 15 = 35.2°$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots° = \frac{1.8}{\text{BX}}$ ***(M1)***

Note: Award ***A1*** for their correct angle $\text{C}\hat{\text{B}}\text{D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\text{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

= 2.45 (m) (2.44714...) ***A1***

[5 marks]

A farmer owns a triangular field ABC. The length of side $[\text{AB}]$ is 85 m and side $[\text{AC}]$ is 110 m. The angle between these two sides is 55°.

3a. Find the area of the field. *[3 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Area $= \frac{1}{2} \times 110 \times 85 \times \sin 55^\circ$ **(M1)(A1)**

$= 3830(3829.53\ldots)\text{m}^2$ **A1**

Note: units must be given for the final **A1** to be awarded.

[3 marks]

3b. The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[\text{BC}]$. *[6 marks]*

Find BD. Fully justify any assumptions you make.

Markscheme

$\mathrm{BC}^2 = 110^2 + 85^2 - 2 \times 110 \times 85 \times \cos 55^\circ$ **(M1)A1**

$\mathrm{BC} = 92.7(92.7314\ldots)(\mathrm{m})$ **A1**

METHOD 1

Because the height and area of each triangle are equal they must have the same length base **R1**

D must be placed half-way along BC **A1**

$\mathrm{BD} = \frac{92.731\ldots}{2} \approx 46.4(\mathrm{m})$ **A1**

Note: the final two marks are dependent on the **R1** being awarded.

METHOD 2

Let $\mathrm{C\hat{B}A} = \theta^\circ$

$\frac{\sin\theta}{110} = \frac{\sin 55^\circ}{92.731\ldots}$ **M1**

$\Rightarrow \theta = 76.3^\circ(76.3354\ldots)$

Use of area formula

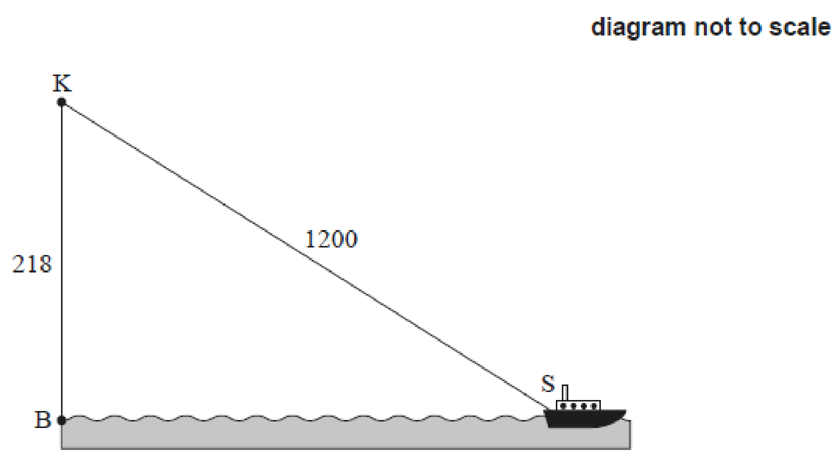
$\frac{1}{2} \times 85 \times \mathrm{BD} \times \sin(76.33\ldots^\circ) = \frac{3829.53\ldots}{2}$ **A1**

$\mathrm{BD} = 46.4(46.365\ldots)(\mathrm{m})$ **A1**

[6 marks]

Kacheena stands at point K, the top of a $218\ \mathrm{m}$ vertical cliff. The base of the cliff is located at point B. A ship is located at point S, $1200\ \mathrm{m}$ from Kacheena.

This information is shown in the following diagram.



4a. Find the angle of elevation from the ship to Kacheena. *[2 marks]*

Markscheme

$\sin\left(\hat{BSK}\right) = \frac{218}{1200}$ **OR** $\frac{\sin\left(\hat{BSK}\right)}{218} = \frac{\sin(90^\circ)}{1200}$ ***(M1)***

Note: Award ***M1*** for a correct trig formula. Accept other variables representing $\hat{BSK}$.

$\left(\hat{BSK} =\right) 10.5^\circ \ (10.4668\ldots)$ ***A1***

Note: Award ***A1*** for the radian answer, $0.182681\ldots$. Award ***M1A0*** if the candidate finds the correct angle of elevation but then uses it to find a complementary angle as their final answer.

[2 marks]

4b. Find the horizontal distance from the base of the cliff to the ship. *[2 marks]*

Markscheme

$SB^2 + 218^2 = 1200^2$ **OR** $\cos(10.4468\ldots) = \frac{SB}{1200}$ **OR** $\tan(10.4468\ldots) = \frac{218}{SB}$ **OR** $\frac{BS}{\sin(79.5331\ldots^\circ)} = \frac{1200}{\sin(90^\circ)}$ ***(M1)***

$1800\,(m) \ \left(\sqrt{1392476},\ 1180.03\ldots\right)$ ***A1***

[2 marks]

4c. Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. *[2 marks]*

Markscheme

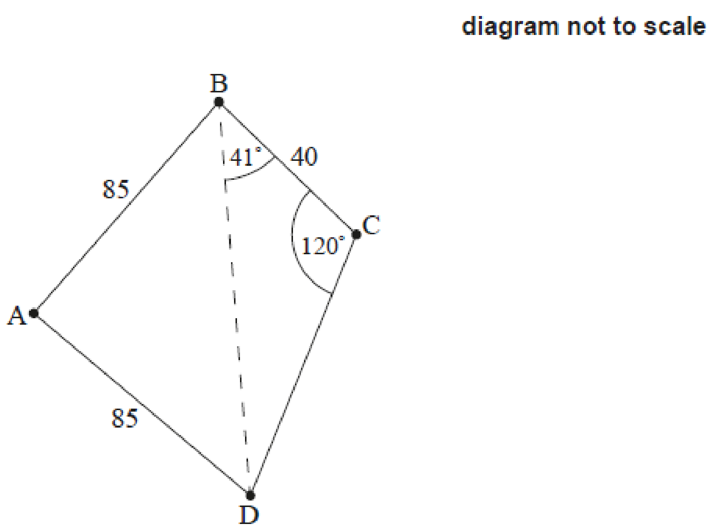
1.18×10^3 ***A1A1***

Note: Award ***A1*** for 1.18
Award ***A1*** for 10^3
Accept their rounded answer to part (b).
Award ***A0A0*** for answers of the type: 11.8×10^2

[2 marks]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$\text{AB} = 85\text{ m},\ \text{AD} = 85\text{ m},\ \text{BC} = 40\text{ m},\ \text{C}\hat{\text{B}}\text{D} = 41^\circ,\ \text{B}\hat{\text{C}}\text{D} = 120^\circ$



5a. Write down the value of angle BDC. *[1 mark]*

Markscheme

19° ***A1***

[1 mark]

5b. Hence use triangle BDC to find the length of path BD. *[3 marks]*

Markscheme

$\frac{\mathrm{BD}}{\sin 120^\circ} = \frac{40}{\sin 19^\circ}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted sine rule for BCD, ***A1*** for their correct substitution.

$(\mathrm{BD} =)\ 106\ \mathrm{m} \quad (106.401\ldots)$ ***A1***

[3 marks]

5c. Calculate the size of angle $\mathrm{B\hat{A}D}$, correct to five significant figures. *[3 marks]*

Markscheme

METHOD 1 (cosine rule)

$\cos \mathrm{BAD} = \frac{85^2+85^2-106.401\ldots^2}{2\times85\times85}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted cosine rule, ***A1*** for their correct substitution.

77.495 ***A1***

Note: Accept an answer of 77.149 from use of 3 sf answer from part (a). The final answer must be correct to five significant figures.

METHOD 2 (right angled trig/isosceles triangles)

$\sin\left(\frac{\mathrm{BAD}}{2}\right) = \frac{53.2008\ldots}{85}$ ***(A1)(M1)***

Note: Award ***A1*** for $53.2008\ldots$ seen. Award ***M1*** for correctly substituted trig ratio. Follow through from part (a).

$77.495\ldots$ ***A1***

Note: Use of 3 sf answer from part (a), results in 77.149.

[3 marks]

The size of angle $\mathrm{B\hat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\hat{A}D} = 77^\circ$ for the rest of this question.

5d. Find the area bounded by the path BD, and fences AB and AD. *[3 marks]*

Markscheme

EITHER

$(\text{Area} =)\ \frac{1}{2} \times 85 \times 85 \times \sin(77^\circ)$ ***(M1)(A1)***

Note: Award ***M1*** for substituted area formula, ***A1*** for correct substitution.
Award at most ***(M1)(A1)A0*** if an angle other than 77° is used.

OR

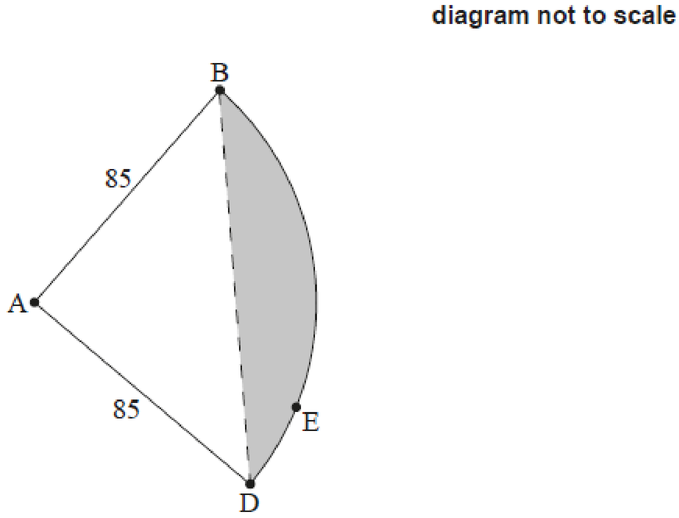
$(\text{Area} =)\ \frac{1}{2} \times (2 \times 85 \times \sin(38.5^\circ)) \times (85 \times \cos(38.5^\circ))$
(M1)(A1)

Note: Award ***M1*** for substituted area formula $A = \frac{1}{2}bh$, ***A1*** for correct substitution.

$3520\ \mathrm{m}^2 \quad (3519.91\ldots)$ ***A1***

[3 marks]

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.



5e. Write down the distance from A to E. *[1 mark]*

Markscheme

$85\ \mathrm{m}$ ***A1***

[1 mark]

5f. Find the perimeter of the proposed park, ABED. *[3 marks]*

Markscheme

$85 + 85 + \frac{77}{360} \times 2\pi \times 85$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted into $\frac{\theta}{360} \times 2\pi \times r$, ***M1*** for addition of AB and AD.

$284\ \mathrm{m}\ \ (284.231\ldots)$ ***A1***

[3 marks]

5g. Find the area of the shaded region in the proposed park. *[3 marks]*

Markscheme

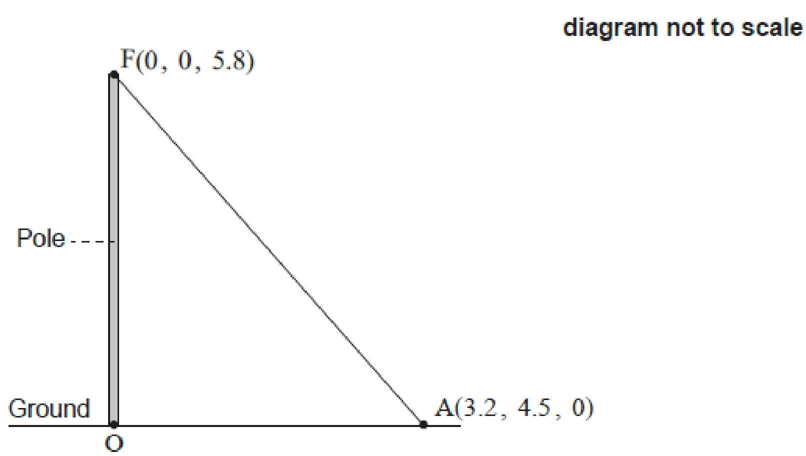
$\frac{77}{360} \times \pi \times (85)^2 - 3519.91\ldots$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted area of sector formula, ***M1*** for subtraction of their area from part (c).

$1330\ \mathrm{m}^2\ \ (1334.93\ldots)$ ***A1***

[3 marks]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0,\ 0,\ 5.8)$. All units are in metres.



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates $(3.2,\ 4.5,\ 0)$. The rope forms a straight line from A to F.

6a. Find the length of the rope connecting A to F. *[2 marks]*

Markscheme

$\sqrt{3.2^2 + 4.5^2 + 5.8^2}$ ***(M1)***

$= 8.01 \quad (8.00812\ldots)\text{ m}$ ***A1***

[2 marks]

6b. Find $\mathrm{F\hat{A}O}$, the angle the rope makes with the ground. *[2 marks]*

Markscheme

$\mathrm{F\hat{A}O} = \sin^{-1}\left(\frac{5.8}{8.00812\ldots}\right)$ **OR** $\cos^{-1}\left(\frac{5.52177\ldots}{8.00812\ldots}\right)$ **OR** $\tan^{-1}\left(\frac{5.8}{5.52177\ldots}\right)$ ***(M1)***

$46.4^\circ \quad (46.4077\ldots^\circ)$ ***A1***

[2 marks]