Mock exam review - probability *[57 marks]*

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

1a. Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. *[4 marks]*

Markscheme

$0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***

[4 marks]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

1b. Find the value of k such that the game is fair. *[3 marks]*

Markscheme

$0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$ ***(M1)(M1)***

Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k =)\ 24$ (points) ***A1***

[3 marks]

Elsie, a librarian, wants to investigate the length of time, T minutes, that people spent in her library on a particular day.

2a. State whether the variable T is discrete or continuous. *[1 mark]*

Markscheme

continuous ***A1***

[1 mark]

Elsie's data for 160 people who visited the library on that particular day is shown in the following table.

T (minutes)	$0 \le T < 20$	$20 \le T < 40$	$40 \le T < 60$	$60 \le T < 80$	$80 \le T < 100$
Frequency	50	62	k	14	8

2b. Find the value of k. *[2 marks]*

Markscheme

$160 - 50 - 62 - 14 - 8$ ***(M1)***

$(k =)\ 26$ ***A1***

[2 marks]

2c. Write down the modal class. *[1 mark]*

Markscheme

$20 \leq T < 40$ ***A1***

[1 mark]

2d. Write down the mid-interval value for this class. *[1 mark]*

Markscheme

30 ***A1***

[1 mark]

2e. Use Elsie's data to calculate an estimate of the mean time that people spent in the library. *[2 marks]*

Markscheme

33.5 minutes ***A2***

Note: ***FT*** from their value of k and their mid-interval value. Follow through from part (c)(ii) but only if mid-interval value lies in their interval.

[2 marks]

2f. Using the table, write down the maximum possible number of people who spent 35 minutes or less in the library on that day. *[1 mark]*

Markscheme

112 ***A1***

[1 mark]

Elsie assumes her data to be representative of future visitors to the library.

2g. Find the probability a visitor spends at least 60 minutes in the library. *[2 marks]*

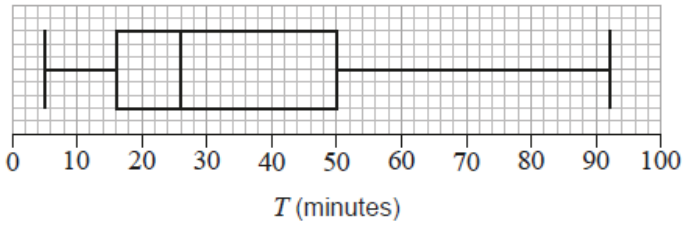
Markscheme

$\frac{22}{160}$ $\left[0.138,\ 0.1375,\ 13.75\%,\ \frac{11}{80}\right]$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

[2 marks]

The following box and whisker diagram shows the times, in minutes, that the 160 visitors spent in the library.



2h. Write down the median time spent in the library. *[1 mark]*

Markscheme

26 minutes ***A1***

[1 mark]

2i. Find the interquartile range. *[2 marks]*

Markscheme

$50 - 16$ ***(M1)***

Note: Award ***M1*** for both correct quartiles seen.

34 minutes ***A1***

[2 marks]

2j. Hence show that the longest time that a person spent in the library is not an outlier. *[3 marks]*

Markscheme

correct substitution into outlier formula ***(M1)***

$50 + 1.5 \times 34$

$= 101$ ***A1***

$92 < 101$ **OR** highest value on diagram < 101 ***R1***

not an outlier ***AG***

Note: Award ***R1*** for their correct comparison. Follow through from their part (h). Award ***R0*** if their conclusion is "it is an outlier", this contradicts Elsie's belief.

[3 marks]

Elsie believes the box and whisker diagram indicates that the times spent in the library are not normally distributed.

2k. Identify one feature of the box and whisker diagram which might support Elsie's belief. *[1 mark]*

Markscheme

EITHER

the diagram is not symmetric or equivalent

e.g the median is not in the center of the box or
the lengths of the whiskers are (very) different or (positive or right) skew

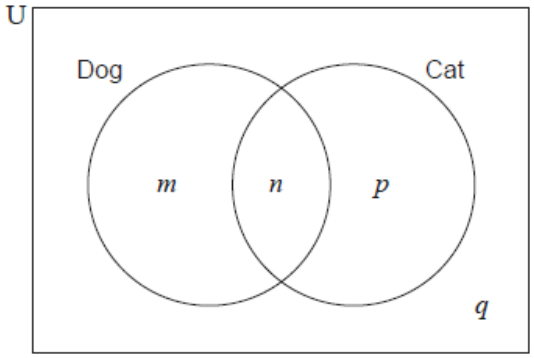
OR

the mean and median are (very) different; ***A1***

[1 mark]

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



Find the value of

3a. m. *[1 mark]*

Markscheme

$(m =)\ 54\%$ **A1**

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

3b. n. *[1 mark]*

Markscheme

$(n =)\ 14\%$ **A1**

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

3c. p. *[1 mark]*

Markscheme

$(p =)\ 22\%$ **A1**

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

3d. q. *[1 mark]*

Markscheme

$(q =)\ 10\%$ **A1**

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[1 mark]

3e. Find the percentage of students who have a dog or a cat or both. *[1 mark]*

Markscheme

$90\,(\%)$ **A1**

Note: Award ***A0*** for a decimal answer.

[1 mark]

Find the probability that a randomly chosen student

3f. has a dog but does not have a cat. *[1 mark]*

Markscheme

$0.54\ \left(\frac{54}{100},\ \frac{27}{50},\ 54\%\right)$ **A1**

[1 mark]

3g. has a dog given that they do not have a cat. *[2 marks]*

Markscheme

$\frac{54}{64}$ $\left(0.844,\ \frac{27}{32},\ 84.4\%,\ 0.84375\right)$ ***A1A1***

Note: Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[2 marks]

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events “being a school captain” and “having a dog” are independent.

Use Tim’s model to find the probability that in the next 10 years

3h. 5 school captains have a dog. *[2 marks]*

Markscheme

recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10,\ 0.68)$

$(\mathrm{P}(X = 5) =)\ 0.123\ \ (0.122940\ldots,\ 12.3\%)$ ***A1***

[2 marks]

3i. more than 3 school captains have a dog. *[2 marks]*

Markscheme

$1 - \mathrm{P}(X \leq 3)$ **OR** $\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(4 \leq X \leq 10)$ ***(M1)***

0.984 $(0.984497\ldots, 98.4\%)$ ***A1***

[2 marks]

3j. exactly 9 school captains in succession have a dog. *[3 marks]*

Markscheme

$(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

0.0199 $(0.0198957\ldots, 1.99\%)$ ***A1***

[3 marks]

John randomly chooses 10 students from the survey.

3k. State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. *[1 mark]*

Markscheme

EITHER

the probability is not constant ***A1***

OR

the events are not independent ***A1***

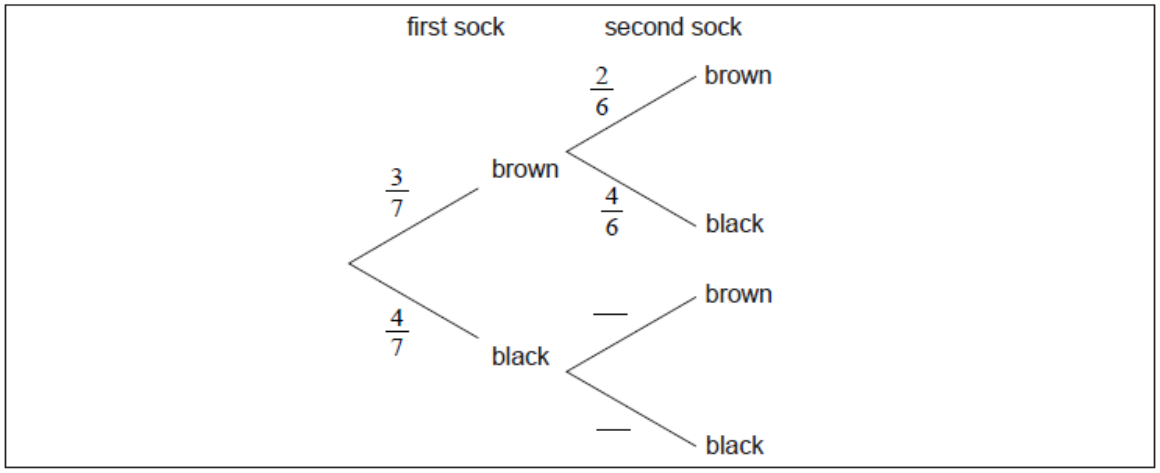
OR

the events should be modelled by the hypergeometric distribution instead ***A1***

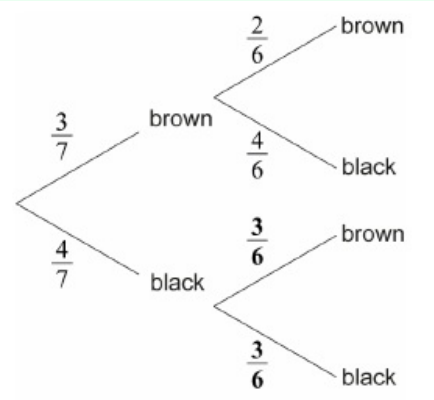
[1 mark]

Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

4a. Complete the tree diagram. *[1 mark]*



Markscheme



A1

Note: Award ***A1*** for both missing probabilities correct.

[1 mark]

4b. Find the probability that Karl takes two socks of the same colour. *[2 marks]*

Markscheme

multiplying along branches and then adding outcomes ***(M1)***

$\frac{3}{7} \times \frac{2}{6} + \frac{4}{7} \times \frac{3}{6}$

$= \frac{18}{42} \left(= \frac{3}{7} \approx 0.429\ (42.9\%)\right)$ ***A1***

[2 marks]

4c. Given that Karl has two socks of the same colour find the probability that he has two brown socks. *[3 marks]*

Markscheme

use of conditional probability formula **M1**

$\frac{\left(\frac{3}{7}\times\frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$ **A1**

$= \frac{6}{18}\ \left(= \frac{1}{3}\right)\ \left(\frac{252}{756},\ 0.333,\ 33.3\%\right)$ **A1**

[3 marks]

A factory produces bags of sugar with a labelled weight of 500 g. The weights of the bags are normally distributed with a mean of 500 g and a standard deviation of 3 g.

5a. Write down the percentage of bags that weigh more than 500 g. *[1 mark]*

Markscheme

50% **A1**

Note: Do not accept 0.5 or $\frac{1}{2}$.

[1 mark]

A bag that weighs less than 495 g is rejected by the factory for being underweight.

5b. Find the probability that a randomly chosen bag is rejected for being underweight. *[2 marks]*

Markscheme

$0.0478\ \ (0.0477903\ldots,\ 4.78\%)$ **A2**

[2 marks]

5c. A bag that weighs more than k grams is rejected by the factory for being overweight. The factory rejects 2% of bags for being overweight. *[3 marks]*

Find the value of k.

Markscheme

$\mathrm{P}(X < k) = 0.98$ **OR** $\mathrm{P}(X > k) = 0.02$ ***(M1)***

Note: Award ***(M1)*** for a sketch with correct region identified.

$506 \text{ g} \quad (506.161\ldots)$ ***A2***

[3 marks]

Roy is a member of a motorsport club and regularly drives around the Port Campbell racetrack.

The times he takes to complete a lap are normally distributed with mean 59 seconds and standard deviation 3 seconds.

6a. Find the probability that Roy completes a lap in less than 55 seconds. *[2 marks]*

Markscheme

$\mathrm{P}(T < 55)$ ***(M1)***

$0.0912 \quad (0.0912112\ldots)$ ***A1***

Note: Award ***M1*** for a correct calculator notation such as $\text{normal cdf}(0,\ 55,\ 59,\ 3)$ or $\text{normal cdf}\left(-1^{99},\ 55,\ 59,\ 3\right)$.

[2 marks]

Roy will complete a 20 lap race. It is expected that 8.6 of the laps will take more than t seconds.

6b. Find the value of t. *[3 marks]*

Markscheme

correct use of expected value

$8.6 = 20 \times p$ **OR** $(p =)\ 0.43$ seen ***(M1)***

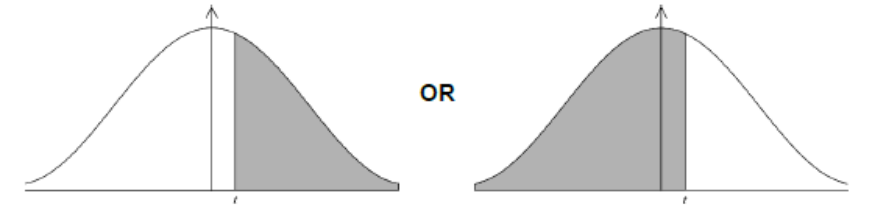
EITHER

correct probability statement

$\mathrm{P}(T > t) = 0.43$ **OR** $P(T < t) = 0.57$ ***(M1)***

OR

t indicated on sketch to communicate correct area ***(M1)***



THEN

$(t =)\ 59.5\ (\text{seconds})\ (59.5291\ldots)$ ***A1***

[3 marks]