Mock exam review - sequences, financial maths *[99 marks]*

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

1a. Find the repayment made each quarter. *[3 marks]*

Markscheme

N = 24
I % = 14
PV = −14000
FV = 0
P/Y = 4
C/Y = 4 ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept PV = 14000.

(€)871.82 ***A1***

[3 marks]

1b. Find the total amount paid for the car. *[2 marks]*

Markscheme

4 × 6 × 871.82 ***(M1)***

(€) 20923.68 ***A1***

[2 marks]

1c. Find the interest paid on the loan. *[2 marks]*

Markscheme

20923.68 − 14000 ***(M1)***

(€) 6923.68 ***A1***

[2 marks]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

1d. Find the amount to be borrowed for this option. *[2 marks]*

Markscheme

0.9 × 14000 (= 14000 − 0.10 × 14000) ***M1***

(€) 12600.00 ***A1***

[2 marks]

1e. Find the annual interest rate, r. *[3 marks]*

Markscheme

N = 72

PV = 12600

PMT = −250

FV = 0

P/Y = 12

C/Y = 12 ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept PV = −12600 provided PMT = 250.

12.56(%) ***A1***

[3 marks]

1f. State which option Bryan should choose. Justify your answer. *[2 marks]*

Markscheme

EITHER

Bryan should choose Option A ***A1***

no deposit is required ***R1***

Note: Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1***.

OR

Bryan should choose Option B ***A1***

cost of Option A (6923.69) > cost of Option B ($72 \times 250 - 12600 = 5400$) ***R1***

Note: Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

[2 marks]

1g. Bryan's car depreciates at an annual rate of 25 % per year. *[3 marks]*

Find the value of Bryan's car six years after it is purchased.

Markscheme

$14\,000\left(1-\frac{25}{100}\right)^6$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula. Award ***A1*** for correct substitutions.

= (€)2491.70 ***A1***

OR

N = 6

I% = −25

PV = ±14 000

P/Y = 1

C/Y = 1 ***(A1)(M1)***

Note: Award ***A1*** for PV = ±14 000, ***M1*** for other entries correct.

(€)2491.70 ***A1***

[3 marks]

Give your answers to this question correct to two decimal places.

Gen invests $2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

2a. Calculate the value of her savings after 10 years. *[2 marks]*

Markscheme

$2400(1.04)^{10} = \$3552.59$ ***M1A1***

[2 marks]

2b. The rate of inflation during this 10 year period is 1.5% per year. *[3 marks]*

Calculate the real value of her savings after 10 years.

Markscheme

real interest rate = $4 - 1.5 = 2.5\%$ ***A1***

$2400(1.025)^{10} = \$3072.20$ ***M1A1***

[3 marks]

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of $4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

3a. Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. *[3 marks]*

Markscheme

Use of finance solver ***M1***

$N = 360$, $I = 5\%$, Pmt = 4000, FV = 0, PpY = 12, CpY = 1 ***A1***

$755000 (correct to 3 s.f.) ***A1***

[3 marks]

3b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. *[3 marks]*

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

Markscheme

$N = 384$, $I = 5\%$, PV = 0, FV = 754638, PpY = 12, CpY = 1 ***M1A1***

\$817 per month (correct to 3 s.f.) ***A1***

[3 marks]

Paul wants to buy a car. He needs to take out a loan for \$7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay \$200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

4a. the number of months it will take for Paul to repay the loan. *[3 marks]*

Markscheme

evidence of using Finance solver on GDC ***M1***

$N = 39.8$ ***A1***

It will take 40 months ***A1***

[3 marks]

4b. the total amount that Paul has to pay. *[2 marks]*

Markscheme

$40 \times 200 = \$8000$ ***M1A1***

[2 marks]

Use option 2 to calculate

4c. the amount Paul pays each month. *[2 marks]*

Markscheme

Monthly payment = \$316 (\$315.70) ***M1A1***

[2 marks]

4d. the total amount that Paul has to pay. *[2 marks]*

Markscheme

$24 \times 315.7 = \$7580 \ (\$7576.80)$ ***M1A1***

[2 marks]

Give a reason why Paul might choose

4e. option 1. *[1 mark]*

Markscheme

The monthly repayment is lower, he might not be able to afford \$316 per month. ***R1***

[1 mark]

4f. option 2. *[1 mark]*

Markscheme

the total amount to repay is lower. ***R1***

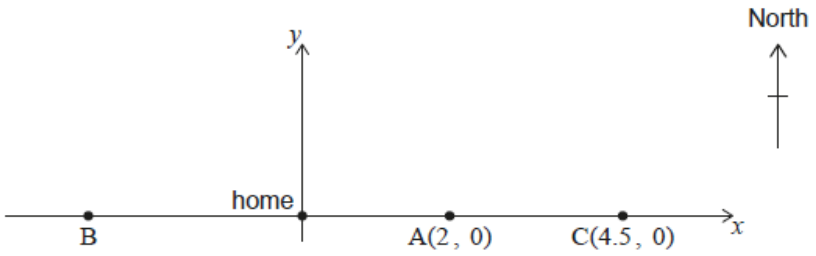
[1 mark]

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5,\ 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

5a. Show that the common ratio, r, is -1.5. *[2 marks]*

Markscheme

$4.5 = 2(r)^{3-1}$ ***(M1)***

$r = \pm 1.5$, ***R1***

(Some x-values are negative or direction from house changes each day)

$r = -1.5$ ***AG***

Note: Award ***M0R0AG*** for a verification approach $4.5 = 2(-1.5)^{3-1}$.

[2 marks]

On the 6th day, Kristi runs to point F.

5b. Find the location of point F. *[2 marks]*

Markscheme

$2(-1.5)^{6-1}$ ***(M1)***

EITHER

$(-15.2,\ 0)$ $(-15.1875\ldots,\ 0)$ ***A1***

OR

$x = -15.2\ \text{km}$ ***A1***

OR

15.2 km west (of the origin) ***A1***

Note: Award ***(M1)A0*** for an answer of "$-15.2\,(\text{km})$" without indicating that it is the x-value.

[2 marks]

5c. Find the total distance Kristi runs during the first 7 days of training. *[3 marks]*

Markscheme

choosing, $r = 1.5$ ***(A1)***

$\frac{2\left((1.5)^{7}-1\right)}{1.5-1}$ ***(M1)***

Note: Award ***M1*** for an attempt at a substituted GP formula with $n = 7$.
Award ***A0M1A0*** for substitution of $r = -1.5$, with $n = 7$ (this can be implied from a final answer of $14.4687\ldots$).

$64.3\ldots\ \text{km}$ $(64.3437\ldots)$ ***A1***

[3 marks]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

6a. Find the number of trees to be planted in the 15th month. *[3 marks]*

Markscheme

use of the n^{th} term of an arithmetic sequence formula ***(M1)***

$u_{15} = 85 + (15-1)\times 30$ ***(A1)***

505 ***A1***

[3 marks]

6b. Find the total number of trees to be planted in the first 15 months. *[2 marks]*

Markscheme

use of the sum of n terms of an arithmetic sequence formula ***(M1)***

$S_{15} = \frac{15}{2}(85 + 505)$ **OR** $\frac{15}{2}(2\times 85 + (15-1)\times 30)$

$4430 \ \ (4425)$ ***A1***

[2 marks]

6c. Find the mean number of trees planted per month during the first 15 months. *[2 marks]*

Markscheme

$\frac{4425}{15}$ **OR** $85+(8-1)\times 30$ ***(M1)***

295 ***A1***

Note: Accept $295.333\ldots$ from use of 3sf value from part (b).

[2 marks]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR}\ 500$ into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR}\ 10\ 000$.

7a. Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. *[4 marks]*

Markscheme

$I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\ 000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***

$(k =)18$ ***A1***

Note: Award ***(M0)(A0)(A0)A0*** for a final answer of 17 with no working. The final answer must be an integer.

[4 marks]

7b. For this value of k, find the interest earned in the savings account. *[3 marks]*

Express your answer correct to the nearest EUR.

Markscheme

$10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

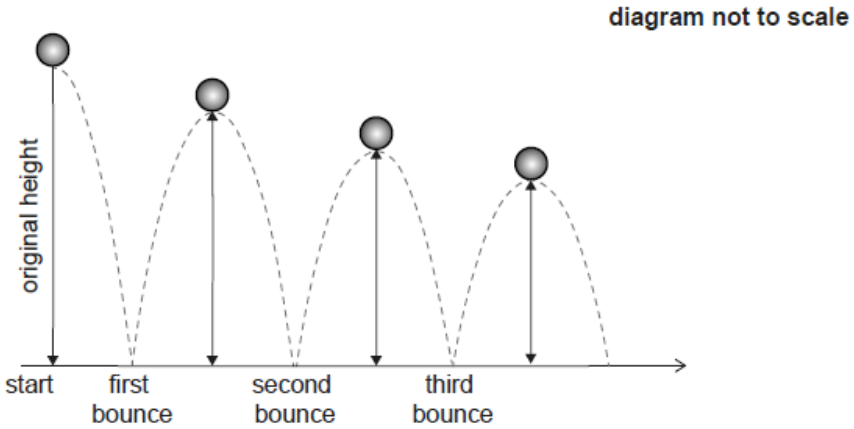
Note: Award ***(A1)*** for 10389 $(10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of k. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

$589\ \text{EUR}$ ***A1***

Note: Final answer must be to the nearest euro.

[3 marks]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



8a. Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\ \text{cm}$, to the nearest cm. *[2 marks]*

Markscheme

use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68\ \text{m}$

$= 68\ \text{cm}$ ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68\ \text{cm}$ ***AG***

[2 marks]

8b. Find the number of times, after the first bounce, that the maximum height reached is greater than $10\ \text{cm}$. *[2 marks]*

Markscheme

EITHER

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If $1.8\ \text{m}$ (or $180\ \text{cm}$) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If $1.53\ \text{m}$ (or $153\ \text{cm}$) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114\ \text{m}$ and $(0.85)^{18}(1.8) = 0.0966\ \text{m}$ ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

8c. Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. *[3 marks]*

Markscheme

EITHER

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8\times 0.85)\left(1-0.85^3\right)}{1-0.85}\ (= 3.935925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$18 + 2(3.935925)$

$= 9.67\ \text{m}\ \ (9.67185\ldots\ \text{m})$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)\left(1-0.85^4\right)}{1-0.85}\ (= 5.735925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67\ \text{m}\ \ (9.67185\ldots\ \text{m})$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8)+(0.85)^2(1.8)+(0.85)^3(1.8)\ \ (= 3.935925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8)+2(0.85)^2(1.8)+2(0.85)^3(1.8)$

$= 9.67\ \text{m}\ \ (9.67185\ldots\ \text{m})$ ***A1***

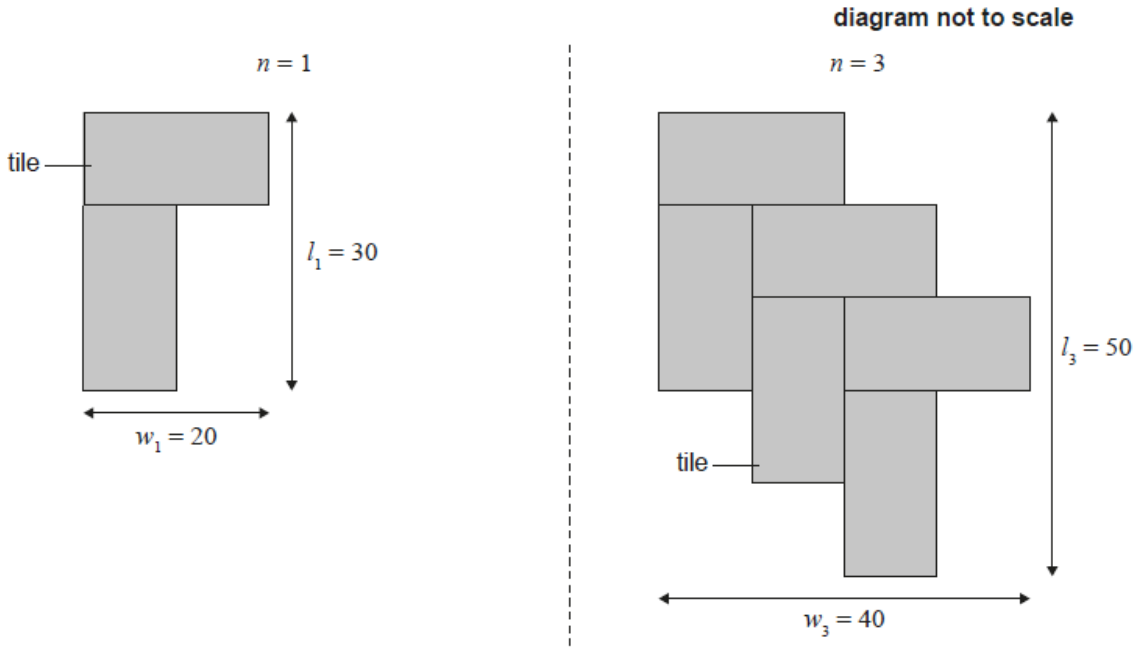
Note: Answers may be given in cm.

[3 marks]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\ \text{cm}$ wide and $20\ \text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

9a. a. *[1 mark]*

Markscheme

30 ***A1***

[1 mark]

9b. b. *[1 mark]*

Markscheme

40 ***A1***

[1 mark]

Write down an expression in terms of n for

9c. w_n. *[2 marks]*

Markscheme

arithmetic formula chosen ***(M1)***

$w_n = 20 + (n-1)10 \quad (= 10 + 10n)$ ***A1***

[2 marks]

9d. l_n. *[1 mark]*

Markscheme

arithmetic formula chosen

$l_n = 30 + (n-1)10 \quad (= 20 + 10n)$ ***A1***

[1 mark]

Eddie's lawn has a length $740\ \text{cm}$.

9e. Show that Eddie needs 144 tiles. *[2 marks]*

Markscheme

$740 = 30 + (n-1)10$ **OR** $740 = 20 - 10n$ ***M1***

$n = 72$ ***A1***

144 tiles ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

[2 marks]

9f. Find the value of w_n for this path. *[1 mark]*

Markscheme

$w_{72} = 730$ ***A1***

[1 mark]

9g. Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. *[3 marks]*

Markscheme

$(10 \times 20) \times 144$ ***(M1)***

$= 28800$ ***(A1)***

$2.88 \times 10^4 \text{ cm}^2$ ***A1***

Note: Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

[3 marks]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

9h. Find the cost of a single pack of five tiles. *[3 marks]*

Markscheme

EITHER

1 square metre $= 100\text{ cm} \times 100\text{ cm}$ ***(M1)***

(so, 50 tiles) and hence 10 packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10\text{ packs}}$)

OR

area covered by one pack of tiles is $(0.2\text{ m} \times 0.1\text{ m} \times 5 =)\ 0.1\text{ m}^2$ ***(A1)***

24.5×0.1 ***(M1)***

THEN

$\$2.45$ per pack (of 5 tiles) ***A1***

[3 marks]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

9i. Find the minimum number of packs of tiles Eddie will need to order. *[3 marks]*

Markscheme

$\frac{1.08 \times 144}{5}\ (= 31.104)$ ***(M1)(M1)***

Note: Award ***M1*** for correct numerator, ***M1*** for correct denominator.

32 (packs of tiles) ***A1***

[3 marks]

There is a fixed delivery cost of $\$35$.

9j. Find the total cost for Eddie's order. *[2 marks]*

Markscheme

$35 + (32 \times 2.45)$ ***(M1)***

$\$113$ (113.4) ***A1***

[2 marks]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

10a. fed to the dog per day. *[3 marks]*

Markscheme

EITHER

$115.5 = u_1 + (3-1) \times d \quad (115.5 = u_1 + 2d)$

$108 = u_1 + (8-1) \times d \quad (108 = u_1 + 7d)$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

1.5 (cups/day) ***A1***

Note: Answer must be written as a positive value to award ***A1***.

OR

$(d =) \frac{115.5-108}{5}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting a calculation using the difference between term 3 and term 8; ***A1*** for a correct substitution.

$(d =) 1.5$ (cups/day) ***A1***

[3 marks]

10b. remaining in the bag at the end of the first day. *[1 mark]*

Markscheme

$(u_1 =) 118.5$ (cups) ***A1***

[1 mark]

10c. Calculate the number of days that Scott can feed his dog with one bag of food. *[2 marks]*

Markscheme

attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$0 = 118.5 + (n-1) \times (-1.5)$

$(n =)$ 80 days ***A1***

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

10d. Determine the amount that Scott expects to spend on dog food in 2025. *[3 marks]*
Round your answer to the nearest dollar.

Markscheme

$(t_5 =)\ 625 \times 1.064^{(5-1)}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution

$\$801$ ***A1***

Note: The answer must be rounded to a whole number to award the final ***A1***.

[3 marks]

10e. *[1 mark]*

Calculate the value of $\sum_{n=1}^{10} \left(625 \times 1.064^{(n-1)} \right)$.

Markscheme

$(S_{10} =)$ (\$) 8390 $(8394.39\ldots)$ ***A1***

[1 mark]

10f. Describe what the value in part (d)(i) represents in this context. *[2 marks]*

Markscheme

EITHER

the total cost (of dog food) ***R1***

for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

OR

the total cost (of dog food) ***R1***

from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

[2 marks]

10g. Comment on the appropriateness of modelling this scenario with a geometric sequence. *[1 mark]*

Markscheme

EITHER
According to the model, the cost of dog food per year will eventually be too high to keep a dog.

OR
The model does not necessarily consider changes in inflation rate.

OR
The model is appropriate as long as inflation increases at a similar rate.

OR
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.

OR
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either “model” is mentioned specifically, or other mathematical terms such as “increasing” or “discrete quantities” are seen. Do not accept a contextual argument in isolation, e.g. “The dog will eventually die”.

[1 mark]