Mock exam review - sequences, financial maths *[99 marks]*

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

1a. Find the repayment made each quarter. *[3 marks]*

1b. Find the total amount paid for the car. *[2 marks]*

1c. Find the interest paid on the loan. *[2 marks]*

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

1d. Find the amount to be borrowed for this option. *[2 marks]*

1e. Find the annual interest rate, r. *[3 marks]*

1f. State which option Bryan should choose. Justify your answer. *[2 marks]*

1g. Bryan's car depreciates at an annual rate of 25 % per year. *[3 marks]*

Find the value of Bryan's car six years after it is purchased.

Give your answers to this question correct to two decimal places.

Gen invests $2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

2a. Calculate the value of her savings after 10 years. *[2 marks]*

2b. The rate of inflation during this 10 year period is 1.5% per year. *[3 marks]*

Calculate the real value of her savings after 10 years.

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of $4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

3a. Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. *[3 marks]*

3b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. *[3 marks]*

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

Paul wants to buy a car. He needs to take out a loan for $7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay $200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

4a. the number of months it will take for Paul to repay the loan. *[3 marks]*

4b. the total amount that Paul has to pay. *[2 marks]*

Use option 2 to calculate

4c. the amount Paul pays each month. *[2 marks]*

4d. the total amount that Paul has to pay. *[2 marks]*

Give a reason why Paul might choose

4e. option 1. *[1 mark]*

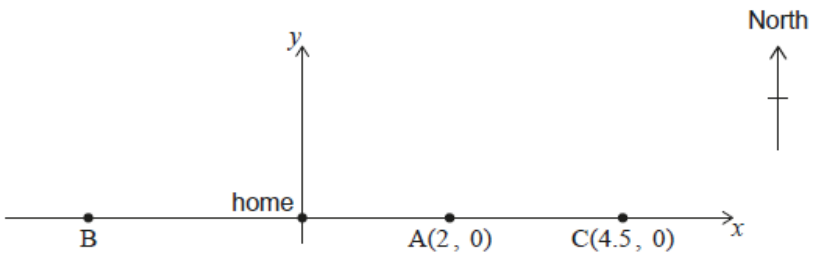
4f. option 2. *[1 mark]*

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5,\ 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

5a. Show that the common ratio, r, is -1.5. *[2 marks]*

On the $6\mathrm{th}$ day, Kristi runs to point F.

5b. Find the location of point F. *[2 marks]*

5c. Find the total distance Kristi runs during the first 7 days of training. *[3 marks]*

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

6a. Find the number of trees to be planted in the 15th month. *[3 marks]*

6b. Find the total number of trees to be planted in the first 15 months. *[2 marks]*

6c. Find the mean number of trees planted per month during the first 15 months. *[2 marks]*

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR } 500$ into the savings account.

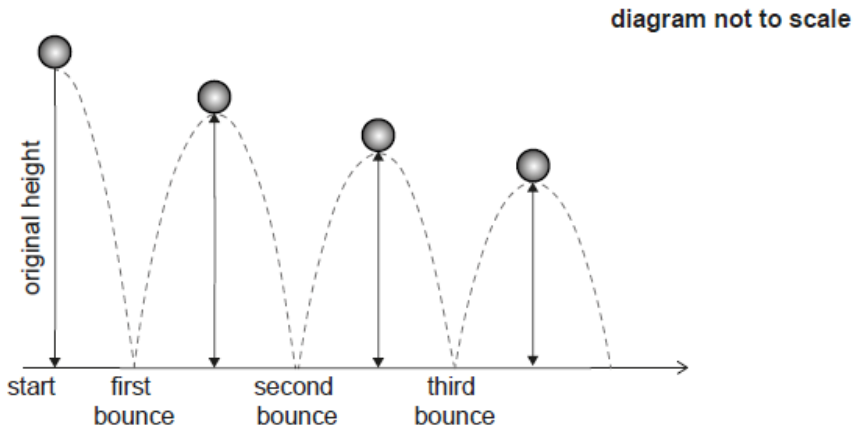
At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR } 10\ 000$.

7a. Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. *[4 marks]*

7b. For this value of k, find the interest earned in the savings account. *[3 marks]*

Express your answer correct to the nearest EUR.

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



8a. Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\ \text{cm}$, to the nearest cm. *[2 marks]*

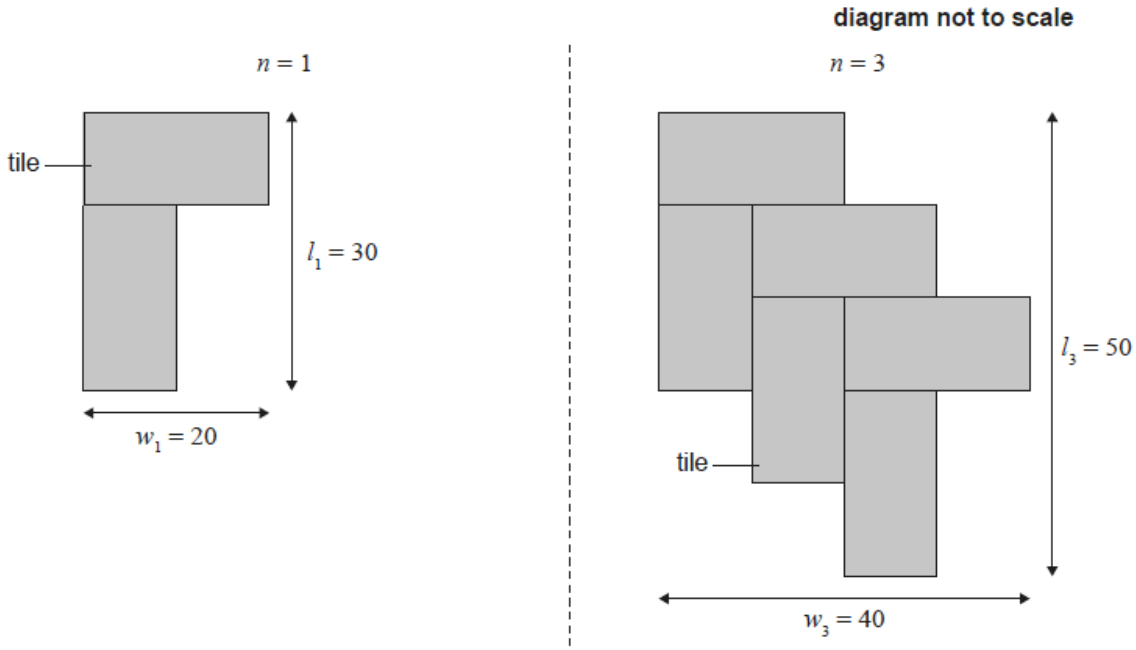
8b. Find the number of times, after the first bounce, that the maximum height reached is greater than $10\ \text{cm}$. *[2 marks]*

8c. Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. *[3 marks]*

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\ \text{cm}$ wide and $20\ \text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

9a. a. *[1 mark]*

9b. b. *[1 mark]*

Write down an expression in terms of n for

9c. w_n. *[2 marks]*

9d. l_n. *[1 mark]*

Eddie's lawn has a length $740\ \text{cm}$.

9e. Show that Eddie needs 144 tiles. *[2 marks]*

9f. Find the value of w_n for this path. *[1 mark]*

9g. Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. *[3 marks]*

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

9h. Find the cost of a single pack of five tiles. *[3 marks]*

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

9i. Find the minimum number of packs of tiles Eddie will need to order. *[3 marks]*

There is a fixed delivery cost of $\$35$.

9j. Find the total cost for Eddie's order. *[2 marks]*

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

10a. fed to the dog per day. *[3 marks]*

10b. remaining in the bag at the end of the first day. *[1 mark]*

10c. Calculate the number of days that Scott can feed his dog with one bag of food. *[2 marks]*

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

10d. Determine the amount that Scott expects to spend on dog food in 2025. *[3 marks]*
Round your answer to the nearest dollar.

10e. *[1 mark]*
Calculate the value of $\sum_{n=1}^{10}\left(625 \times 1.064^{(n-1)}\right)$.

10f. Describe what the value in part (d)(i) represents in this context. *[2 marks]*

10g. Comment on the appropriateness of modelling this scenario with a geometric sequence. *[1 mark]*