

# Volumes 2 [98 marks]

The function  $f$  is defined by  $f(x) = e^{2x}(3x - 4)$ , where  $x \in \mathbb{R}$ .

1a. Find  $f'(x)$ .

[3 marks]

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1b. Hence or otherwise, find the coordinates of the point on the graph of  $y = f(x)$  where the tangent is parallel to the line  $y = x$ .

[3 marks]

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The region enclosed by the curve  $y = f(x)$ , the  $x$ -axis and the  $y$ -axis is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

1c. Find the volume of this solid.

*[4 marks]*

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Consider a function  $g$  such that  $g(0) = 1$  and  $g'(0) = 2$ .

Find the value of

1d.  $(f \circ g)(0)$ .

[2 marks]

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1e.  $(f \circ g)'(0)$ .

[3 marks]

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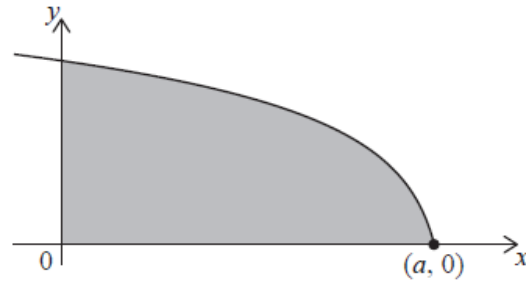
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Let  $f(x) = \sqrt{12 - 2x}$ ,  $x \leq a$ . The following diagram shows part of the graph of  $f$ . The shaded region is enclosed by the graph of  $f$ , the  $x$ -axis and the  $y$ -axis.



The graph of  $f$  intersects the  $x$ -axis at the point  $(a, 0)$ .

2a. Find the value of  $a$ .

[2 marks]

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2b. Find the volume of the solid formed when the shaded region is revolved  $360^\circ$  about the  $x$ -axis. [5 marks]

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Consider the function defined by  $f(x) = \frac{kx-5}{x-k}$ , where  $x \in \mathbb{R} \setminus \{k\}$  and  $k^2 \neq 5$ .

3a. State the equation of the vertical asymptote on the graph of  $y = f(x)$ . [1 mark]

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3b. State the equation of the horizontal asymptote on the graph of  $y = f(x)$ . [1 mark]

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Consider the case where  $k = 3$ .

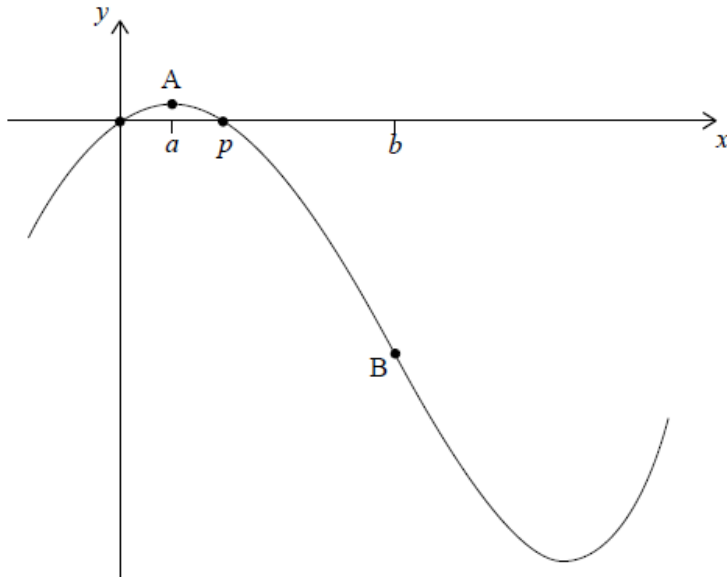
- 3d. Sketch the graph of  $y = f(x)$ , stating clearly the equations of any asymptotes and the coordinates of any points of intersections with the coordinate axes. *[3 marks]*







Let  $f(x) = x^4 - 54x^2 + 60x$ , for  $-1 \leq x \leq 6$ . The following diagram shows the graph of  $f$ .



There are  $x$ -intercepts at  $x = 0$  and at  $x = p$ . There is a maximum at point A where  $x = a$ , and a point of inflexion at point B where  $x = b$ .

4a. Find the value of  $p$ .

[2 marks]

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4b. Write down the coordinates of A.

[2 marks]

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4c. Find the equation of the tangent to the graph of  $f$  at A.

[2 marks]

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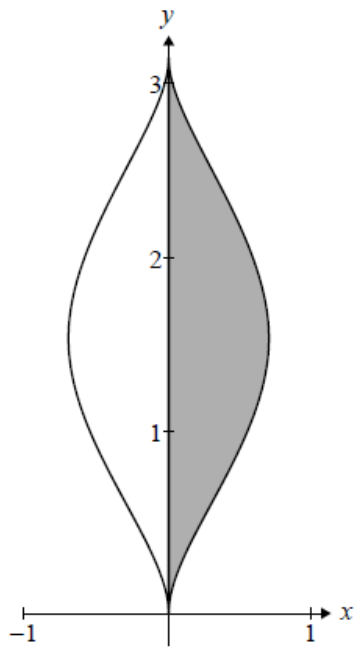
4d. Find the coordinates of B.

[5 marks]

A large rectangular box containing 20 horizontal dotted lines for writing the answer.



The following diagram shows part of the graph of  $2x^2 = \sin^3 y$  for  $0 \leq y \leq \pi$ .



5a. Using implicit differentiation, find an expression for  $\frac{dy}{dx}$ . *[4 marks]*

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Let  $f(x) = 6 - \ln(x^2 + 2)$ , for  $x \in \mathbb{R}$ . The graph of  $f$  passes through the point  $(p, 4)$ , where  $p > 0$ .

7a. Find the value of  $p$ .

[2 marks]

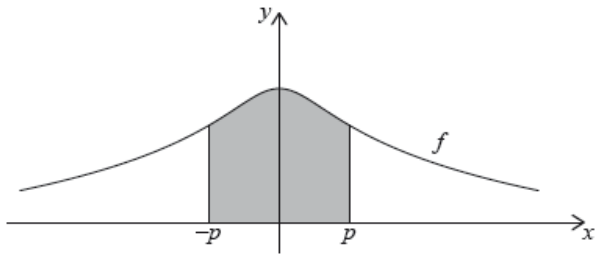
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7b. The following diagram shows part of the graph of  $f$ .

[3 marks]



The region enclosed by the graph of  $f$ , the  $x$ -axis and the lines  $x = -p$  and  $x = p$  is rotated  $360^\circ$  about the  $x$ -axis. Find the volume of the solid formed.

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Consider the curve defined by the equation  $4x^2 + y^2 = 7$ .

8. Find the volume of the solid formed when the region bounded by the curve, the  $x$ -axis for  $x \geq 0$  and the  $y$ -axis for  $y \geq 0$  is rotated through  $2\pi$  about the  $x$ -axis. *[3 marks]*

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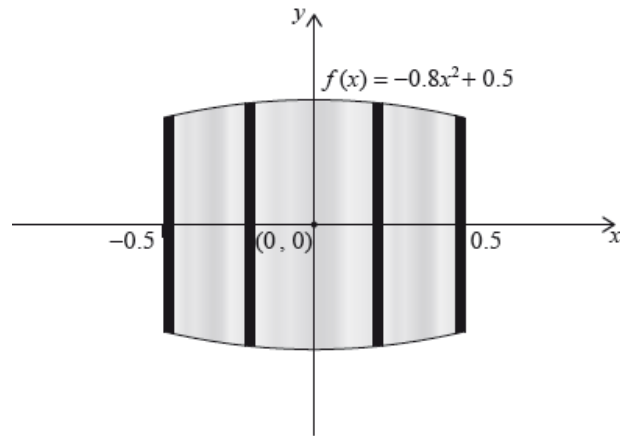
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All lengths in this question are in metres.

Let  $f(x) = -0.8x^2 + 0.5$ , for  $-0.5 \leq x \leq 0.5$ . Mark uses  $f(x)$  as a model to create a barrel. The region enclosed by the graph of  $f$ , the  $x$ -axis, the line  $x = -0.5$  and the line  $x = 0.5$  is rotated  $360^\circ$  about the  $x$ -axis. This is shown in the following diagram.



9a. Use the model to find the volume of the barrel.

[3 marks]

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9b. The empty barrel is being filled with water. The volume  $V \text{ m}^3$  of water in [3 marks] the barrel after  $t$  minutes is given by  $V = 0.8(1 - e^{-0.1t})$ . How long will it take for the barrel to be half-full?

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