Mathematics: analysis and approaches

Practice paper 2 HL — Total 110

Section A [56 marks]

1. [Maximum mark: 4]

A data set consisting of 16 test scores has mean 14.5. One test score of 9 requires a second marking and is removed from the data set.

Find the mean of the remaining 15 test scores.

2. [Maximum mark: 5]

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by

$$v = 4t^2 - 6t + 9 - 2\sin(4t),\ 0 \le t \le 1.$$

The particle's acceleration is zero at $t = T$.

(a) Find the value of T. [2]

Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$.

(b) Show that $s_2 > s_1$. [3]

3. [Maximum mark: 8]

The following table shows the systolic blood pressures, p mmHg, and the ages, t years, of 6 patients at a medical clinic.

Patient	P1	P2	P3	P4	P5	P6
t (years)	40	72	35	47	21	61
p (mmHg)	105	145	100	130	95	132

(a) (i) Determine the value of Pearson's product-moment correlation coefficient, r, for these data. [2]

(ii) Interpret, in context, the value of r found in part (a) (i). [1]

The relationship between t and p can be modelled by the regression line of p on t with equation $p = at + b$.

(b) Find the equation of the regression line of p on t. [2]

A 50-year-old patient enters the medical clinic for his appointment.

(c) Use the equation from part (b) to predict this patient's systolic blood pressure. [2]

A 16-year-old male patient enters the medical clinic for his appointment.

(d) Explain why the regression equation from part (b) should not be used to predict this patient's systolic blood pressure. [1]

4. [Maximum mark: 5]

The quadratic equation $(k-1)x^2 + 2x + (2k-3) = 0$, where $k \in \mathbb{R}$, has real distinct roots.

Find the range of possible values for k.

5. [Maximum mark: 7]

Consider the curves $y = x^2 \sin x$ and $y = -1 - \sqrt{1 + 4(x+2)^2}$ for $-\pi \le x \le 0$.

(a) Find the x- coordinates of the points of intersection of the two curves. [3]

(b) Find the area, A, of the region enclosed by the two curves. [4]

6. [Maximum mark: 7]

The curve C has equation $e^{2y} = x^3 + y$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2}{2e^{2y} - 1}$. [3]

The tangent to C at the point P is parallel to the y-axis.

(b) Find the x-coordinate of P. [4]

7. [Maximum mark: 8]

Consider the identity $\frac{2+7x}{(1+2x)(1-x)} \equiv \frac{A}{1+2x} + \frac{B}{1-x}$, where $A, B \in \mathbb{Z}$.

(a) Find the value of A and the value of B. [3]

(b) Hence, expand $\frac{2+7x}{(1+2x)(1-x)}$ in ascending powers of x, up to and including the term in x^2. [4]

(c) Give a reason why the series expansion found in part (b) is not valid for $x = \frac{3}{4}$. [1]

8. [Maximum mark: 6]

Prove by contradiction that $\log_2 5$ is an irrational number.

9. [Maximum mark: 6]

A biased coin is weighted such that the probability, p, of obtaining a tail is 0.6. The coin is tossed repeatedly and independently until a tail is obtained.

Let E be the event "obtaining the first tail on an even numbered toss".

Find $P(E)$.

Section B [54 marks]

10. [Maximum mark: 15]

The time, *T* minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

(a) By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. [4]

Use $\mu = 32.29$ in the remainder of the question.

(b) Find the 86th percentile time to complete the jigsaw puzzle. [2]

(c) Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. [2]

Six randomly chosen people complete the jigsaw puzzle.

(d) Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. [3]

Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle.

(e) Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle. [4]

11. [Maximum mark: 17]

The points $\mathrm{A}(5,-2,5)$, $\mathrm{B}(5,4,-1)$, $\mathrm{C}(-1,-2,-1)$ and $\mathrm{D}(7,-4,-3)$ are the vertices of a right-pyramid.

(a) Find the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [2]

(b) Use a vector method to show that $\mathrm{B\hat{A}C} = 60°$. [3]

(c) Show that the Cartesian equation of the plane Π that contains the triangle ABC is $-x + y + z = -2$. [3]

The line L passes through the point D and is perpendicular to Π.

(d) (i) Find a vector equation of the line L.

(ii) Hence determine the minimum distance, $d_{\min}$, from D to Π. [5]

(e) Find the volume of right-pyramid ABCD. [4]

12. [Maximum mark: 22]

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f\left(\frac{y}{x}\right),\ x > 0.$$

(a) Use the substitution $y = vx$ to show that $\int \frac{\mathrm{d}v}{f(v) - v} = \ln x + C$ where C is an arbitrary constant. [3]

The curve $y = f(x)$ for $x > 0$ has a gradient function given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 + 3xy + 2x^2}{x^2}.$$

The curve passes through the point $(1, -1)$.

(b) By using the result from part (a) or otherwise, solve the differential equation and hence show that the curve has equation $y = x(\tan(\ln x) - 1)$. [9]

(c) The curve has a point of inflexion at (x_1, y_1) where $e^{-\frac{\pi}{2}} < x_1 < e^{\frac{\pi}{2}}$. Determine the coordinates of this point of inflexion. [6]

(d) Use the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 + 3xy + 2x^2}{x^2}$ to show that the points of zero gradient on the curve lie on two straight lines of the form $y = mx$ where the values of m are to be determined. [4]