International Baccalaureate
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Mathematics: analysis and approaches

Practice paper 1 SL markscheme **Total 80**

Section A [35 marks]

1.

METHOD 1

$f(x)=\int 3\sqrt{x}\ \mathrm{d}x$ **(A1)**

attempts to integrate **(M1)**

$f(x)=2x^{\frac{3}{2}}+C\ \left(=2x\sqrt{x}+C\right)$ **A1**

uses $f(1)=3$ to obtain $3=2(1)^{\frac{3}{2}}+C$ and so $C=1$ **M1**

substitutes $x=4$ into their expression for $f(x)$ **(M1)**

so $f(4)=17$ **A1**

METHOD 2

$\int_1^4 f'(x)\mathrm{d}x=\int_1^4 3\sqrt{x}\ \mathrm{d}x$ **(A1)**

attempts to integrate both sides **(M1)**

$\left[f(x)\right]_1^4=\left[2x^{\frac{3}{2}}\right]_1^4$ **A1**

$f(4)-f(1)=16-2$ **M1**

uses $f(1)=3$ to find their value of $f(4)$ **(M1)**

$f(4)-3=16-2$

so $f(4)=17$ **A1**

Total [6 marks]

2.

METHOD 1

$2\ln x-\ln 9=4$

uses $m\ln x=\ln x^m$ **(M1)**

$\ln x^2-\ln 9=4$

uses $\ln a-\ln b=\ln\frac{a}{b}$ **(M1)**

$\ln\frac{x^2}{9}=4$

$\frac{x^2}{9}=e^4$ **A1**

$x^2=9e^4\Rightarrow x=\sqrt{9e^4}$ ($x>0$) **A1**

$x=3e^2$ ($p=3,q=2$) **A1**

METHOD 2

expresses 4 as $4\ln e$ and uses $\ln x^m=m\ln x$ **(M1)**

$2\ln x=2\ln 3+4\ln e\ (\ln x=\ln 3+2\ln e)$ **A1**

uses $2\ln e=\ln e^2$ and $\ln a+\ln b=\ln ab$ **(M1)**

$\ln x=\ln\left(3e^2\right)$ **A1**

$x=3e^2$ ($p=3,q=2$) **A1**

METHOD 3

expresses 4 as $4\ln e$ and uses $m\ln x=\ln x^m$ **(M1)**

$\ln x^2=\ln 3^2+\ln e^4$ **A1**

uses $\ln a+\ln b=\ln ab$ **(M1)**

$\ln x^2=\ln\left(3^2e^4\right)$

$x^2=3^2e^4\Rightarrow x=\sqrt{3^2e^4}$ ($x>0$) **A1**

so $x=3e^2\ (x>0)$ ($p=3,q=2$) **A1**

Total [5 marks]

3.

uses $\sum \mathrm{P}(X = x)(=1)$ **(M1)**

$k^2 + (7k+2) + (-2k) + (3k^2)(=1)$

$4k^2 + 5k + 1(=0)$ **A1**

EITHER

attempts to factorize their quadratic **M1**

$(k+1)(4k+1) = 0$

OR

attempts use of the quadratic formula on their equation **M1**

$k = \dfrac{-5 \pm \sqrt{5^2 - 4(4)(1)}}{8} \quad \left(= \dfrac{-5 \pm 3}{8}\right)$

THEN

$k = -1, -\dfrac{1}{4}$ **A1**

rejects $k = -1$ as this value leads to invalid probabilities, for example, $\mathrm{P}(X = 2) = -5 < 0$ **R1**

so $k = -\dfrac{1}{4}$ **A1**

Note: Award **R0A1** if $k = -\dfrac{1}{4}$ is stated without a valid reason given for rejecting $k = -1$.

Total [6 marks]

4.

(a) **EITHER**

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1 + u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1 + (3u_1 + 8)}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

(b) $d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

[4 marks]

Total [6 marks]

5.

$(f \circ g)(x) = ax + b - 2$ **(M1)**

$(f \circ g)(2) = -3 \Rightarrow 2a + b - 2 = -3 \ (2a + b = -1)$ **A1**

$(g \circ f)(x) = a(x-2) + b$ **(M1)**

$(g \circ f)(1) = 5 \Rightarrow -a + b = 5$ **A1**

a valid attempt to solve their two linear equations for a and b **M1**

so $a = -2$ and $b = 3$ **A1**

Total [6 marks]

6.

attempts either product rule or quotient rule differentiation **M1**

EITHER

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3x^2 + bx}{(x+2)^2} + \frac{6x+b}{x+2}$ **A1**

OR

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+2)(6x+b) - (3x^2 + bx)}{(x+2)^2}$ **A1**

Note: Award **A0** if the denominator is incorrect. Subsequent marks can be awarded.

THEN

sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ **M1**

$(x+2)(6x+b) - (3x^2 + bx) = 0$

$3x^2 + 12x + 2b = 0$ **A1**

(exactly two points of zero gradient requires) $12^2 - (4)(3)(2b) > 0$ **M1**

$b < 6$ **A1**

Total [6 marks]

Section B [45 marks]

7.

(a) $P = 2x + 2y$ **(A1)**

$= 2x + 2\left(4 - x^2\right)$ **A1**

so $P = -2x^2 + 2x + 8$ **AG**

[2 marks]

(b) **METHOD 1**

EITHER

uses the axis of symmetry of a quadratic **(M1)**

$x = -\dfrac{2}{2(-2)}$

OR

forms $\dfrac{\mathrm{d}P}{\mathrm{d}x} = 0$ **(M1)**

$-4x + 2 = 0$

THEN

$x = \dfrac{1}{2}$ **A1**

substitutes their value of x into $y = 4 - x^2$ **(M1)**

$y = 4 - \left(\dfrac{1}{2}\right)^2$

$y = \dfrac{15}{4}$ **A1**

so the dimensions of rectangle ORST of maximum perimeter are $\dfrac{1}{2}$ by $\dfrac{15}{4}$

EITHER

substitutes their value of x into $P=-2x^2+2x+8$ **(M1)**

$$P=-2\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)+8$$

OR

substitutes their values of x and y into $P=2x+2y$ **(M1)**

$$P=2\left(\frac{1}{2}\right)+2\left(\frac{15}{4}\right)$$

$P=\frac{17}{2}$ **A1**

so the maximum perimeter is $\frac{17}{2}$

METHOD 2

attempts to complete the square **M1**

$P=-2\left(x-\frac{1}{2}\right)^2+\frac{17}{2}$ **A1**

$x=\frac{1}{2}$ **A1**

substitutes their value of x into $y=4-x^2$ **(M1)**

$$y=4-\left(\frac{1}{2}\right)^2$$

$y=\frac{15}{4}$ **A1**

so the dimensions of rectangle ORST of maximum perimeter are $\frac{1}{2}$ by $\frac{15}{4}$

$P=\frac{17}{2}$ **A1**

so the maximum perimeter is $\frac{17}{2}$

[6 marks]

(c) substitutes $y = 4 - x^2$ into $A = xy$ **(M1)**

$A = x\left(4 - x^2\right)\ \left(= 4x - x^3\right)$ **A1**

[2 marks]

(d) $\frac{\mathrm{d}A}{\mathrm{d}x} = 4 - 3x^2$ **A1**

attempts to solve their $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ for x **(M1)**

$4 - 3x^2 = 0$

$\Rightarrow x = \frac{2}{\sqrt{3}}\left(= \frac{2\sqrt{3}}{3}\right)\ (x > 0)$ **A1**

substitutes their (positive) value of x into $y = 4 - x^2$ **(M1)**

$y = 4 - \left(\frac{2}{\sqrt{3}}\right)^2$

$y = \frac{8}{3}$ **A1**

[5 marks]

(e) $A = \frac{16}{3\sqrt{3}}\left(= \frac{16\sqrt{3}}{9}\right)$ **A1**

[1 mark]

Total [16 marks]

8.

(a) for example,

a reflection in the x-axis (in the line $y=0$) **A1**

a horizontal translation (shift) 3 units to the left **A1**

a vertical translation (shift) down by 1 unit **A1**

Note: Award **A1** for each correct transformation applied in a correct position in the sequence. Do not accept use of the "move" for a translation.

Note: Award **A1A1A1** for a correct alternative sequence of transformations. For example,

a vertical translation (shift) down by 1 unit, followed by a horizontal translation (shift) 3 units to the left and then a reflection in the line $y=-1$.

[3 marks]

(b) range is $f(x)\le -1$ **A1**

Note: Correct alternative notations include $]-\infty,-1]$, $(-\infty,-1]$ or $y\le -1$.

[1 mark]

(c) $-1-\sqrt{y+3}=x$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$\sqrt{y+3}=-x-1\left(=-(x+1)\right)$ **A1**

$y+3=(x+1)^2$ **A1**

so $f^{-1}(x)=(x+1)^2-3$ $\left(f^{-1}(x)=x^2+2x-2\right)$ **A1**

domain is $x\le -1$ **A1**

Note: Correct alternative notations include $]-\infty,-1]$ or $(-\infty,-1]$.

[5 marks]

(d) the point of intersection lies on the line $y = x$

EITHER

$(x+1)^2 - 3 = x$ **M1**

attempts to solve their quadratic equation **M1**

for example, $(x+2)(x-1) = 0$ or $x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \dfrac{-1 \pm 3}{2}\right)$

OR

$-1 - \sqrt{x+3} = x$ **M1**

$\left(-1 - \sqrt{x+3}\right)^2 = x^2 \Rightarrow 2\sqrt{x+3} + x + 4 = x^2$

substitutes $2\sqrt{x+3} = -2(x+1)$ to obtain $-2(x+1) + x + 4 = x^2$

attempts to solve their quadratic equation **M1**

for example, $(x+2)(x-1) = 0$ or $x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \dfrac{-1 \pm 3}{2}\right)$

THEN

$x = -2, 1$ **A1**

as $x \le -1$, the only solution is $x = -2$ **R1**

so the coordinates of the point of intersection are $(-2, -2)$ **A1**

Note: Award **R0A1** if $(-2, -2)$ is stated without a valid reason given for rejecting $(1, 1)$.

[5 marks]

Total [15 marks]

9.

(a) attempts to find $h(0)$ **(M1)**

$h(0)=0.4\cos(0)+1.8(=2.2)$

2.2 (m) (above the ground) **A1**

[2 marks]

(b) **EITHER**

uses the minimum value of $\cos(\pi t)$ which is -1 **M1**

$0.4(-1)+1.8$ (m)

OR

the amplitude of motion is 0.4 (m) and the mean position is 1.8 (m) **M1**

OR

finds $h'(t)=-0.4\pi\sin(\pi t)$, attempts to solve $h'(t)=0$ for t and determines that the minimum height above the ground occurs at $t=1,3,\ldots$ **M1**

$0.4(-1)+1.8$ (m)

THEN

1.4 (m) (above the ground) **A1**

[2 marks]

(c) **EITHER**

the ball is released from its maximum height and returns there a period later **R1**

the period is $\frac{2\pi}{\pi}(=2)$ (s) **A1**

OR

attempts to solve $h(t)=2.2$ for t **M1**

$\cos(\pi t)=1$

$t=0,2,...$ **A1**

THEN

so it takes 2 seconds for the ball to return to its initial position for the first time **AG**

[2 marks]

(d) $0.4\cos(\pi t)+1.8=1.8+0.2\sqrt{2}$ **(M1)**

$0.4\cos(\pi t)=0.2\sqrt{2}$

$\cos(\pi t)=\frac{\sqrt{2}}{2}$ **A1**

$\pi t=\frac{\pi}{4},\frac{7\pi}{4}$ **(A1)**

Note: Accept extra correct positive solutions for πt.

$t=\frac{1}{4},\frac{7}{4}$ $(0\le t\le 2)$ **A1**

Note: Do not award **A1** if solutions outside $0\le t\le 2$ are also stated.

the ball is less than $1.8+0.2\sqrt{2}$ metres above the ground for $\frac{7}{4}-\frac{1}{4}$(s)

1.5 (s) **A1**

[5 marks]

(e) **EITHER**

attempts to find $h'(t)$ **(M1)**

OR

recognizes that $h'(t)$ is required **(M1)**

THEN

$h'(t) = -0.4\pi\sin(\pi t)$ **A1**

attempts to evaluate their $h'\left(\frac{1}{3}\right)$ **(M1)**

$h'\left(\frac{1}{3}\right) = -0.4\pi\sin\frac{\pi}{3}$

$= -0.2\pi\sqrt{3}\ \left(\text{ms}^{-1}\right)$ **A1**

Note: Accept equivalent correct answer forms where $p \in \mathbb{Q}$. For example, $-\frac{1}{5}\pi\sqrt{3}$.

[4 marks]

Total [15 marks]