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Mathematics: analysis and approaches

Practice paper 1 SL — Total 80

Section A [35 marks]

1. [Maximum mark: 6]

The derivative of a function f is given by $f'(x) = 3\sqrt{x}$.

Given that $f(1) = 3$, find the value of $f(4)$.

2. [Maximum mark: 5]

Solve the equation $2\ln x = \ln 9 + 4$. Give your answer in the form $x = p\mathrm{e}^q$ where $p, q \in \mathbb{Z}^+$.

3. [Maximum mark: 6]

The following table shows the probability distribution of a discrete random variable X where $x = 1, 2, 3, 4$.

x	1	2	3	4
$\mathrm{P}(X = x)$	k^2	$7k + 2$	$-2k$	$3k^2$

Find the value of k, justifying your answer.

4. [Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1, 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

5. [Maximum mark: 6]

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a, b \in \mathbb{R}$.

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b.

6. [Maximum mark: 6]

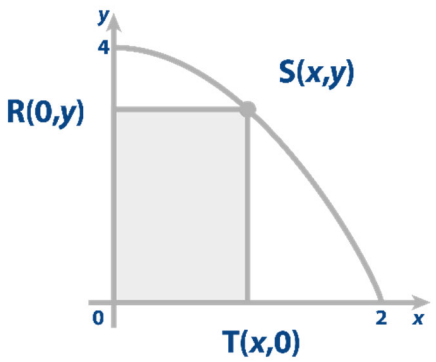
Consider the function $f(x) = \dfrac{3x^2 + bx}{x + 2}$ where $x \neq -2$ and $b \in \mathbb{R}$.

Find the set of values for *b* such that the graph of f has exactly two points with a gradient of zero.

Section B [45 marks]

7. [Maximum mark: 16]

The following diagram shows the graph of $y = 4 - x^2$, $0 \leq x \leq 2$ and rectangle ORST. The rectangle has a vertex at the origin O, a vertex on the y- axis at the point $\mathrm{R}(0, y)$, a vertex on the x- axis at the point $\mathrm{T}(x, 0)$ and a vertex at point $\mathrm{S}(x, y)$ on the graph.



Let P represent the perimeter of rectangle ORST.

(a) Show that $P = -2x^2 + 2x + 8$. [2]

(b) Find the dimensions of rectangle ORST that has maximum perimeter and determine the value of the maximum perimeter. [6]

Let A represent the area of rectangle ORST.

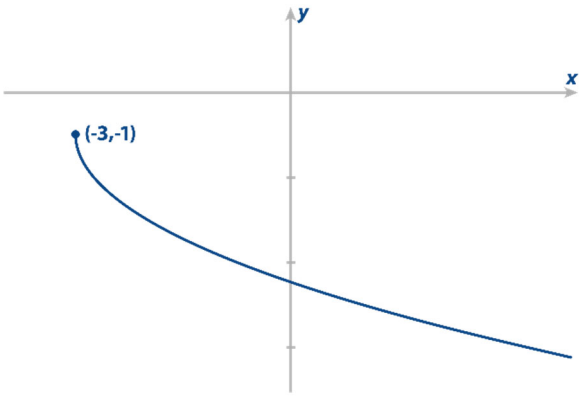
(c) Find an expression for A in terms of x. [2]

(d) Find the dimensions of rectangle ORST that has maximum area. [5]

(e) Determine the maximum area of rectangle ORST. [1]

8. [Maximum mark: 14]

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



(a) Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. [3]

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

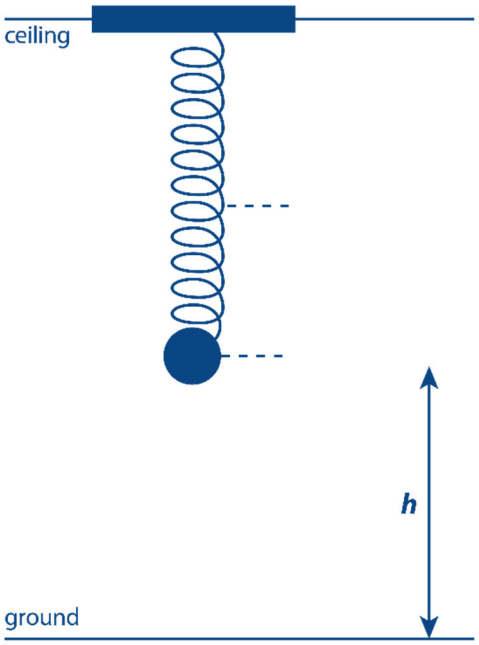
(b) State the range of f. [1]

(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

(d) Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. [5]

9. [Maximum mark: 15]

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.



The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $h(t) = 0.4\cos(\pi t) + 1.8$ where $t \geq 0$.

(a) Find the height of the ball above the ground when it is released. [2]

(b) Find the minimum height of the ball above the ground. [2]

(c) Show that the ball takes 2 seconds to return to its initial height above the ground for the first time. [2]

(d) For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground. [5]

(e) Find the rate of change of the ball's height above the ground when $t = \frac{1}{3}$. Give your answer in the form $p\pi\sqrt{q}$ ms^{-1} where $p \in \mathbb{Q}$ and $q \in \mathbb{Z}^+$. [4]