International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Mathematics: analysis and approaches

Practice paper 2 SL markscheme — Total 80

Section A [36 marks]

1.

$$\frac{\sum_{i=1}^{16} x_i}{16} = 14.5$$ **(M1)**

Note: Award **M1** for use of $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$.

$\Rightarrow \sum_{i=1}^{16} x_i = 232$ **(A1)**

new $\bar{x} = \frac{232-9}{15}$ **(A1)**

$= 14.9 \left(= 14.8\overline{6}, = \frac{223}{15} \right)$ **A1**

Note: Do not accept 15.

Total [4 marks]

2.

(a) **EITHER**

uses the arc length formula **(M1)**

arc length is $3(2\pi - \theta)$ **A1**

OR

length of arc AB is 3θ **A1**

the sum of the lengths of arc AB and arc APB is 6π **A1**

THEN

so arc APB has length $6\pi - 3\theta$ **AG**

[2 marks]

(b) uses the cosine rule **(M1)**

$L^2 = 3^2 + 3^2 - 2(3)(3)\cos\theta$ **A1**

so $L = \sqrt{18 - 18\cos\theta}$ **AG**

[2 marks]

(c) $6\pi - 3\theta = 2\sqrt{18 - 18\cos\theta}$ **A1**

attempts to solve for θ **(M1)**

$\theta = 2.49$ **A1**

[3 marks]

Total [7 marks]

3.

(a) attempts either graphical or symbolic means to find the value of t when $\frac{dv}{dt} = 0$ **(M1)**

$T = 0.465$ (s) **A1**

[2 marks]

(b) attempts to find the value of either $s_1 = \int_0^{0.46494...} v \, dt$ or $s_2 = \int_{0.46494...}^{1} v \, dt$ **(M1)**

$s_1 = 3.02758...$ and $s_2 = 3.47892...$ **A1A1**

Note: Award as above for obtaining, for example, $s_2 - s_1 = 0.45133...$ or $\frac{s_2}{s_1} = 1.14907....$

Note: Award a maximum of **M1A1A0FT** for use of an incorrect value of T from part (a).

so $s_2 > s_1$ **AG**

[3 marks]

Total [5 marks]

4.

(a) (i) $r = 0.946$ **A2**

(ii) the value of r shows a (very) strong positive correlation between age and (systolic) blood pressure **A1**

[3 marks]

(b) $p = 1.05t + 69.3$ **A1A1**

Note: Only award marks for an equation. Award **A1** for $a = 1.05$ and **A1** for $b = 69.3$. Award **A1A0** for $y = 1.05x + 69.3$.

[2 marks]

(c) 122 (mmHg) **(M1)A1**

[2 marks]

(d) the regression equation should not be used because it involves extrapolation **A1**

[1 mark]

Total [8 marks]

5.

attempts to find an expression for the discriminant, Δ, in terms of k **(M1)**

$\Delta = 4 - 4(k-1)(2k-3)$ $\left(= -8k^2 + 20k - 8\right)$ **(A1)**

Note: Award **M1A1** for finding $x = \dfrac{-2 \pm \sqrt{4 - 4(k-1)(2k-3)}}{2(k-1)}$.

attempts to solve $\Delta > 0$ for k **(M1)**

Note: Award **M1** for attempting to solve $\Delta = 0$ for k.

$\frac{1}{2} < k < 2$ **A1A1**

Note: Award **A1** for obtaining critical values $k = \frac{1}{2}, 2$ and **A1** for correct inequality signs.

[5 marks]

6.

(a) attempts to solve $x^2 \sin x = -1 - \sqrt{1 + 4(x+2)^2}$ **(M1)**

$x = -2.76, -1.54$ **A1A1**

Note: Award **A1A0** if additional solutions outside the domain are given.

[3 marks]

(b) $A = \int_{-2.762...}^{-1.537...} \left(-1 - \sqrt{1 + 4(x+2)^2} - x^2 \sin x\right) \mathrm{d}x$ (or equivalent) **(M1)(A1)**

Note: Award **M1** for attempting to form an integrand involving "top curve" – "bottom curve".

so $A = 1.47$ **A2**

[4 marks]

Total [7 marks]

Section B [44 marks]

7.

(a) uses $H_n = H_1 + (n-1)d$ with $H_1 = 70\ 000$ and $d = 2400$ **(M1)**

$H_n = 70\ 000 + 2400(n-1)$ **A1**

so $H_n = 2400n + 67\ 600$ **AG**

[2 marks]

(b) $r = 1.03$ **A1**

[1 mark]

(c) (i) evidence of use of an appropriate table or graph or GDC numerical solve feature to find the value of N such that $J_n > H_n$ **(M1)**

EITHER

for example, an excerpt from an appropriate table

N	H_n	J_n
11	94 000	94 074

(A1)

OR

for example, use of a GDC numerical solve feature to obtain $N = 10.800...$

(A1)

Note: Award **A1** for an appropriate graph. Condone use of a continuous graph.

THEN

$N = 11$ **A1**

(ii) $H_{11} = 94\ 000(\$)$ **A1**

$J_{11} = 94\ 074(\$)$ **A1**

Helen's annual salary is $\$94\ 000$ and Jane's annual salary is $\$94\ 074$

Note: Award **A1** for a correct H_{11} value and **A1** for a correct J_{11} value seen in part (c) (i).

[5 marks]

(d) at the start of the 10th year, Jane will have worked for 9 years so the value of S_9 is required **R1**

Note: Award **R1** if S_9 is seen anywhere.

uses $S_n = \dfrac{J_1(r^n - 1)}{r - 1}$ with $J_1 = 70\ 000$, $r = 1.03$ and $n = 9$ **(M1)**

Note: Award **M1** if $n = 10$ is used.

$S_9 = \dfrac{70\ 000\left((1.03)^9 - 1\right)}{1.03 - 1} = 711\ 137.42...$ **(A1)**

$= 711\ 137(\$)$ **A1**

Jane's total earnings are $\$711\ 137$ (correct to the nearest dollar)

[4 marks]

Total [12 marks]

8.

(a) $T \sim \mathrm{N}\left(\mu, 8.6^2\right)$

$\mathrm{P}\left(T \le 36.8\right) = 0.7$ **(A1)**

states a correct equation, for example, $\dfrac{36.8-\mu}{8.6} = 0.5244...$ **A1**

attempts to solve their equation **(M1)**

$\mu = 36.8 - (0.5244...)(8.6) \quad (= 32.2902...)$ **A1**

the solution to the equation is $\mu = 32.29$, correct to two decimal places **AG**

[4 marks]

(b) let $t_{0.86}$ be the 86th percentile

attempts to use the inverse normal feature of a GDC to find $t_{0.86}$ **(M1)**

$t_{0.86} = 41.6$ (mins) **A1**

[2 marks]

(c) evidence of identifying the correct area under the normal curve **(M1)**

Note: Award **M1** for a clearly labelled sketch.

$\mathrm{P}\left(T > 30\right) = 0.605$ **A1**

[2 marks]

(d) let X represent the number of people out of the six who take more than 30 minutes to complete the jigsaw puzzle

$X \sim \mathrm{B}\left(6, 0.6049...\right)$ **(M1)**

for example, $\mathrm{P}\left(X=5\right) + \mathrm{P}\left(X=6\right)$ or $1 - \mathrm{P}\left(X \le 4\right)$ **(A1)**

$\mathrm{P}\left(X \ge 5\right) = 0.241$ **A1**

[3 marks]

(e) recognizes that $\mathrm{P}(T > 30 \mid T \geq 25)$ is required **(M1)**

Note: Award **M1** for recognizing conditional probability.

$= \dfrac{\mathrm{P}(T > 30 \cap T \geq 25)}{\mathrm{P}(T \geq 25)}$ **(A1)**

$= \dfrac{\mathrm{P}(T > 30)}{\mathrm{P}(T \geq 25)} = \dfrac{0.6049\ldots}{0.8016\ldots}$ **M1**

$= 0.755$ **A1**

[4 marks]

Total [15 marks]

9.

(a) when $t = 0, T = 100 \Rightarrow 100 = T_0 \mathrm{e}^0$ **A1**

so $T_0 = 100$ **AG**

[1 mark]

(b) correct substitution of $t = 10, T = 70$ **M1**

$70 = 100\mathrm{e}^{-10k}$ or $\mathrm{e}^{-10k} = \dfrac{7}{10}$

EITHER

$-10k = \ln \dfrac{7}{10}$ **A1**

$\ln \dfrac{7}{10} = -\ln \dfrac{10}{7}$ or $-\ln \dfrac{7}{10} = \ln \dfrac{10}{7}$ **A1**

OR

$\mathrm{e}^{10k} = \dfrac{10}{7}$ **A1**

$10k = \ln \dfrac{10}{7}$ **A1**

THEN

$k = \frac{1}{10}\ln\frac{10}{7}$ **AG**

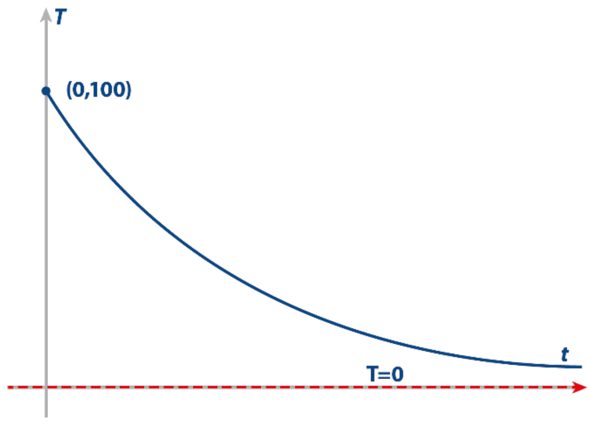
[3 marks]

(c) substitutes $t = 15$ into T **(M1)**

$T = 58.6\left(°\mathrm{C}\right)$ **A1**

[2 marks]

(d)



a decreasing exponential **A1**

starting at $(0,100)$ labelled on the graph or stated **A1**

$T \to 0$ as $t \to \infty$ **A1**

horizontal asymptote $T = 0$ labelled on the graph or stated **A1**

Note: Award **A0** for stating $y = 0$ as the horizontal asymptote.

[4 marks]

(e) $100e^{-kt} = 50$ where $k = \frac{1}{10}\ln\frac{10}{7}$ **A1**

EITHER

uses an appropriate graph to attempt to solve for t **(M1)**

OR

manipulates logs to attempt to solve for t e.g. $\ln\frac{1}{2} = \left(-\frac{1}{10}\ln\frac{10}{7}\right)t$ **(M1)**

$t = \frac{\ln 2}{\frac{1}{10}\ln\frac{10}{7}} = 19.433...$ **A1**

THEN

temperature will be $50°C$ after 19 minutes and 26 seconds **A1**

[4 marks]

(f) **METHOD 1**

substitutes $T_0 = 100$, $t = 10$ and $T = 70$ into $T = T_0 a^{\frac{t}{10}}$ **(M1)**

$70 = 100a^{\frac{10}{10}}$ **A1**

$a = \frac{7}{10}$ **A1**

METHOD 2

$100a^{\frac{t}{10}} = 100e^{-kt}$ where $k = \frac{1}{10}\ln\frac{10}{7}$

EITHER

$e^{-k} = a^{\frac{1}{10}} \Rightarrow a = e^{-10k}$ **(M1)**

OR

$a = \left(e^{\left(-\frac{1}{10}\ln\frac{10}{7}\right)t}\right)^{\frac{10}{t}}$ **(M1)**

THEN

$a = \mathrm{e}^{-\ln\frac{10}{7}} \left(= \mathrm{e}^{\ln\frac{7}{10}} \right)$ **A1**

$a = \dfrac{7}{10}$ **A1**

[3 marks]

Total [17 marks]