

Mathematics: analysis and approaches

Practice paper 2 SL

Total 80

Section A [36 marks]

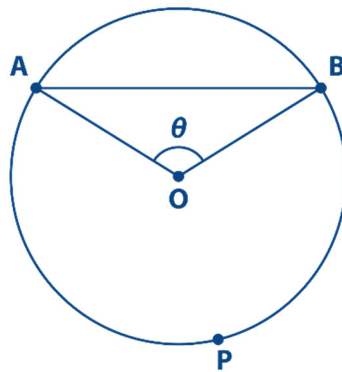
1. [Maximum mark: 4]

A data set consisting of 16 test scores has mean 14.5. One test score of 9 requires a second marking and is removed from the data set.

Find the mean of the remaining 15 test scores.

2. [Maximum mark: 7]

The following diagram shows a circle with centre O and radius 3.



Points A , P and B lie on the circumference of the circle.

Chord $[AB]$ has length L and $\widehat{AOB} = \theta$ radians.

(a) Show that arc APB has length $6\pi - 3\theta$. [2]

(b) Show that $L = \sqrt{18 - 18\cos\theta}$. [2]

Arc APB is twice the length of chord $[AB]$.

(c) Find the value of θ . [3]

3. [Maximum mark: 5]

A particle moves in a straight line such that its velocity, v ms^{-1} , at time t seconds is given by

$$v = 4t^2 - 6t + 9 - 2 \sin(4t), \quad 0 \leq t \leq 1.$$

The particle's acceleration is zero at $t = T$.

(a) Find the value of T . [2]

Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$.

(b) Show that $s_2 > s_1$. [3]

4. [Maximum mark: 8]

The following table shows the systolic blood pressures, p mmHg, and the ages, t years, of 6 male patients at a medical clinic.

Patient	P1	P2	P3	P4	P5	P6
t (years)	40	72	35	47	21	61
p (mmHg)	105	145	100	130	95	132

(a) (i) Determine the value of Pearson's product-moment correlation coefficient, r , for these data. [2]

(ii) Interpret, in context, the value of r found in part (a) (i). [1]

The relationship between t and p can be modelled by the regression line of p on t with equation $p = at + b$.

(b) Find the equation of the regression line of p on t . [2]

A 50-year-old male patient enters the medical clinic for his appointment.

(c) Use the regression equation from part (b) to predict this patient's systolic blood pressure. [2]

A 16-year-old male patient enters the medical clinic for his appointment.

- (d) Explain why the regression equation from part (b) should not be used to predict this patient's systolic blood pressure. [1]

5. [Maximum mark: 5]

The quadratic equation $(k-1)x^2 + 2x + (2k-3) = 0$, where $k \in \mathbb{R}$, has real distinct roots.

Find the range of possible values for k .

6. [Maximum mark: 7]

Consider the curves $y = x^2 \sin x$ and $y = -1 - \sqrt{1 + 4(x+2)^2}$ for $-\pi \leq x \leq 0$.

- (a) Find the x -coordinates of the points of intersection of the two curves. [3]
(b) Find the area, A , of the region enclosed by the two curves. [4]

Section B [44 marks]

7. [Maximum mark: 12]

Helen and Jane both commence new jobs each starting on an annual salary of \$70,000. At the start of each new year, Helen receives an annual salary increase of \$2400.

Let $\$H_n$ represent Helen's annual salary at the start of her n th year of employment.

- (a) Show that $H_n = 2400n + 67\,600$. [2]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary, $\$J_n$, at the start of her n th year of employment is given by

$$J_n = 70\,000(1.03)^{n-1}.$$

- (b) Given that J_n follows a geometric sequence, state the value of the common ratio, r . [1]
- (c) At the start of year N , Jane's annual salary exceeds Helen's annual salary for the first time.
- (i) Find the value of N .
- (ii) For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [5]
- (d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

8. [Maximum mark: 15]

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

- (a) By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. [4]

Use $\mu = 32.29$ in the remainder of the question.

- (b) Find the 86th percentile time to complete the jigsaw puzzle. [2]
- (c) Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. [2]

Six randomly chosen people complete the jigsaw puzzle.

- (d) Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. [3]

Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle.

- (e) Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle. [4]

9.

[Maximum mark: 17]

The temperature $T^{\circ}\text{C}$ of water t minutes after being poured into a cup can be modelled by $T = T_0 e^{-kt}$ where $t \geq 0$ and T_0, k are positive constants.

The water is initially boiling at 100°C . When $t = 10$, the temperature of the water is 70°C .

- (a) Show that $T_0 = 100$. [1]
- (b) Show that $k = \frac{1}{10} \ln \frac{10}{7}$. [3]

- (c) Find the temperature of the water when $t = 15$. [2]
- (d) Sketch the graph of T versus t , clearly indicating any asymptotes with their equations and stating the coordinates of any points of intersection with the axes. [4]
- (e) Find the time taken for the water to have a temperature of 50°C . Give your answer correct to the nearest second. [4]

The model for the temperature of the water can also be expressed in the form $T = T_0 a^{\frac{t}{10}}$ for $t \geq 0$ and a is a positive constant.

- (f) Find the exact value of a . [3]