1. (a) For his Mathematical Studies project, Marty set out to discover if stress was related to the amount of time that students spent travelling to or from school. The results of one of his surveys are shown in the table below.

Travel time (t mins)   Number of students

↓	high stress	moderate stress	low stress
$t \le 15$	9	5	18
$15 < t \le 30$	17	8	28
$30 < t$	18	6	7

He used a χ^2 test at the 5% level of significance to find out if there was any relationship between student stress and travel time.

(i) Write down the null and alternative hypotheses for this test.

(2)

(ii) Write down the table of expected values. Give values to the nearest integer.

(3)

(iii) Show that there are 4 degrees of freedom.

(1)

(iv) Calculate the χ^2 statistic for this data.

(2)

The χ^2 critical value for 4 degrees of freedom at the 5% level of significance is 9.488.

(v) What conclusion can Marty draw from this test? Give a reason for your answer.

(2)

(Total 10 marks)

2. Tom performs a chi-squared test to see if there is any association between the time to prepare for a penalty kick (short time, medium time and long time) and the outcome (scores a goal, doesn't score a goal). Tom performs this test at the 10% level.

(a) Write down the null hypothesis.

(b) Find the number of degrees of freedom for this test.

(c) The p-value for this test is 0.073. What conclusion can Tom make? Justify your answer.

(Total 6 marks)

3. Oral tests are conducted by three examiners A, B and C separately. The results of the examination are classified as Credit, Pass or Fail. A χ^2 test is applied to the data collected in order to test whether or not the examiners differ in their standard of awards.

(a) State the null hypothesis, H_0, for this data.

(b) Write down the number of degrees of freedom.

Of the 135 students who sit the exam, 30 get Credit and 45 are tested by examiner A.

(c) Calculate the expected number of students who get a Credit and are tested by examiner A.

Using a 5% level of significance, the p-value is found to be 0.0327 correct to 3 s.f.

(d) State whether H_0 should be accepted. Justify your answer.

(Total 6 marks)

4. A survey was conducted in a company to determine whether position in upper management was independent of gender. The results of this survey are tabulated below.

	Managers	Junior executives	Senior executives	**Totals**
Male	95	130	75	**300**
Female	65	110	25	**200**
Totals	**160**	**240**	**100**	**500**

The table below shows the expected number of males and females at each level, if they were represented proportionally to the total numbers of males and females employed.

	Managers	Junior executives	Senior executives	**Totals**
Male	a	c	60	**300**
Female	b	d	40	**200**
Totals	**160**	**240**	**100**	**500**

(a) (i) Show that the expected number of **Male Managers** (*a*) is 96.

    (ii) Hence find the values of *b, c* and *d*.

(5)

(b) (i) Write a suitable null hypothesis for this data.

    (ii) Write a suitable alternate hypothesis for this data.

(2)

(c) (i) Perform a chi-squared test of independence for this data to show the value of χ^2 is 12.8 to 3 significant figures.

    (ii) Calculate the number of degrees of freedom, and write down the critical value of χ^2 at the 5% significance level.

    (iii) What conclusion can be drawn regarding gender and position in upper management? The critical value for this test is 5.991.

(6)
(Total 13 marks)

5. Some of the customers in each café were given survey forms to complete to find out if they were satisfied with the standard of service they received.

	Pete's Eats	Alan's Diner	Sarah's Snackbar	Total
Dissatisfied	16	8	16	40
Satisfied	26	20	34	80
Total	42	28	50	120

One of the survey forms was chosen at random, find the probability that

(a)   the form showed "Dissatisfied";

(2)

(b)   the form showed "Satisfied" and was completed at Sarah's Snackbar;

(2)

(c)   the form showed "Dissatisfied", given that it was completed at Alan's Diner.

(2)

A χ^2 test at the 5 % significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés. The critical value is 5.991.

(d)   Write down the null hypothesis, H_0, for the χ^2 test.

(1)

(e)   Write down the number of degrees of freedom for the test.

(1)

(f)   Using your graphic display calculator, find χ^2_{calc}.

(2)

(g)   State, giving a reason, the conclusion to the test.

(2)
(Total 12 marks)

6. A university required all Science students to study one language for one year.
A survey was carried out at the university amongst the 150 Science students. These students all studied one of either French, Spanish or Russian. The results of the survey are shown below.

	French	**Spanish**	**Russian**
Female	9	29	12
Male	31	40	29

Ludmila decides to use the χ^2 test at the 5 % level of significance to determine whether the choice of language is independent of gender. The critical value is 5.991.

(a) State Ludmila's null hypothesis.

(1)

(b) Write down the number of degrees of freedom.

(1)

(c) Find the expected frequency for the females studying Spanish.

(2)

(d) Use your graphic display calculator to find the χ^2 test statistic for this data.

(2)

(e) State whether Ludmila accepts the null hypothesis. Give a reason for your answer.

(2)
(Total 8 marks)

7. Members of a certain club are required to register for one of three games, billiards, snooker or darts.

The number of club members of each gender choosing each game in a particular year is shown in the table below.

	Billiards	Snooker	Darts
Male	39	16	8
Female	21	14	17

(a) Use a χ^2 (Chi-squared) test at the 5% significance level to test whether choice of games is independent of gender. State clearly the null and alternative hypotheses tested, the expected values, and the number of degrees of freedom used. The critical value is 5.991.

(13)

The following year the choice of games was widened and the figures for that year are as follows:

	Billiards	Snooker	Darts	Fencing
Male	4	15	8	10
Female	10	21	17	37

(b) If the χ^2 test were applied to this new set of data,

   (i) why would it be necessary to combine billiards with another game?

   (ii) which other game would you combine with billiards and why?

(2)

A club member is to be selected at random.

(c) What is the probability that the club member selected is a

    (i) female who chose billiards or snooker?

    (ii) male or female who chose darts or fencing?

(2)
(Total 17 marks)