1. In a class of 20 students, 12 study Biology, 15 study History and 2 students study neither Biology nor History.

(a) Illustrate this information on a Venn diagram.

(2)

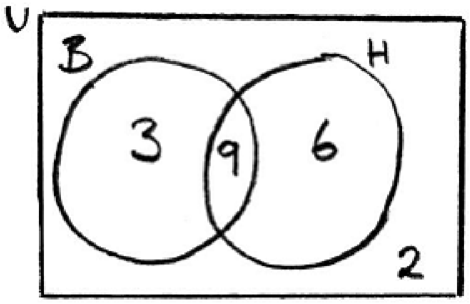
(b) Find the probability that a randomly selected student from this class is studying both Biology and History.

(1)

(c) Given that a randomly selected student studies Biology, find the probability that this student also studies History.

(1)
(Total 4 marks)

(a)



A1A1

Note: Award A1 for a diagram with two intersecting regions and at least the value of the intersection.

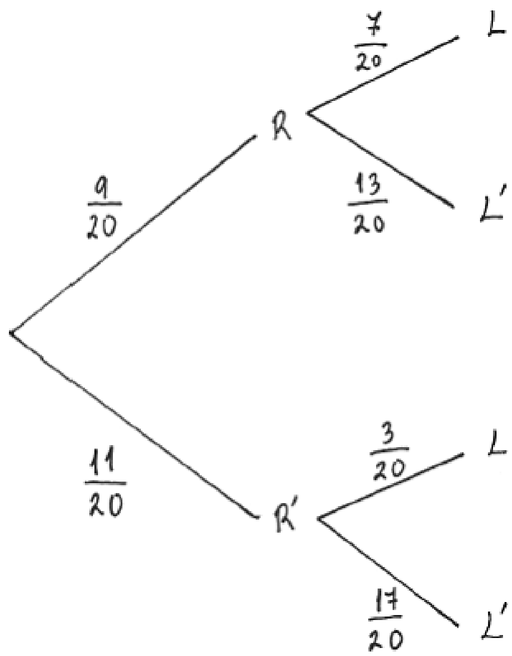
(b) $\frac{9}{20}$ A1

(c) $\frac{9}{12}\left(=\frac{3}{4}\right)$ A1

[4]

2. Jenny goes to school by bus every day. When it is not raining, the probability that the bus is late is $\frac{3}{20}$. When it is raining, the probability that the bus is late is $\frac{7}{20}$. The probability that it rains on a particular day is $\frac{9}{20}$. On one particular day the bus is late. Find the probability that it is not raining on that day.

(Total 5 marks)



(A1)

$\mathrm{P}(R' \cap L) = \frac{11}{20} \times \frac{3}{20}$ A1

$\mathrm{P}(L) = \frac{9}{20} \times \frac{7}{20} + \frac{11}{20} \times \frac{3}{20}$ A1

$P(R'|L) = \frac{\mathrm{P}(R' \cap L)}{\mathrm{P}(L)}$ (M1)

$= \frac{33}{96} \left(= \frac{11}{32} \right)$ A1

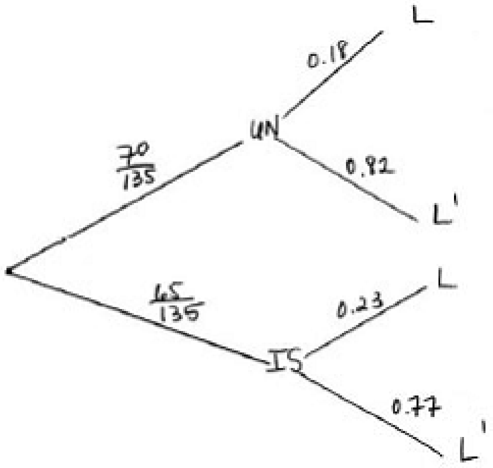
[5]

3. Only two international airlines fly daily into an airport. UN Air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18% probability of losing their luggage and passengers flying with IS Air have a 23% probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost.

Find the probability that she travelled with IS Air.

(Total 6 marks)

METHOD 1



(M1)

Let P(I) be the probability of flying IS Air, P(U) be the probability flying UN Air and P(L) be the probability of luggage lost.

$$P(I \mid L) = \frac{P(I \cap L)}{P(L)}$$

$$\left(\text{or Bayes' formula}, P(I \mid L) = \frac{P(L \mid I)P(I)}{P(L \mid I)P(I) + P(L \mid U)P(U)}\right)$$ (M1)

$$= \frac{0.23 \times \frac{65}{135}}{0.18 \times \frac{70}{135} + 0.23 \times \frac{65}{135}}$$ A1A1A1

$$= \frac{299}{551} \quad (= 0.543, \text{accept } 0.542)$$ A1

METHOD 2

Expected number of suitcases lost by UN Air is $0.18 \times 70 = 12.6$ M1A1

Expected number of suitcases lost by IS Air is $0.23 \times 65 = 14.95$ A1

$$P(I \mid L) = \frac{14.95}{12.6 + 14.95}$$ M1A1

$$= 0.543$$ A1

[6]

4. In a population of rabbits, 1 % are known to have a particular disease. A test is developed for the disease that gives a positive result for a rabbit that **does** have the disease in 99 % of cases. It is also known that the test gives a positive result for a rabbit that **does not** have the disease in 0.1 % of cases. A rabbit is chosen at random from the population.

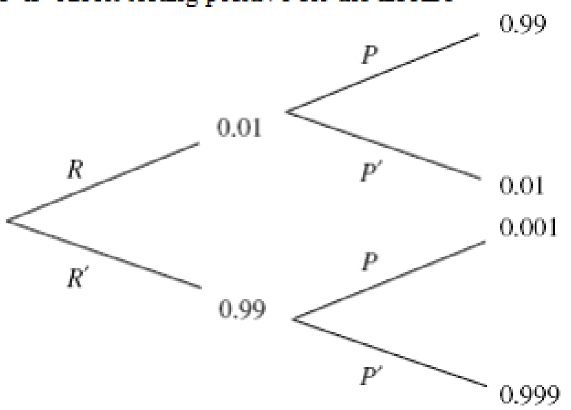
(a) Find the probability that the rabbit tests positive for the disease.

(2)

(b) Given that the rabbit tests positive for the disease, show that the probability that the rabbit does not have the disease is less than 10 %.

(3)
(Total 5 marks)

R is 'rabbit with the disease'
P is 'rabbit testing positive for the disease'



(a) $\mathrm{P}(P) = \mathrm{P}(R \cap P) + \mathrm{P}(R' \cap P)$
$= 0.01 \times 0.99 + 0.99 \times 0.001$ M1
$= 0.01089\ (= 0.0109)$ A1

Note: Award M1 for a correct tree diagram with correct probability values shown.

(b) $\mathrm{P}(R'|P) = \dfrac{0.001 \times 0.99}{0.001 \times 0.99 + 0.01 \times 0.99} \left(= \dfrac{0.00099}{0.01089} \right)$ M1A1

$\dfrac{0.00099}{0.01089} < \dfrac{0.001}{0.01} = 10\ \%$ (or other valid argument) R1

[5]

5. Bag A contains 2 red and 3 green balls.

(a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen.

(2)

Bag B contains 4 red and n green balls.

(b) Two balls are chosen without replacement from this bag. If the probability that two red balls are chosen is $\frac{2}{15}$, show that $n = 6$.

(4)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(3)

(d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die.

(4)
(Total 13 marks)

(a) $\mathrm{P}(RR) = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right)$ (M1)

$= \frac{1}{10}$ A1 N2

(b) $\mathrm{P}(RR) = \frac{4}{4+n} \times \frac{3}{3+n} = \frac{2}{15}$ A1

Forming equation $12 \times 15 = 2(4 + n)(3 + n)$ (M1)

$12 + 7n + n^2 = 90$ A1

$\Rightarrow n^2 + 7n - 78 = 0$ A1

$n = 6$ AG N0

(c) **EITHER**

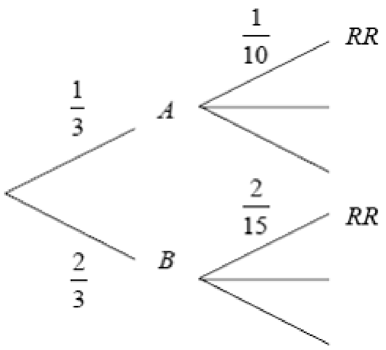
$P(A) = \frac{1}{3}$ $\quad P(B)\frac{2}{3}$ A1

$P(RR) = P(A \cap RR) + P(B \cap RR)$ (M1)

$= \left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{15}\right)$

$= \frac{11}{90}$ A1 N2

OR



A1

$P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{15}$ M1

$= \frac{11}{90}$ A1 N2

(d) P(1or 6) = P(A) M1

$P(A \mid RR) = \frac{P(A \cap RR}{P(RR)}$ (M1)

$= \frac{\left[\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)\right]}{\frac{11}{90}}$ M1

$= \frac{3}{11}$ A1 N2

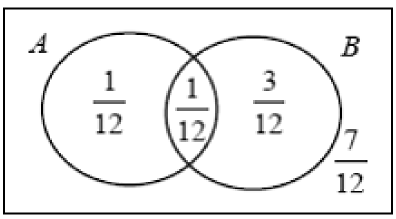
[13]

6. If $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{3}$, and $P(A \cup B) = \frac{5}{12}$, what is $P(A' / B')$?

(Total 6 marks)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ M1

$= \frac{2}{12} + \frac{4}{12} - \frac{5}{12} = \frac{1}{12}$ A1



M1A1

$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{\frac{7}{12}}{\frac{8}{12}} = \frac{7}{8}$ M1A1

[6]

7. Two players, A and B, alternately throw a fair six–sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six.

(Total 7 marks)

\

P (six in first throw) = $\frac{1}{6}$ (A1)

P (six in third throw) = $\frac{25}{36}\times\frac{1}{6}$ (M1)(A1)

P (six in fifth throw) = $\left(\frac{25}{36}\right)^2\times\frac{1}{6}$

P(A obtains first six) = $\frac{1}{6}+\frac{25}{36}\times\frac{1}{6}+\left(\frac{25}{36}\right)^2\times\frac{1}{6}+\dots$ (M1)

recognizing that the common ratio is $\frac{25}{36}$ (A1)

P(A obtains first six) = $\frac{\frac{1}{6}}{1-\frac{25}{36}}$ (by summing the infinite GP) M1

$=\frac{6}{11}$ A1

[7]

8. An influenza virus is spreading through a city. A vaccination is available to protect against the virus. If a person has had the vaccination, the probability of catching the virus is 0.1; without the vaccination, the probability is 0.3. The probability of a randomly selected person catching the virus is 0.22.

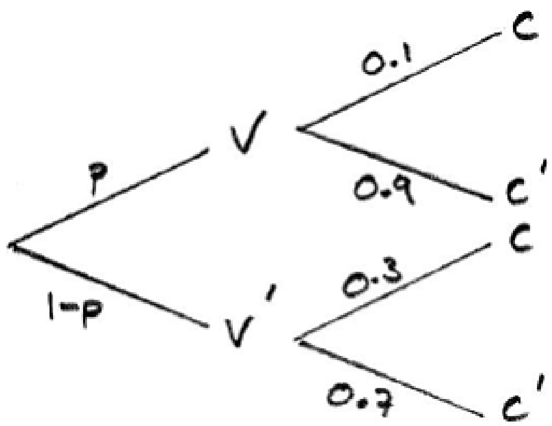
(a) Find the percentage of the population that has been vaccinated.

(3)

(b) A randomly chosen person catches the virus. Find the probability that this person has been vaccinated.

(2)
(Total 5 marks)

(a)



using the law of total probabilities: (M1)

$0.1p + 0.3(1-p) = 0.22$ A1

$0.1p + 0.3 - 0.3p = 0.22$

$0.2p = 0.08$

$p = \dfrac{0.08}{0.2} = 0.4$

$p = 40\%$ (accept 0.4) A1

(b) required probability $= \dfrac{0.4 \times 0.1}{0.22}$ M1

$= \dfrac{2}{11}$ (0.182) A1

[5]

9. At a nursing college, 80 % of incoming students are female. College records show that 70 % of the incoming females graduate and 90 % of the incoming males graduate.
A student who graduates is selected at random. Find the probability that the student is male, giving your answer as a fraction in its lowest terms.

(Total 5 marks)

$\mathrm{P}\ M|\ G = \dfrac{\mathrm{P}(M \cap G)}{\mathrm{P}(G)}$ (M1)

$= \dfrac{0.2 \times 0.9}{0.2 \times 0.9 + 0.8 \times 0.7}$ M1A1A1

$= \dfrac{0.18}{0.74}$

$= \dfrac{9}{37}$ A1

[5]

10. Let A and B be events such that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \mid B) = 0.6$.

Find $\mathrm{P}(B)$.

(Total 6 marks)

EITHER

Using $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ (M1)

$0.6\mathrm{P}(B) = \mathrm{P}(A \cap B)$ A1

Using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ to obtain
$0.8 = 0.6 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ A1

Substituting $0.6\mathrm{P}(B) = \mathrm{P}(A \cap B)$ into above equation M1

OR

As $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ then A and B are independent events M1R1

Using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A) \times \mathrm{P}(B)$ A1

to obtain $0.8 = 0.6 + \mathrm{P}(B) - 0.6 \times \mathrm{P}(B)$ A1

THEN

$0.8 = 0.6 + 0.4\mathrm{P}(B)$ A1

$\mathrm{P}(B) = 0.5$ A1 N1

[6]

11. Events A and B are such that $P(A) = 0.3$ and $P(B) = 0.4$.

(a) Find the value of $P(A \cup B)$ when

(i) A and B are mutually exclusive;

(ii) A and B are independent.

(4)

(b) Given that $P(A \cup B) = 0.6$, find $P(A \mid B)$.

(3)
(Total 7 marks)

(a) (i) $P(A \cup B) = P(A) + P(B) = 0.7$ A1

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$= P(A) + P(B) - P(A)P(B)$ (M1)

$= 0.3 + 0.4 - 0.12 = 0.58$ A1

(b) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.3 + 0.4 - 0.6 = 0.1$ A1

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ (M1)

$= \dfrac{0.1}{0.4} = 0.25$ A1

[7]

12. There are 30 students in a class, of which 18 are girls and 12 are boys. Four students are selected at random to form a committee. Calculate the probability that the committee contains

(a) two girls and two boys;

(3)

(b) students all of the same gender.

(3)
(Total 6 marks)

(a) Total number of ways of selecting 4 from 30 = $\binom{30}{4}$ (M1)

Number of ways of choosing $2B$ $2G$ = $\binom{12}{2}\binom{18}{2}$ (M1)

$P(2B \text{ or } 2G) = \dfrac{\binom{12}{2}\binom{18}{2}}{\binom{30}{4}} = 0.368$ A1 N2

(b) Number of ways of choosing $4B$ = $\binom{12}{4}$, choosing $4G$ = $\binom{18}{4}$ A1

$P(4B \text{ or } 4G) = \dfrac{\binom{12}{4}+\binom{18}{4}}{\binom{30}{4}}$ (M1)

$= 0.130$ A1 N2

[6]