Example 1 Self Tutor

Find the equation of the tangent to $f(x) = \sqrt{x^2+5}$ at the point $(2, 3)$.

$f(x) = \sqrt{x^2+5} = (x^2+5)^{\frac{1}{2}}$

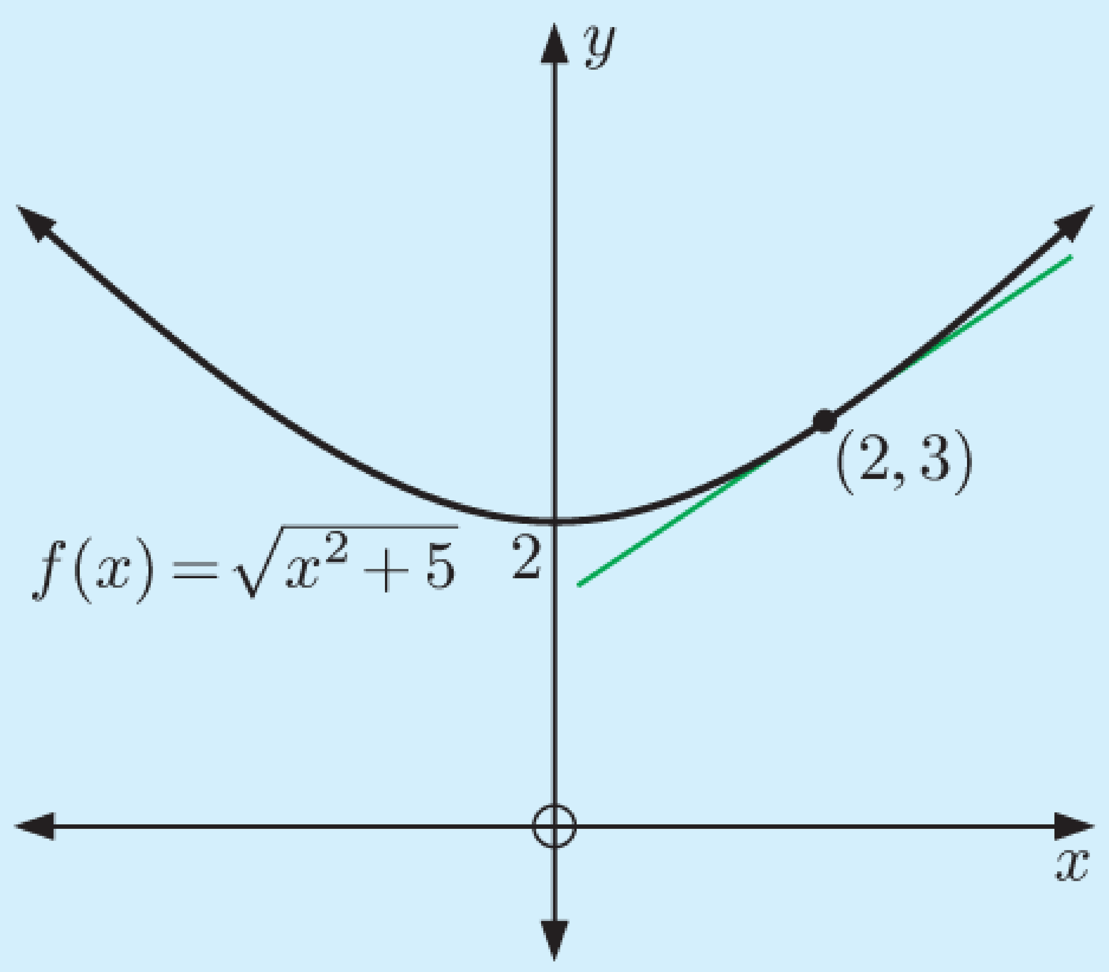
$\therefore\ f'(x) = \frac{1}{2}(x^2+5)^{-\frac{1}{2}}(2x)$ {chain rule}

$= \dfrac{x}{\sqrt{x^2+5}}$

$\therefore\ f'(2) = \dfrac{2}{\sqrt{2^2+5}} = \dfrac{2}{3}$

So, the tangent has equation $y = \frac{2}{3}(x-2)+3$

$\therefore\ y = \frac{2}{3}x + \frac{5}{3}$



Example 2 Self Tutor

Find the equations of any horizontal tangents to $y = x^3 - 12x + 2$.

Since $y = x^3 - 12x + 2$, $\dfrac{dy}{dx} = 3x^2 - 12$

Horizontal tangents have gradient 0,

so $3x^2 - 12 = 0$

$\therefore\ 3(x^2-4) = 0$

$\therefore\ 3(x+2)(x-2) = 0$

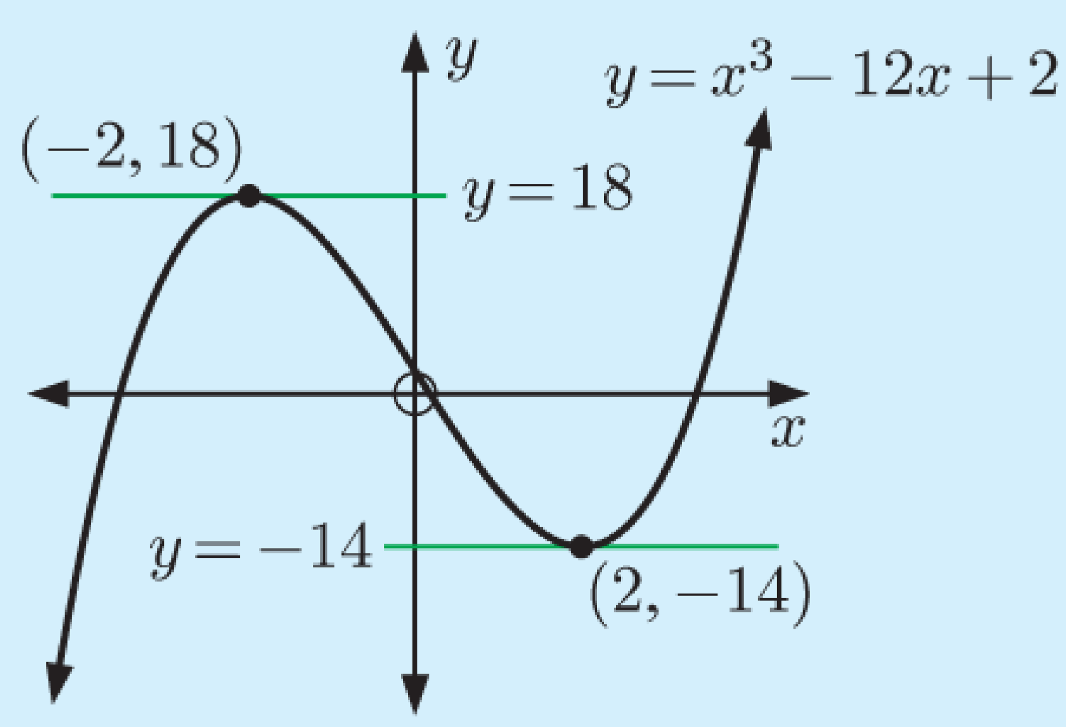
$\therefore\ x = -2$ or 2

When $x = 2$, $y = 8 - 24 + 2 = -14$

When $x = -2$, $y = -8 + 24 + 2 = 18$

$\therefore$ the points of contact are $(2, -14)$ and $(-2, 18)$

$\therefore$ the tangents are $y = -14$ and $y = 18$.

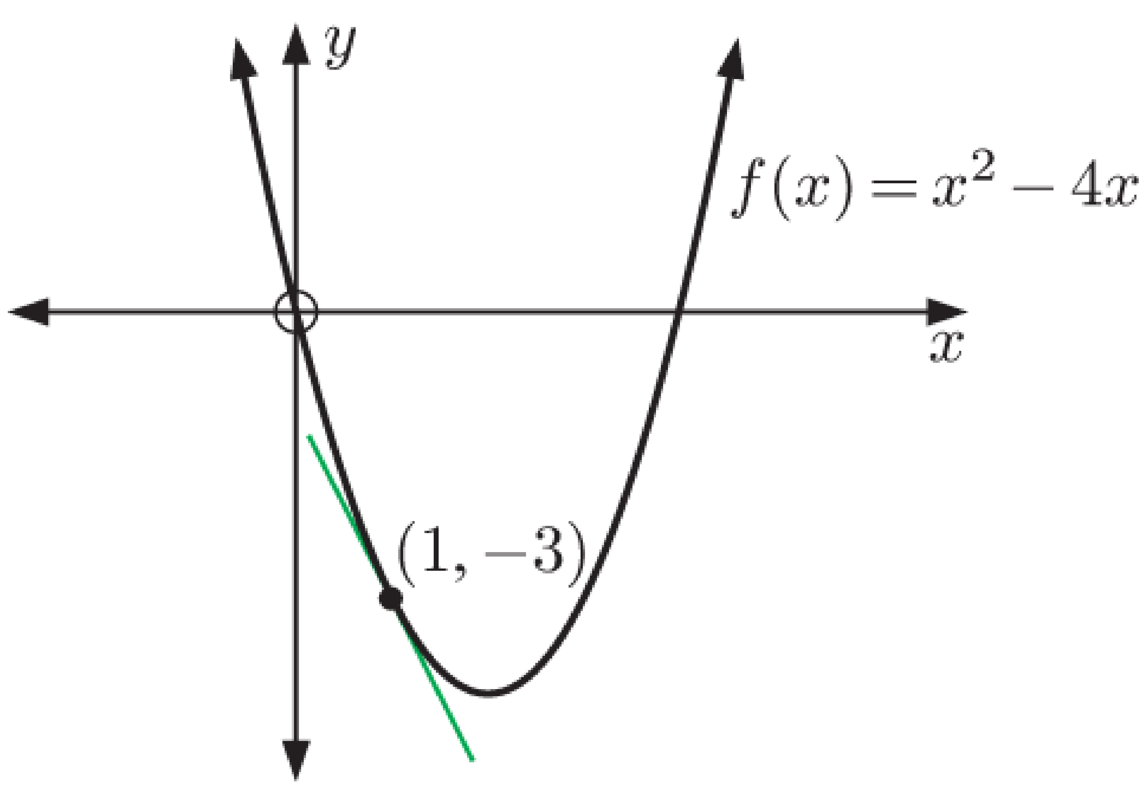


EXERCISE 18A

1 The graph of $f(x) = x^2 - 4x$ is shown alongside.

a Find $f'(x)$.

b Hence find the equation of the illustrated tangent.



2 Find the equation of the tangent to:

a $y = x - 2x^2 + 3$ at $x = 2$

b $y = \sqrt{x} + 1$ at $x = 4$

c $y = x^3 - 5x$ at $x = 1$

d $y = \dfrac{4}{\sqrt{x}}$ at $(1, 4)$

e $y = \dfrac{3}{x} - \dfrac{1}{x^2}$ at $(-1, -4)$

f $y = 3x^2 - \dfrac{1}{x}$ at $x = -1$

g $y = \dfrac{1}{(x^2+1)^2}$ at $(1, \frac{1}{4})$

h $y = \dfrac{1}{\sqrt{3-2x}}$ at $x = -3$.

GRAPHING PACKAGE

Check your answers using technology.

3 Find the equations of any horizontal tangents to:

a $y = 2x^3 + 3x^2 - 12x + 1$

b $y = -x^3 + 3x^2 + 9x - 4$

c $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

4 The tangent to $y = 2x^3 + kx^2 - 3$ at the point where $x = 2$ has gradient 4.

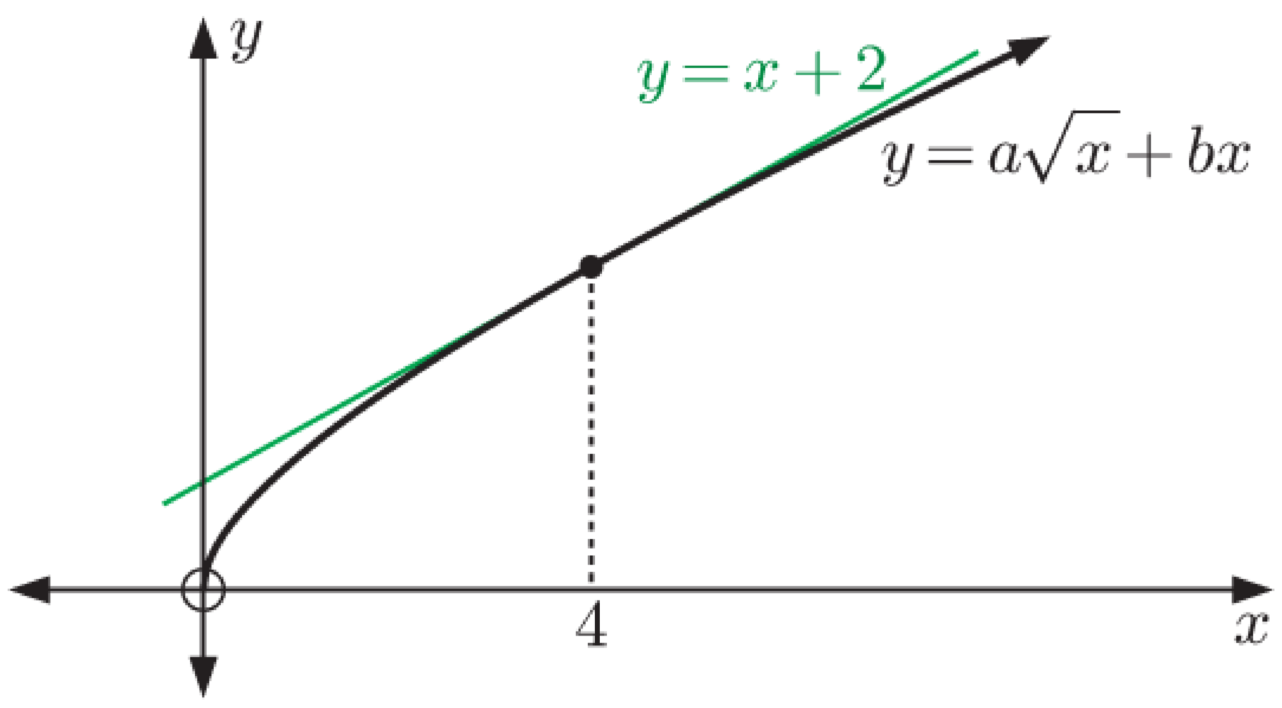
a Find k.

b Hence find the equation of this tangent.

5 Find the equation of another tangent to $y = 1 - 3x + 12x^2 - 8x^3$ which is parallel to the tangent at $(1, 2)$.

6 Consider the curve $y = x^2 + ax + b$ where a and b are constants. The tangent to this curve at the point where $x = 1$ is $2x + y = 6$. Find the values of a and b.

7 Find the values of a and b.



8 Show that the equation of the tangent to $y = 2x^2 - 1$ at the point where $x = a$, is $4ax - y = 2a^2 + 1$.

9 Consider the function $f(x) = x^2 + \dfrac{4}{x^2}$. Show that there is a unique horizontal tangent to the curve which touches the curve *twice*.

10 Consider the curve $y = a\sqrt{1 - bx}$ where a and b are constants. The tangent to this curve at the point where $x = -1$ is $3x + y = 5$. Find the values of a and b.

11 The tangent to $f(x) = (2x-1)^4$ at $x = k$ has gradient 8.

a Find k.

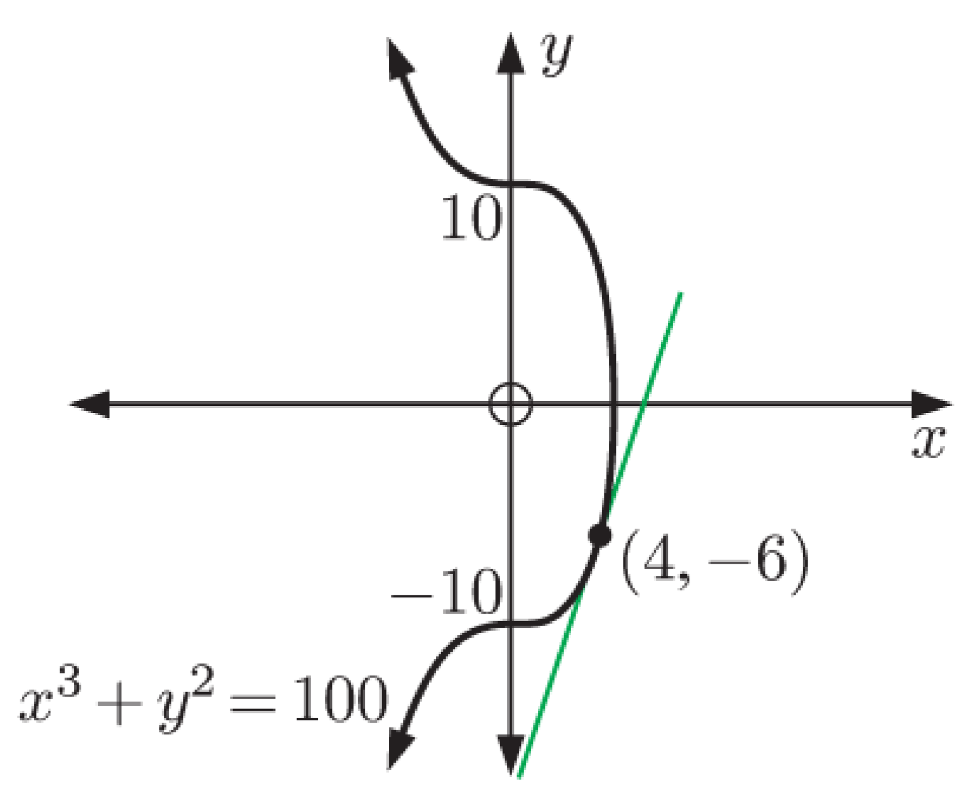
b Find the equation of this tangent.

c Hence find the x-intercept of this tangent.

12 The graph of $x^3 + y^2 = 100$ is shown alongside.

a Find $\dfrac{dy}{dx}$.

b Find the equation of the illustrated tangent.



13 **a** Find the tangents to the unit circle $x^2 + y^2 = 1$ at the points where $x = \frac{1}{2}$.

b At which point on the x-axis do these tangents intersect?

Example 3 **Self Tutor**

Show that the equation of the tangent to $y = \ln x$ at $y = -1$ is $y = ex - 2$.

When $y = -1$, $\ln x = -1$

$\therefore\ x = e^{-1} = \frac{1}{e}$

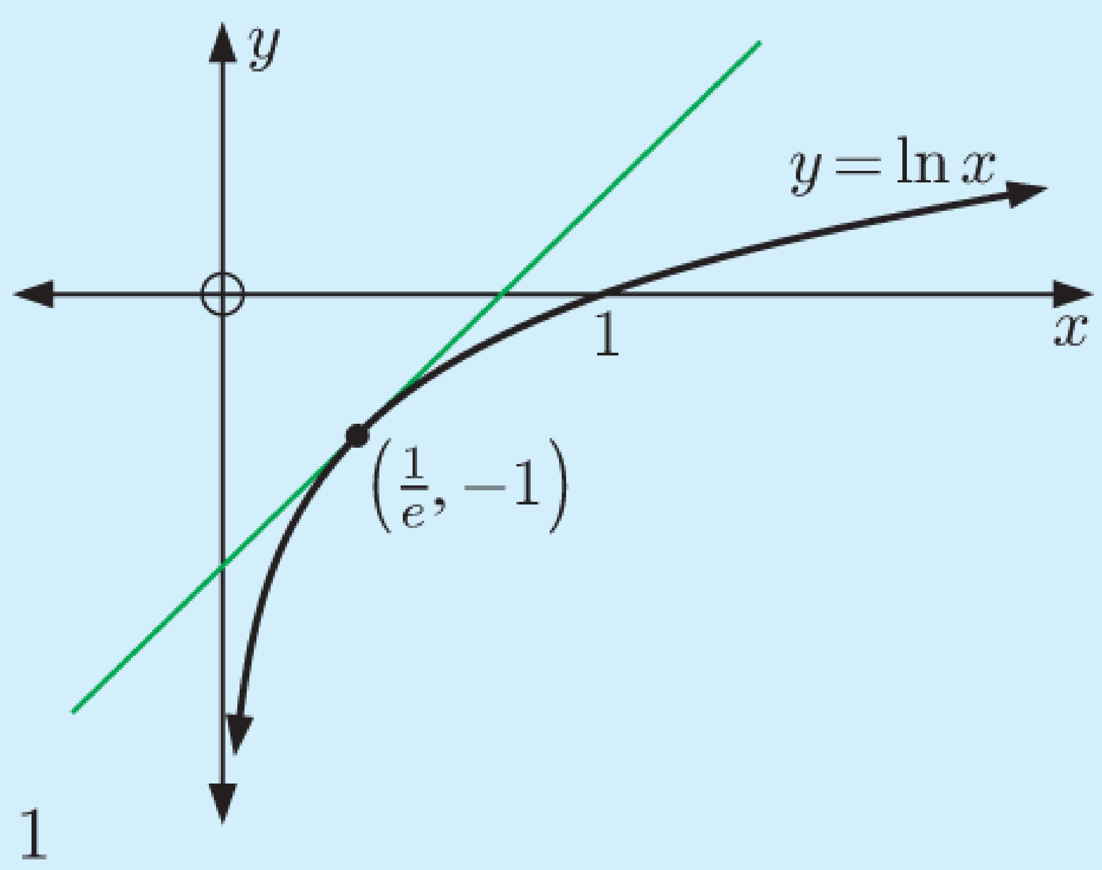
$\therefore$ the point of contact is $\left(\frac{1}{e}, -1\right)$.

Now $f(x) = \ln x$ has derivative $f'(x) = \dfrac{1}{x}$

$\therefore$ the tangent at $\left(\frac{1}{e}, -1\right)$ has gradient $\dfrac{1}{\frac{1}{e}} = e$

$\therefore$ the tangent has equation $y = e\left(x - \frac{1}{e}\right) - 1$

which is $y = ex - 2$



14 Find the equation of the tangent to:

a $f(x) = e^{-x}$ at $x = 2$

b $y = \ln(2 - x)$ at $x = -1$

c $y = (x + 2)e^x$ at $x = 1$

d $y = \ln\sqrt{x}$ at $y = -1$

e $y = e^{3x-5}$ at $y = e$.

15 Consider $f(x) = \ln(x(x-2))$.

a State the domain of $f(x)$.

b Find $f'(x)$.

c Find the equation of the tangent to $y = f(x)$ at the point where $x = 3$.

16 **a** Find the y-intercept of the tangent to $f(x) = x\ln x$ at the point where:

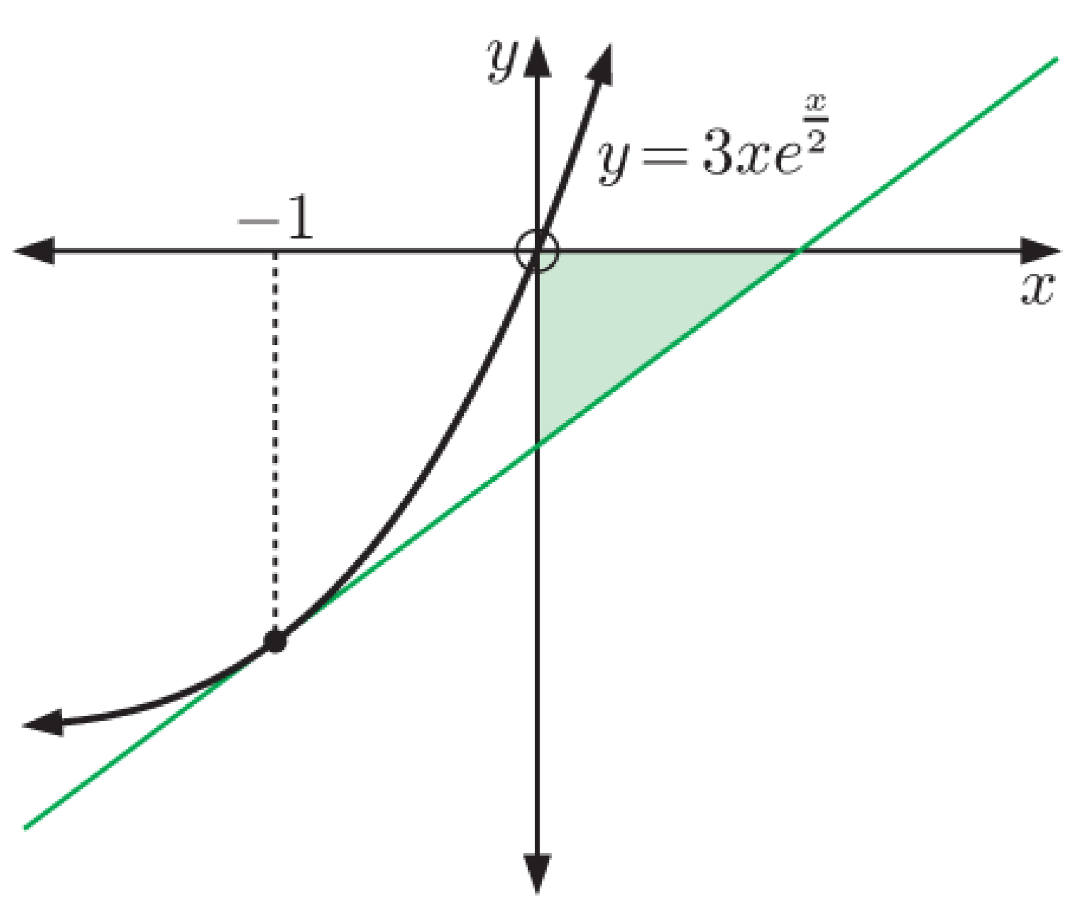
i $x = 1$ **ii** $x = 2$ **iii** $x = 3$.

b Make a conjecture about the y-intercept of the tangent to $f(x) = x\ln x$ at the point where $x = a$, $a > 0$.

c Prove your conjecture algebraically.

17 Find the axes intercepts of the tangent to $y = x^2e^x$ at $x = 1$.

18 Find the exact area of the shaded triangle.



Example 4 — Self Tutor

Find the equation of the tangent to $y = \tan x$ at the point where $x = \frac{\pi}{4}$.

When $x = \frac{\pi}{4}$, $y = \tan\frac{\pi}{4} = 1$

$\therefore$ the point of contact is $(\frac{\pi}{4}, 1)$.

Now $f(x) = \tan x$ has derivative $f'(x) = \sec^2 x$

$\therefore$ the tangent at $(\frac{\pi}{4}, 1)$ has gradient $\dfrac{1}{\cos^2(\frac{\pi}{4})} = \dfrac{1}{\frac{1}{2}} = 2$

$\therefore$ the tangent has equation $y = 2(x - \frac{\pi}{4}) + 1$

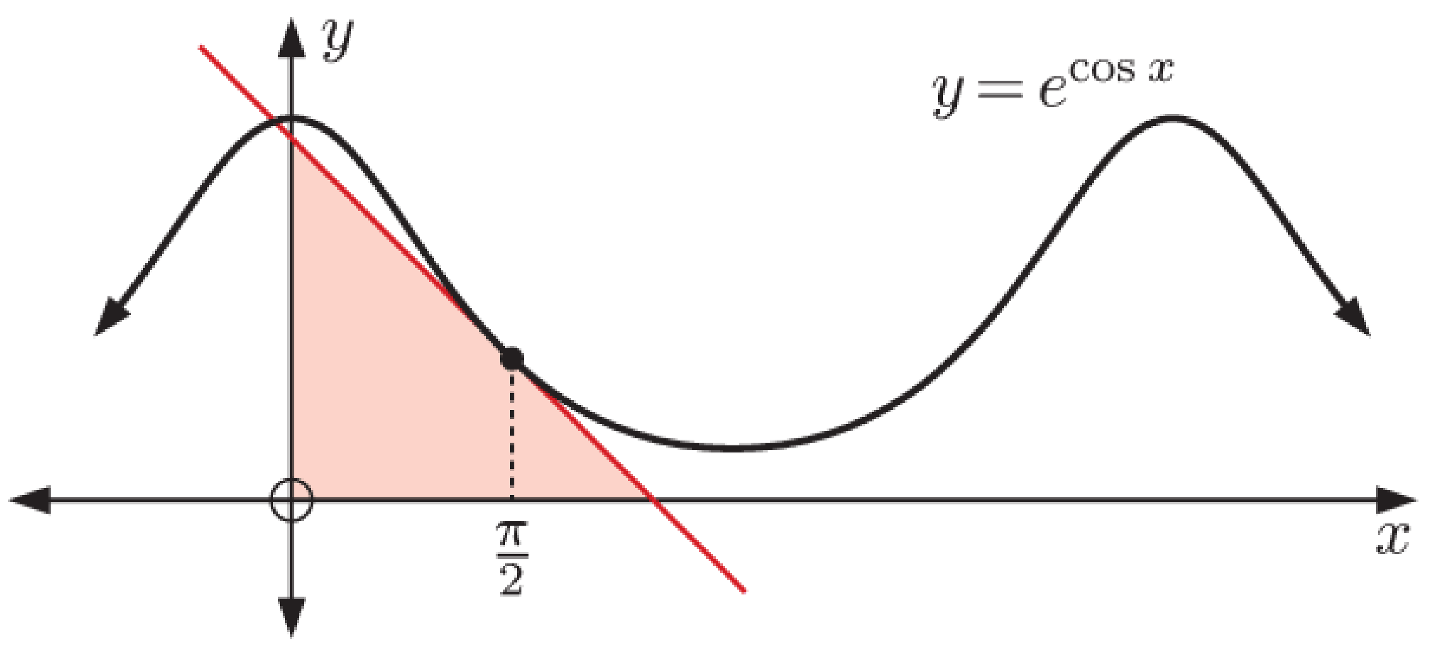
which is $y = 2x + (1 - \frac{\pi}{2})$

19 Find the equation of the tangent to:

- **a** $y = \sin x$ at the origin
- **b** $y = \tan x$ at the origin
- **c** $y = \cos x$ at $x = \frac{\pi}{6}$
- **d** $y = \dfrac{1}{\sin 2x}$ at $x = \frac{\pi}{4}$
- **e** $y = \cos 2x + 3\sin x$ at $x = \frac{\pi}{2}$.

20 Show that the curve with equation $y = \dfrac{\cos x}{1 + \sin x}$ does not have any horizontal tangents.

21 The graph of $y = e^{\cos x}$ is shown alongside. Find the area of the shaded triangle.



22 Find the equation of the tangent to:

- **a** $y = \sec x$ at $x = \frac{\pi}{4}$
- **b** $y = \cot\frac{x}{2}$ at $x = \frac{\pi}{3}$
- **c** $y = \arctan x$ at $x = 1$
- **d** $y = x \arccos\frac{x}{2}$ at $x = -1$

Example 5 — Self Tutor

Find where the tangent to $y = x^3 + x + 2$ at $(1, 4)$ meets the curve again.

Let $f(x) = x^3 + x + 2$

$\therefore$ $f'(x) = 3x^2 + 1$ and $\therefore$ $f'(1) = 3 + 1 = 4$

$\therefore$ the equation of the tangent at $(1, 4)$ is $4x - y = 4(1) - 4$

or $y = 4x$.

The curve meets the tangent again when $x^3 + x + 2 = 4x$

$\therefore$ $x^3 - 3x + 2 = 0$

$\therefore$ $(x-1)^2(x+2) = 0$

$(x-1)^2$ must be a factor of $x^3 - 3x + 2 = 0$ since we are using the *tangent* at $x = 1$.

When $x=-2$, $y=(-2)^3+(-2)+2=-8$

$\therefore$ the tangent meets the curve again at $(-2,\ -8)$.

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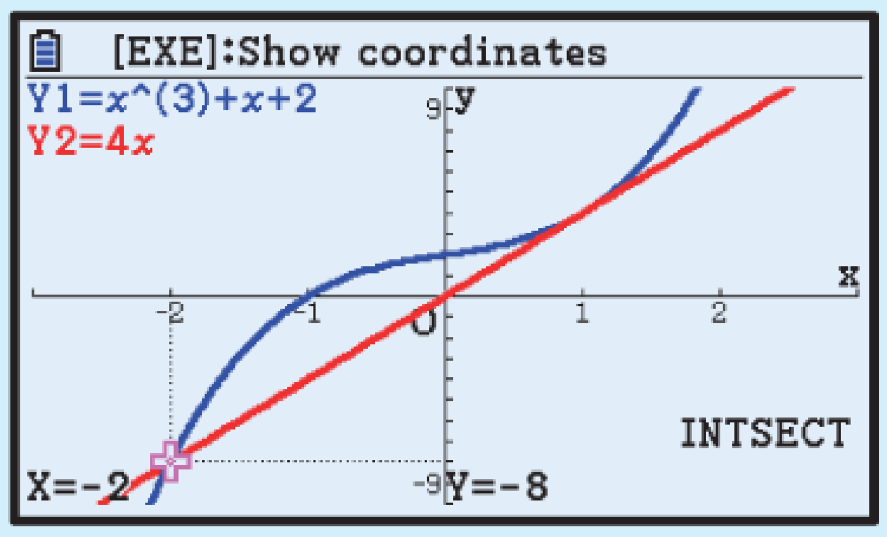

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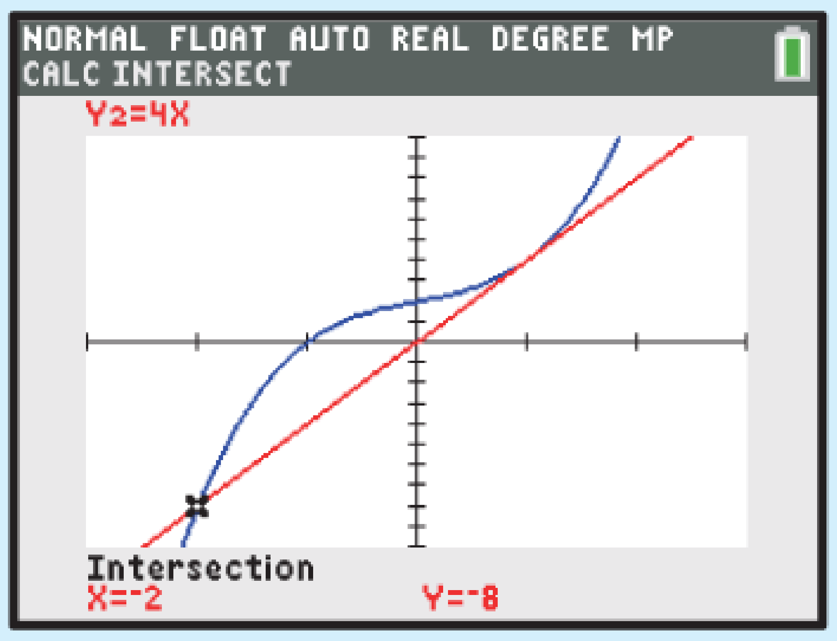

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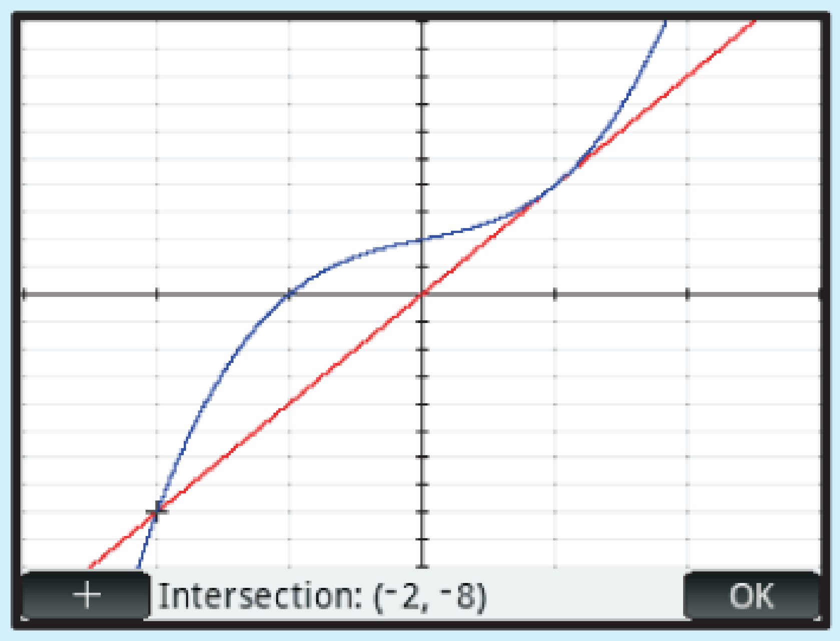


23 Find where the tangent to the curve $y=x^3$ at the point where $x=2$, meets the curve again.

24 Find where the tangent to the curve $y=-x^3+2x^2+1$ at the point where $x=-1$, meets the curve again.

25 Find where the tangent to the curve $y=\frac{1}{x}-\frac{1}{x^2}$ at the point where $x=1$, meets the curve again.

26 Let $P(x)=x^3-3x^2-x+3$.

- **a** Show that $x=1$ is a zero of $P(x)$, and find the three real zeros.
- **b** Sketch the graph of $y=P(x)$.
- **c** Find the equation of the tangent to $y=P(x)$ at the point where $x=2$.
- **d** Find where the tangent in **c** crosses the curve again.
- **e** Suppose a cubic has zeros a, b, and c with $a<b<c$. Prove that the tangent to the cubic at $x=\frac{a+b}{2}$ meets the cubic again at $x=c$.

Example 6 **Self Tutor**

Find the equations of the tangents to $y=-x^2+x+2$ which pass through $(1,\ 3)$.

Let $(a,\ -a^2+a+2)$ be a general point on the curve.

Now $\frac{dy}{dx}=-2x+1$

$\therefore$ the gradient of the tangent when $x=a$ is $-2a+1$

$\therefore$ the equation of the tangent at $(a,\ -a^2+a+2)$ is $y=(-2a+1)(x-a)+(-a^2+a+2)$

which is $y=(1-2a)x+a^2+2$

The tangents which pass through $(1,\ 3)$ must satisfy $(1-2a)(1)+a^2+2=3$

$$\therefore\ a^2-2a=0$$
$$\therefore\ a(a-2)=0$$
$$\therefore\ a=0 \text{ or } 2$$

$\therefore$ two tangents pass through the external point $(1,\ 3)$.

If $a=0$, the tangent has equation $y=x+2$ with point of contact $(0,\ 2)$.

If $a=2$, the tangent has equation $y=-3x+6$ with point of contact $(2,\ 0)$.

27 **a** Find the equation of the tangent to $y = x^2 - x + 9$ at the point where $x = a$.

b Hence find the equations of the two tangents from $(0, 0)$ to the curve. State the coordinates of the points of contact.

28 **a** Find the equation of the tangent to $y = x^2 + 4x$ at the point where $x = a$.

b Hence find the equations of the tangents to $y = x^2 + 4x$ which pass through the external point $(1, -4)$. State the coordinates of the points of contact.

29 Find the equations of the tangents to $y = x^2 - 3x + 1$ which pass through $(1, -10)$.

30 **a** Find the equation of the tangent to $y = e^x$ at the point where $x = a$.

b Hence find the equation of the tangent to $y = e^x$ which passes through the origin.

31 Consider the function $y = 2x^2$.

a Find the equations of the tangents to the function from the external point $(1, -6)$.

b Find the points of contact for the tangents.

c Show that no tangents to the function pass through the point $(1, 4)$.

d Draw a graph of $y = 2x^2$ showing the information above.

32 Consider $f(x) = \dfrac{8}{x^2}$.

a Sketch the graph of the function.

b Find the equation of the tangent at the point where $x = a$.

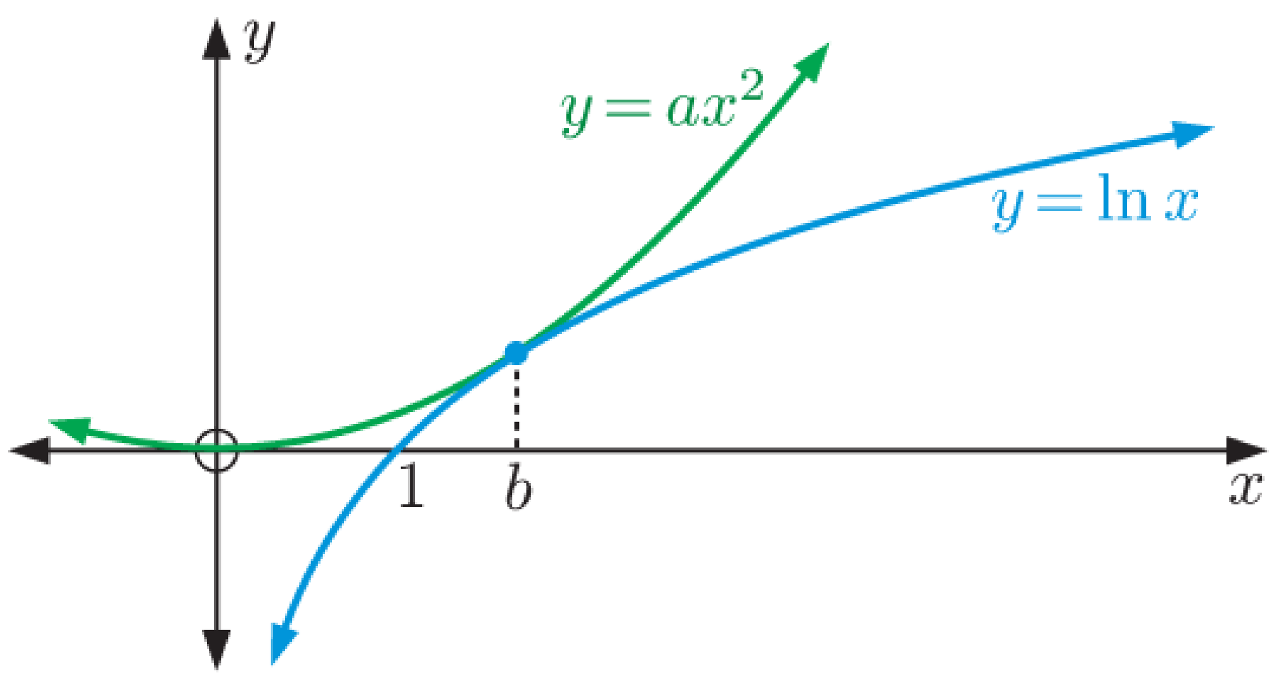
c If the tangent in **b** cuts the x-axis at A and the y-axis at B, find the coordinates of A and B.

d Find the area of triangle OAB and discuss the area of the triangle as $a \to \infty$.

33 The graphs of $y = \sqrt{x + a}$ and $y = \sqrt{2x - x^2}$ have the same gradient at their point of intersection. Find a and the point of intersection.

34 Find, correct to 2 decimal places, the angle between the tangents to $y = 3e^{-x}$ and $y = 2 + e^x$ at their point of intersection.

35 A quadratic of the form $y = ax^2$, $a > 0$, touches the logarithmic function $y = \ln x$ as shown.

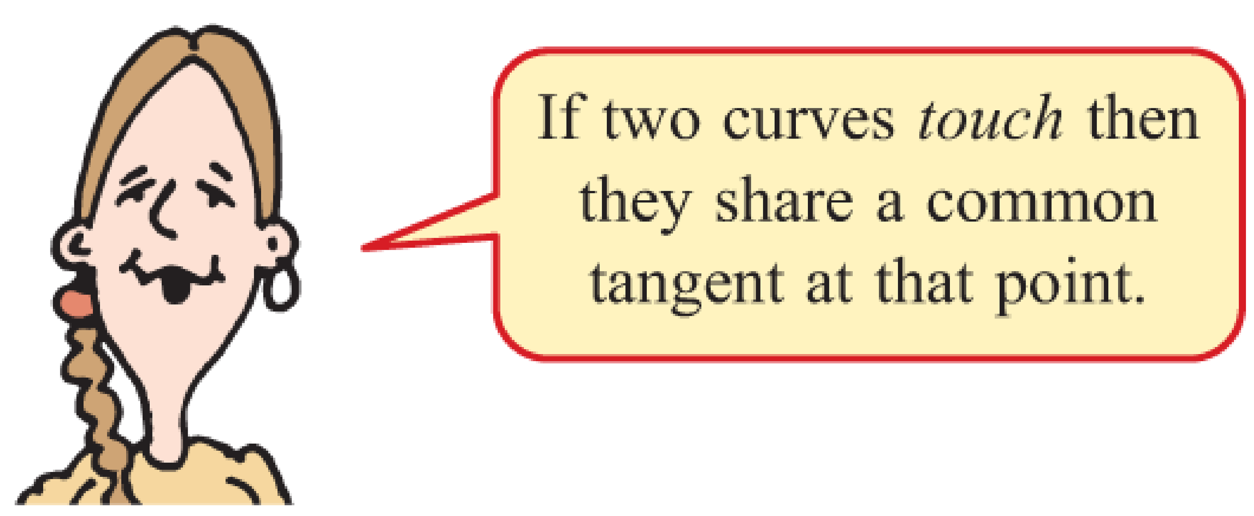


a If the x-coordinate of the point of contact is b, explain why $ab^2 = \ln b$ and $2ab = \dfrac{1}{b}$.

b Deduce that the point of contact is $(\sqrt{e}, \frac{1}{2})$.

c Find the value of a.

d Find the equation of the common tangent.



36 Let $p(x) = ax^2$, $a \neq 0$.

a Find the equations of the tangents to the curve at $x = s$ and $x = t$.

b Prove that the two tangent lines intersect at $x = \dfrac{s+t}{2}$.

c Prove that if the tangent lines are perpendicular then they intersect at $y = -\dfrac{1}{4a}$.

2 **a** $\dfrac{dy}{dx} = 6x^2 - 12x + 7$ **b** $\dfrac{dy}{dx} = -\dfrac{3}{x^2} + \dfrac{15}{x^4}$

c $\dfrac{dy}{dx} = -\dfrac{5}{x^{\frac{4}{3}}}$

3 **a** $f(3) = -17$ **b** $f'(3) = -17$ **c** $f''(3) = -6$

4 **a** $\dfrac{dy}{dx} = 3x^2(1-x^2)^{\frac{1}{2}} - x^4(1-x^2)^{-\frac{1}{2}}$

b $\dfrac{dy}{dx} = \dfrac{(2x-3)(x+1)^{\frac{1}{2}} - \frac{1}{2}(x^2-3x)(x+1)^{-\frac{1}{2}}}{x+1}$

5 **a** $\dfrac{dy}{dx} = e^x + xe^x$ **b** $(1, e)$

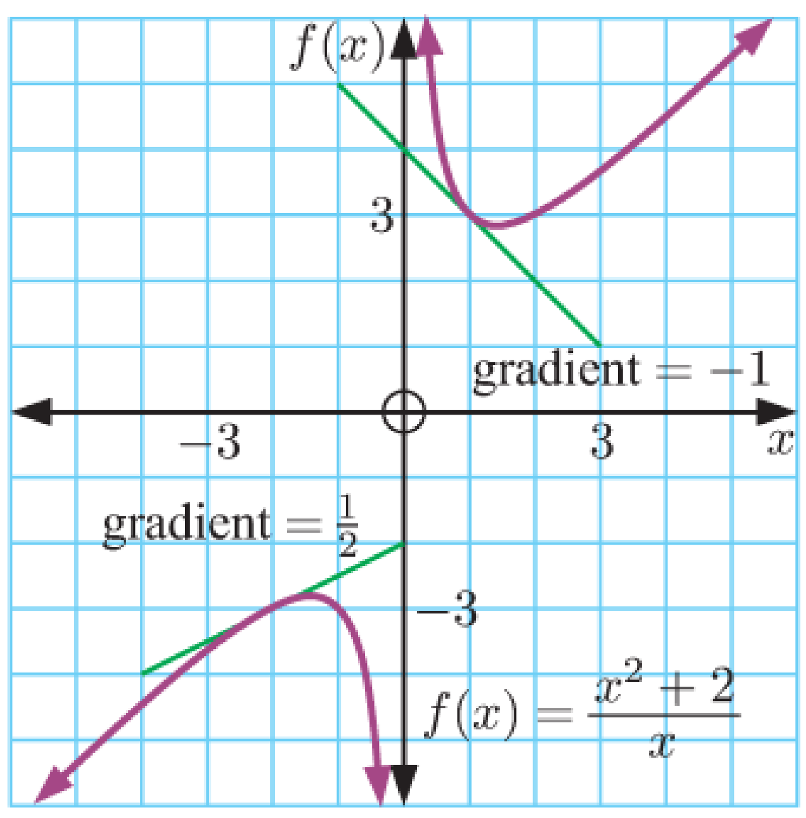
6 **a** $f'(x) = \dfrac{e^x}{e^x+3}$ **b** $f'(x) = \dfrac{3}{x+2} - \dfrac{1}{x}$

c $f'(x) = x^{x^2+1}(2\ln x + 1)$

7 **a** $f'(x) = 1 - \dfrac{2}{x^2}$

b **i** -1
ii $\frac{1}{2}$

c



8 when $x = 1$, $\dfrac{dy}{dx} = 0$

9 **a** $\dfrac{dy}{dx} = \dfrac{3x^2-3}{x^3-3x}$ **b** $\dfrac{dy}{dx} = \dfrac{e^x(x-2)}{x^3}$

c $\dfrac{dy}{dx} = \dfrac{e^{x+y}(y^2+1)}{2y - e^{x+y}(y^2+1)}$

10 **a** $f'(x) = 2^{x^2-5x}\ln 2 \times (2x-5)$

b $f'(x) = \dfrac{\operatorname{cosec} x \sec x}{\ln 10}$

c $f'(x) = -\dfrac{1}{\ln 3}\left(\dfrac{1}{x+5} + \dfrac{1}{x-4}\right)$

11 **a** $f''(x) = 24x^2 - 24x - 18$ **b** $x = -\frac{1}{2}$ or $\frac{3}{2}$

12 **a** $10 - 10\cos 10x$ **b** $\tan x$

c $(5\cos 5x)\ln(2x) + \dfrac{\sin 5x}{x}$

13 **a** $\frac{28}{9}$ **b** 8 **14** $a = 4$, $b = e^2$

15 **a** $-\frac{1}{4}$ **b** **Hint:** Show that $\dfrac{dy}{dx} = -\dfrac{2\sin x + 1}{(\sin x + 2)^2}$.

16 **b** **i** $x = \frac{1}{2}$ **ii** $x \leqslant 0$

18 **a** $\dfrac{dy}{dx} = -\frac{1}{4}\operatorname{cosec}^2\left(\frac{x}{4}\right)$

b $\dfrac{dy}{dx} = x\sec 3x(3x\tan 3x + 2)$

c $\dfrac{dy}{dx} = -e^{-x}\operatorname{cosec}(e^x)[e^x\cot(e^x) + 1]$

d $\dfrac{dy}{dx} = \dfrac{5}{\sqrt{1-25x^2}}$

e $\dfrac{dy}{dx} = 2e^{2x}\left(\arctan 2x + \dfrac{1}{1+4x^2}\right)$

f $\dfrac{dy}{dx} = -\dfrac{\sec^2 x}{\sqrt{1-\tan^2 x}}$

19 **a** $\dfrac{d^2y}{dx^2} = -\dfrac{10}{(1-2x)^3}$ **b** $\dfrac{d^2y}{dx^2} = 6x + \frac{3}{4}x^{-\frac{5}{2}}$

c $\dfrac{d^2y}{dx^2} = -\dfrac{3^x(\ln 3)^2 \times 20}{(20-3^x)^2 \times \ln 2}$

21 **a** $x = -6 \pm \sqrt{33}$ **b** $x = \pm\sqrt{3}$ **c** $x = -3$, 0, or 3

22 **a** $f(x) = -5\sin 4x$ **b** $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$, or $\frac{7\pi}{8}$

23 **a** $\dfrac{dy}{dx} = \dfrac{y^2 - e^x y}{e^x - 2xy}$ **b** 0

27 **a** $f'(x) = e^{ax}(ax+1)$, $f''(x) = ae^{ax}(ax+2)$,
$f'''(x) = a^2e^{ax}(ax+3)$, $f^{(4)}(x) = a^3e^{ax}(ax+4)$

b $f^{(n)}(x) = a^{n-1}e^{ax}(ax+n)$

EXERCISE 18A

1 **a** $f'(x) = 2x - 4$ **b** $y = -2x - 1$

2 **a** $y = -7x + 11$ **b** $y = \frac{1}{4}x + 2$ **c** $y = -2x - 2$
d $y = -2x + 6$ **e** $y = -5x - 9$ **f** $y = -5x - 1$
g $y = \frac{3}{4} - \frac{1}{2}x$ **h** $y = \frac{1}{27}x + \frac{4}{9}$

3 **a** $y = 21$ and $y = -6$ **b** $y = 23$ and $y = -9$
c $y = 2$

4 **a** $k = -5$ **b** $y = 4x - 15$ **5** $y = -3x + 1$

6 $a = -4$, $b = 7$ **7** $a = 2$, $b = \frac{1}{2}$ **10** $a = 4$, $b = 3$

11 **a** $k = 1$ **b** $y = 8x - 7$ **c** $\frac{7}{8}$

12 **a** $\dfrac{dy}{dx} = -\dfrac{3x^2}{2y}$ **b** $y = 4x - 22$

13 **a** $y = -\frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$, $y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$ **b** $(2, 0)$

14 **a** $y = -e^{-2}x + 3e^{-2}$ **b** $y = -\frac{1}{3}x - \frac{1}{3} + \ln 3$

c $y = 4ex - e$ **d** $y = \dfrac{e^2}{2}x - \frac{3}{2}$ **e** $y = 3ex - 5e$

15 **a** Domain is $\{x \mid x < 0 \text{ or } x > 2\}$

b $f'(x) = \dfrac{1}{x} + \dfrac{1}{x-2}$ **c** $y = \frac{4}{3}x - 4 + \ln 3$

16 **a** **i** -1 **ii** -2 **iii** -3 **b** y-intercept is $-a$

17 x-intercept $\frac{2}{3}$, y-intercept $-2e$ **18** $\dfrac{3}{4\sqrt{e}}$ units^2

19 **a** $y = x$ **b** $y = x$ **c** $y = -\frac{1}{2}x + \frac{\pi}{12} + \frac{\sqrt{3}}{2}$
d $y = 1$ **e** $y = 2$

20 **Hint:** Show that there are no tangents which have gradient $= 0$.

21 $\frac{1}{2}\left(\frac{\pi}{2} + 1\right)^2$ units^2

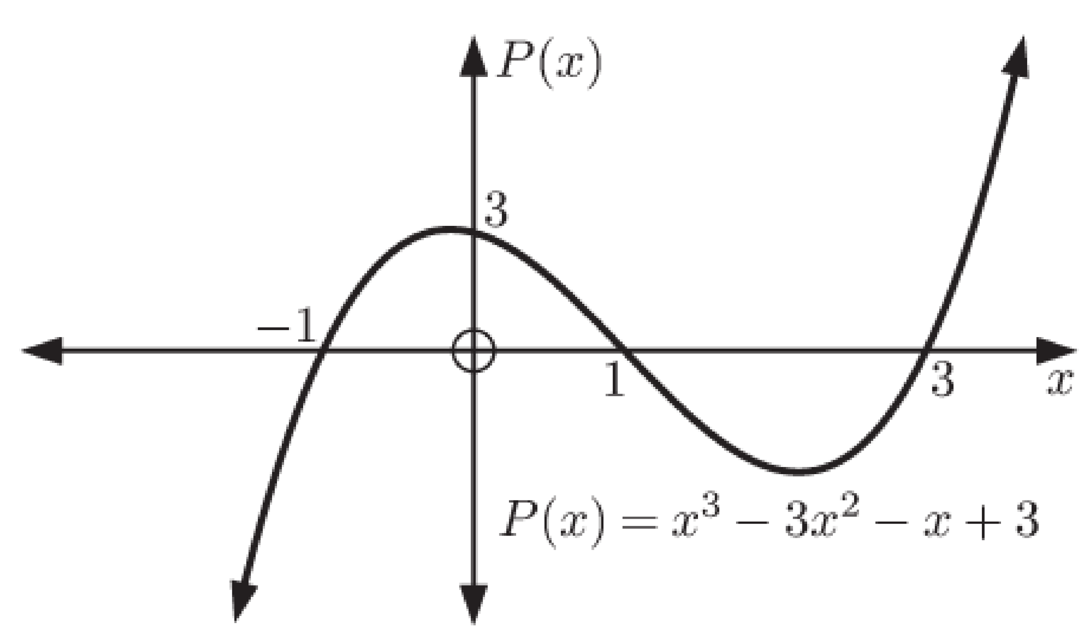
22 **a** $y = \sqrt{2}x - \sqrt{2}\left(\frac{\pi}{4} - 1\right)$ **b** $y = -2x + \frac{2\pi}{3} + \sqrt{3}$

c $y = \frac{1}{2}x + \frac{\pi}{4} - \frac{1}{2}$ **d** $y = \left(\frac{1}{\sqrt{3}} + \frac{2\pi}{3}\right)x + \frac{1}{\sqrt{3}}$

23 $(-4, -64)$ **24** $(4, -31)$ **25** $(-1, -2)$

26 **a** $P(1) = 1 - 3 - 1 + 3 = 0$
The zeros of $P(x)$ are $x = 1, -1$, and 3.

b



c $y = -x - 1$ **d** $(-1, 0)$

e **Hint:** Let $P(x) = \alpha(x-a)(x-b)(x-c)$

27 **a** $y = (2a-1)x - a^2 + 9$

b $y = 5x$, point of contact $(3, 15)$, and $y = -7x$, point of contact $(-3, 21)$

28 **a** $y = (2a+4)x - a^2$

b $y = 12x - 16$, point of contact $(4, 32)$, and $y = -4$, point of contact $(-2, -4)$

29 $y = 5x - 15$ and $y = -7x - 3$

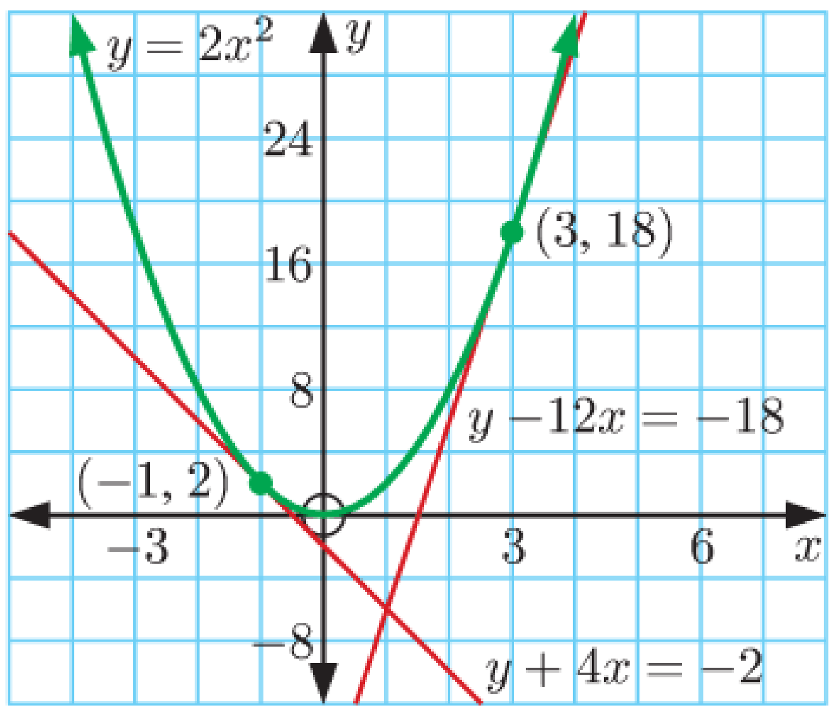
30 **a** $y = e^a x + e^a(1-a)$ **b** $y = ex$

31 **a** $y + 4x = -2$ and $y - 12x = -18$

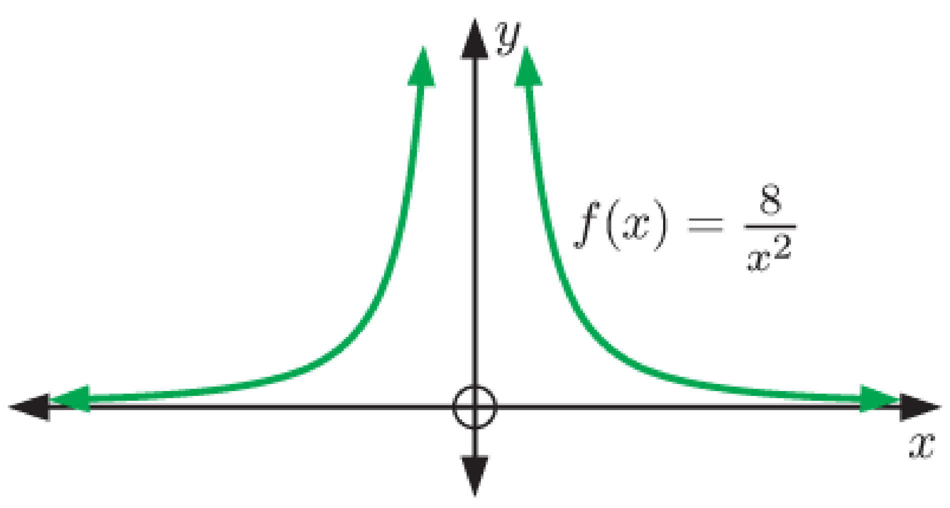
b $(-1, 2)$ for $y + 4x = -2$ and $(3, 18)$ for $y - 12x = -18$

c For a tangent to pass through $(1, 4)$, $4 = 4a - 2a^2$ must have real solutions. But $\Delta < 0$, so no real solutions.

d



32 **a**



b $16x + a^3 y = 24a$ **c** A is $(\frac{3}{2}a, 0)$, B is $\left(0, \dfrac{24}{a^2}\right)$

d area $= \dfrac{18}{|a|}$ units^2; as $a \to \infty$, area $\to 0$

33 $a = \frac{1}{4}$, point of intersection $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

34 $\approx 63.43°$

35 **a** **Hint:** They must have the same y-coordinate at $x = b$ and the same gradient.

c $a = \frac{1}{2e}$ **d** $y = e^{-\frac{1}{2}}x - \frac{1}{2}$

36 **a** $y = 2asx - as^2$, $y = 2atx - at^2$

EXERCISE 18B

1 **a** $x + 8y = 132$ **b** $x + 7y = 26$ **c** $x - 3y = -11$

d $x + 6y = 43$ **e** $64x + 4y = -65$ **f** $x = 2$

g $4x + 57y = 1042$ **h** $x - 2y = -1$

2 **a** $y = 4 - 2x$ **b** $y = -\frac{9}{62}x + \frac{1259}{186}$

3 **a** $\{x \mid x < 2\}$ **c** $3x - 2y = 1$ **d** $8x + 3y = -19$

4 **a** $y = x + 1$ **b** $ex + y = e^2 + 1$ **c** $x + 2ey = 1 + 2e^2$

d $2x + y = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$ **e** $y = 1 - \dfrac{1}{\ln 4}x$ **f** $y = -x$

g $2x - 2\sqrt{3}y = \pi + \sqrt{3}$ **h** $x - \sqrt{2}y = \frac{\pi}{4}$

i $(3\ln 3)x + 2y = 6\ln 3 + \dfrac{2\ln 6}{\ln 3}$

j $x - 2\sqrt{3}y = \frac{\pi}{6} - 4\sqrt{3}$ **k** $\sqrt{6}x + y = \sqrt{6}\pi + \sqrt{2}$

5 $a = 2$, $b = 4$ **6** $(-1, -2)$ and $(2, 1)$

7 $x = 0$ **8** $y = -\sqrt{14}x + 4\sqrt{14}$

9 **a** $\dfrac{dx}{dy} = e^y(y+2)$ **b** $4e^2 x + y = 2 + 12e^4$ **10** $b = 3$

EXERCISE 18C

1 **a** **i** $x \geqslant 0$ **ii** never **b** **i** never **ii** $-2 < x \leqslant 3$

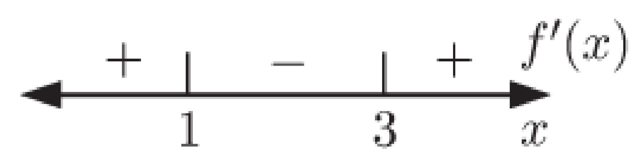
c **i** $x \leqslant 2$ **ii** $x \geqslant 2$ **d** **i** $x \in \mathbb{R}$ **ii** never

e **i** $1 \leqslant x \leqslant 5$ **ii** $x \leqslant 1$, $x \geqslant 5$

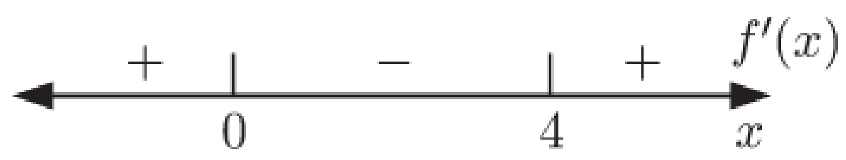
f **i** $2 \leqslant x < 4$, $x > 4$ **ii** $x < 0$, $0 < x \leqslant 2$

2 **a** **i** $x \leqslant 1$, $x \geqslant 3$ **ii** $1 \leqslant x \leqslant 3$

b $f'(x) = 3x^2 - 12x + 9$
$= 3(x-3)(x-1)$



3 **a** $f'(x) = 3x^2 - 12x$
$= 3x(x-4)$



b increasing for $x \leqslant 0$ and $x \geqslant 4$
decreasing for $0 \leqslant x \leqslant 4$

4 **a** increasing for $x \geqslant 0$, decreasing for $x \leqslant 0$

b never increasing, decreasing for all $x \in \mathbb{R}$

c increasing for $x > 0$, never decreasing

d increasing for $x \geqslant -\frac{3}{4}$, decreasing for $x \leqslant -\frac{3}{4}$

e never increasing, decreasing for all $x \neq 0$

f increasing for $x \geqslant 1$, decreasing for $0 \leqslant x \leqslant 1$

g never increasing, decreasing for $x > 0$

h increasing for $x \leqslant 0$, and $x \geqslant 4$,
decreasing for $0 \leqslant x \leqslant 4$

i increasing for $-\sqrt{\frac{2}{3}} \leqslant x \leqslant \sqrt{\frac{2}{3}}$,
decreasing for $x \leqslant -\sqrt{\frac{2}{3}}$, $x \geqslant \sqrt{\frac{2}{3}}$

j increasing for $-\frac{1}{2} \leqslant x \leqslant 3$,
decreasing for $x \leqslant -\frac{1}{2}$, $x \geqslant 3$

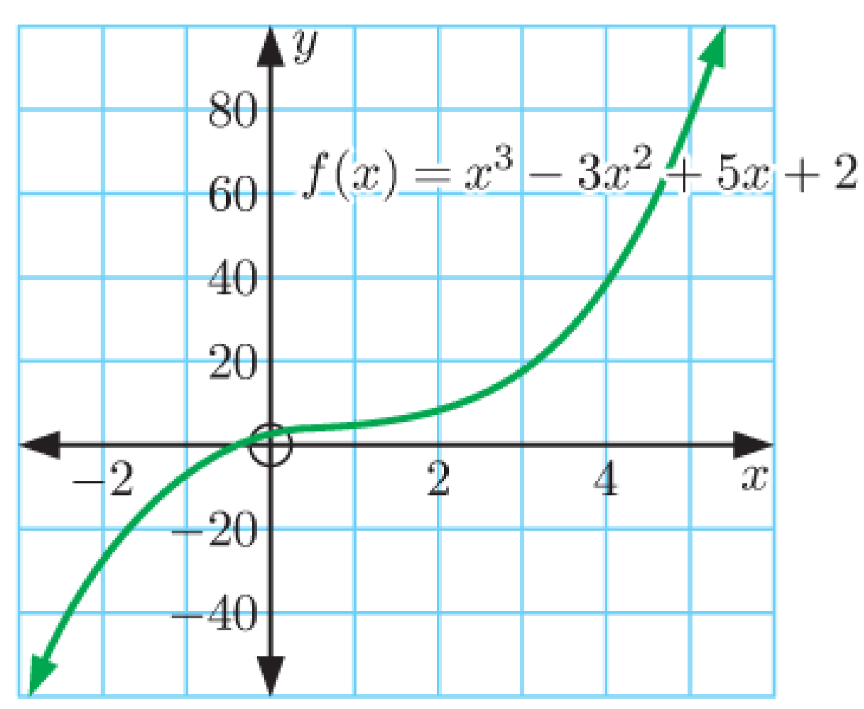
k increasing for $x \geqslant 0$, decreasing for $x \leqslant 0$

l increasing for $x \leqslant 2 - \sqrt{3}$, $x \geqslant 2 + \sqrt{3}$,
decreasing for $2 - \sqrt{3} \leqslant x \leqslant 2 + \sqrt{3}$

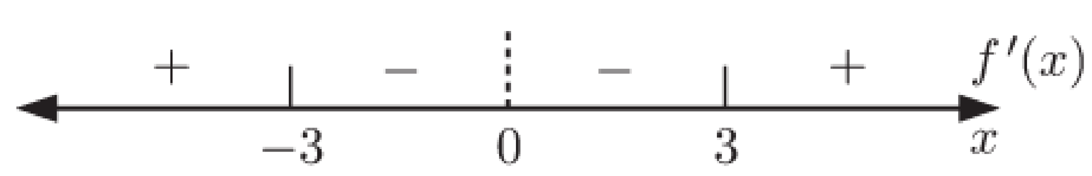
5 **a** $f'(x) = 3x^2 - 6x + 5$

b $\Delta = 36 - 60 < 0$ and $a > 0$
$\therefore$ $f'(x)$ lies entirely above x-axis.
$\therefore$ $f'(x) > 0$ for all x.
$\therefore$ $f(x)$ is increasing for all x.

c



6 **a**



b increasing for $x \leqslant -3$ and $x \geqslant 3$,
decreasing for $-3 \leqslant x < 0$ and $0 < x \leqslant 3$

7 **a**