22 In a game, n small pumpkins are placed 1 metre apart in a straight line. Players start 3 metres before the first pumpkin.



Each player **collects** a single pumpkin by picking it up and bringing it back to the start. The nearest pumpkin is collected first. The player then collects the next nearest pumpkin and the game continues in this way until the signal is given for the end.

Sirma runs to get each pumpkin and brings it back to the start.

- **a** Write down the distance,  $a_1$ , in metres that she has to run in order to **collect** the first pumpkin.
- **b** The distances she runs to **collect** each pumpkin form a sequence  $a_1, a_2, a_3, \ldots$ .

  i Find  $a_2$ .

  ii Find  $a_3$ .
- **c** Write down the common difference, d, of the sequence.

The final pumpkin Sirma collected was 24 metres from the start.

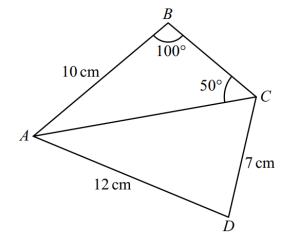
d i Find the total number of pumpkins that Sirma collected.ii Find the total distance that Sirma ran to collect these pumpkins.

Peter also plays the game. When the signal is given for the end of the game he has run 940 metres.

- e Calculate the total number of pumpkins that Peter collected.
- **f** Calculate Peter's distance from the start when the signal is given.
- The quadrilateral ABCD has AB = 10 cm. AD = 12 cm and CD = 7 cm.

The size of angle ABC is  $100^{\circ}$  and the size of angle ACB is  $50^{\circ}$ .

- **a** Find the length of AC in centimetres.
- **b** Find the size of angle ADC.



- After recording the angle of elevation of the top of a tower at an unknown distance from the tower's base, a student walks exactly 20 m directly away from the tower along horizontal ground and records a second angle of elevation. The two angles recorded are 47.7° and 38.2°. Find the height of the tower.
- 14 All that remains intact of an ancient castle is part of the keep wall and a single stone pillar some distance away. The base of the wall and the foot of the pillar are at equal elevations.

From the top of the keep wall, the tip of the pillar is at an angle of depression of 23.5° and the base of the pillar is at an angle of depression of 37.7°.

The wall is known to have a height of 41 m. Find the height of the pillar, to the nearest metre.

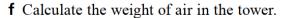
24 The diagram shows an office tower of total height 126 metres. It consists of a square-based pyramid *VABCD* on top of a cuboid *ABCDPQRS*.

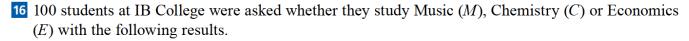
V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1°.

- **a** i Write down the length of *VA* in metres.
  - ii Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA.
- **b** Show that the height of the pyramid is 18.0 metres correct to 3 significant figures.
- **c** Calculate the length of AC in metres.
- **d** Show that the length of *BC* is 19.1 metres correct to 3 significant figures.
- **e** Calculate the volume of the tower.

To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90% of the volume of the tower is occupied by air and they know that 1 m<sup>3</sup> of air weighs 1.2 kg.





10 study all three

15 study Music and Chemistry

17 study Music and Economics

12 study Chemistry and Economics

11 study Music only

6 study Chemistry only

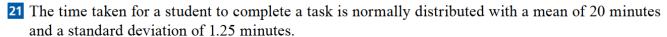
- **a** Draw a Venn diagram to represent the information above.
- **b** Write down the number of students who study Music but not Economics.

There are 22 Economics students in total.

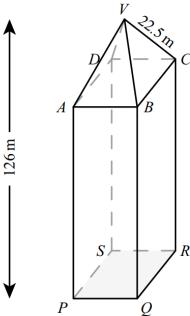
- **c** i Calculate the number of students who study Economics only.
  - ii Find the number of students who study none of these three subjects.

A student is chosen at random from the 100 that were asked above.

- **d** Find the probability that this student
  - i studies Economics
  - ii studies Music and Chemistry but not Economics
  - iii does not study either Music or Economics
  - iv does not study Music given that the student does not study Economics.



- **a** A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes.
- **b** The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k.



23 A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is  $\frac{1}{5}$ . Let X be the number of questions that Mark answers correctly.

**a** i Find E(X).

ii Find the probability that Mark passes the test.

Bill also takes the test. Let Y be the number of questions that Bill answers correctly. The following table is the probability distribution for Y.

y	0	1	2	3	4	5
P(Y=y)	0.67	0.05	a+2b	a-b	2a+b	0.04

**b** i Show that 4a + 2b = 0.24.

ii Given that E(Y) = 1, find a and b.

c Find which student is more likely to pass the test.

The distances thrown by Josie in an athletics competition is modelled by a normal distribution with mean 40 m and standard deviation 5 m. Any distance less than 40 m gets 0 points. Any distance between 40 m and 46 m gets 1 point. Any distance above 46 m gets 4 points.

a Find the expected number of points Josie gets if she throws

i once

ii twice.

**b** What assumptions have you made in **a ii**? Comment on how realistic these assumptions are.

16 Orlaigh wants to borrow £200000. She has two options:

Option A: Pay 5% interest rate for 25 years, compounded quarterly.

Option B: Pay 4% interest rate for 30 years, compounded monthly.

Orlaigh will make the minimum required repayment in each compounding period.

Which option should Orlaigh choose if she wishes to minimize the total amount repaid? Justify your answer.

17 Chad takes out a loan of \$100000 over 20 years. The interest rate is 4.8% compounded monthly.

a How much will Chad pay each month?

**b** If Chad overpays by \$100 each month, how much interest will he save in total? Give your answer to the nearest \$100.

8 A teacher works at two different schools and wants to find out whether the grades his students get depend on the school. She has the following data:

	3	4 or 5	6 or 7
School A	11	44	36
School B	16	83	40

The teachers decides to conduct a  $\chi^2$  test for independence, using the 5% level of significance.

a Write down suitable hypotheses for this test.

**b** Calculate the *p*-value and hence state the conclusion.

9 Francisco and his friends want to test whether performance in running 400 metres improves if they follow a particular training schedule. The competitors are tested before and after the training schedule.

The times taken to run 400 metres, in seconds, before and after training are shown in the following table.

Competitor	A	В	C	D	E
Time before training	75	74	60	69	69
Time after training	73	69	55	72	65

Apply an appropriate test at the 1% significance level to decide whether the training schedule improves competitors' times, stating clearly the null and alternative hypotheses. (It may be assumed that the distributions of the times before and after training are normal.)

10 A toy manufacturer makes a cubical dice with the numbers 1, 2, 3, 4, 5, 6 respectively marked on the six faces. The manufacturer claims that, when it is thrown, the probability distribution of the score X obtained is given by

$$P(X = x) = \frac{x}{21}$$
 for  $x = 1, 2, 3, 4, 5, 6$ .

To check this claim, Pierre throws the dice 420 times with the following results.

x	Frequency			
1	25			
2	46			
3	64			
4	82			
5	99			
6	104			

State suitable hypotheses and, using an appropriate test, determine whether or not the manufacturer's claim can be accepted at the 5% significance level.

Roy is practising archery. He thinks that he has the probability 0.45 of hitting the target. To test this, he takes five shots at a time and records how many times he hit the target. He repeats this 100 times, obtaining the following results.

Outcome	0	1	2	3	4 or 5
Observed	12	18	34	22	14

- a Assuming Roy is correct, write down the distribution of the number of hits out of 5 shots.
- **b** Hence find the expected frequencies.
- **c** Write down the number of degrees of freedom for a  $\chi^2$  goodness of fit test.
- **d** Test using the 5% level of significance whether Roy's belief is correct.