1. The quadratic function $f(x) = p + qx - x^2$ has a maximum value of 5 when $x = 3$.

(a) Find the value of p and the value of q.

(4)

(b) The graph of $f(x)$ is translated 3 units in the positive direction parallel to the x-axis. Determine the equation of the new graph.

(2)
(Total 6 marks)

2. When the function $q(x) = x^3 + kx^2 - 7x + 3$ is divided by $(x + 1)$ the remainder is seven times the remainder that is found when the function is divided by $(x + 2)$.

Find the value of k.

(Total 5 marks)

3. Let $g(x) = \log_5 \left| 2\log_3 x \right|$. Find the product of the zeros of g.

(Total 5 marks)

4. The sum, S_n, of the first n terms of a geometric sequence, whose n^{th} term is u_n, is given by

$$S_n = \frac{7^n - a^n}{7^n}, \text{ where } a > 0.$$

(a) Find an expression for u_n.

(2)

(b) Find the first term and common ratio of the sequence.

(4)

(c) Consider the sum to infinity of the sequence.

(i) Determine the values of a such that the sum to infinity exists.

(ii) Find the sum to infinity when it exists.

(2)
(Total 8 marks)

5. (a) Express the quadratic $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where $a, b, c \in \mathbb{Z}$.

(3)

(b) Describe a sequence of transformations that transforms the graph of $y = x^2$ to the graph of $y = 3x^2 - 6x + 5$.

(3)
(Total 6 marks)

6. Given that $\frac{z}{z+2} = 2 - \text{i}$, $z \in \mathbb{C}$, find z in the form $a + \text{i}b$.

(Total 4 marks)

7. A geometric sequence $u_1, u_2, u_3, \ldots$ has $u_1 = 27$ and a sum to infinity of $\frac{81}{2}$.

(a) Find the common ratio of the geometric sequence.

(2)

An arithmetic sequence $v_1, v_2, v_3, \ldots$ is such that $v_2 = u_2$ and $v_4 = u_4$.

(b) Find the greatest value of N such that $\sum_{n=1}^{N} v_n > 0$.

(5)
(Total 7 marks)

8. (a) Show that $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$.

(2)

(b) Hence find the value of $\cot\frac{\pi}{8}$ in the form $a + b\sqrt{2}$, where $a, b \in \mathbb{Z}$.

(3)
(Total 5 marks)

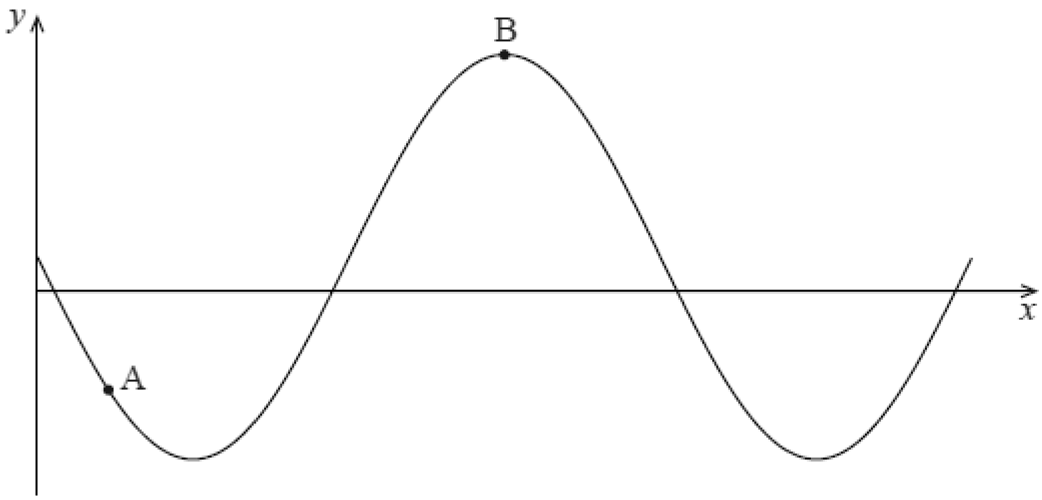
9. Find the area enclosed by the curve $y = \arctan x$, the x-axis and the line $x = \sqrt{3}$.

(Total 6 marks)

10. Show that the points (0, 0) and $(\sqrt{2\pi}, -\sqrt{2\pi})$ on the curve $e^{(x+y)} = \cos(xy)$ have a common tangent.

(Total 7 marks)

11. The diagram below shows a curve with equation $y = 1 + k \sin x$, defined for $0 \le x \le 3\pi$.



The point $A\left(\frac{\pi}{6}, -2\right)$ lies on the curve and B(a, b) is the maximum point.

(a) Show that $k = -6$.

(2)

(b) Hence, find the values of a and b.

(3)
(Total 5 marks)

12. Consider the function $f : x \to \sqrt{\frac{\pi}{4} - \arccos x}$.

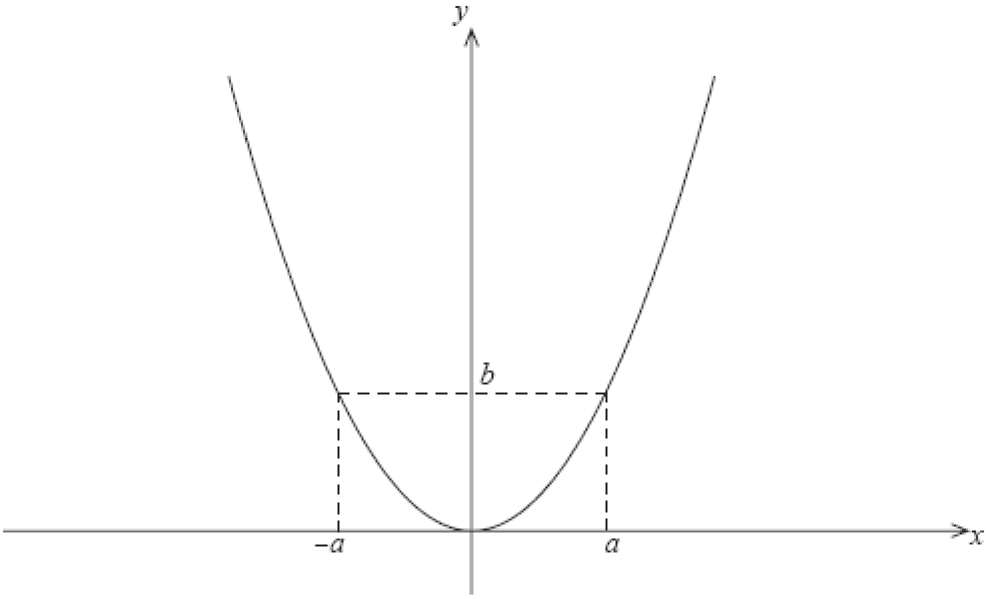
(a) Find the largest possible domain of f.

(4)

(b) Determine an expression for the inverse function, f^{-1}, and write down its domain.

(4)
(Total 8 marks)

13. The diagram below shows the graph of the function $y = f(x)$, defined for all $x \in \mathbb{R}$, where $b > a > 0$.

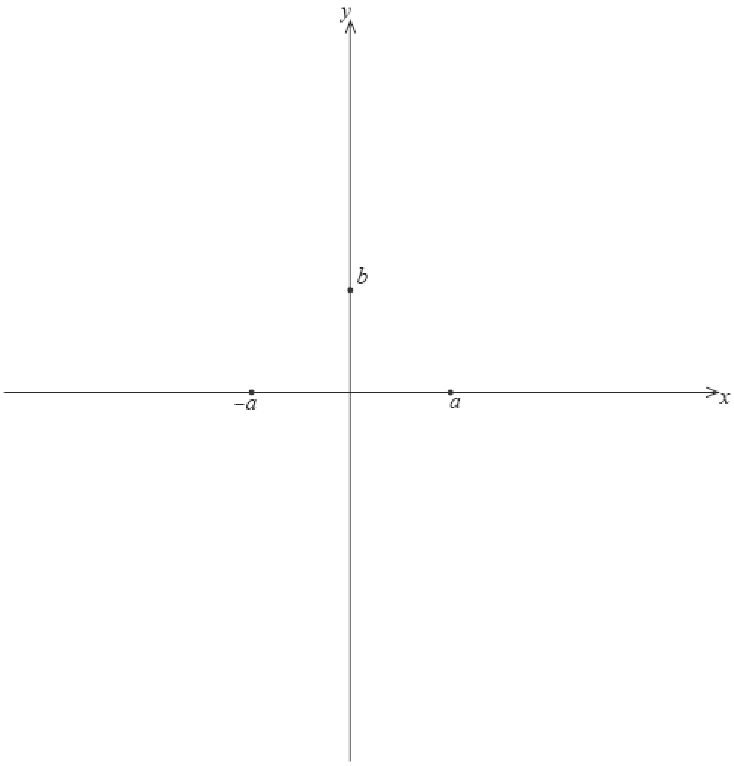


Consider the function $g(x) = \dfrac{1}{f(x-a)-b}$.

(a) Find the largest possible domain of the function g.

(2)

(b) On the axes below, sketch the graph of $y = g(x)$. On the graph, indicate any asymptotes and local maxima or minima, and write down their equations and coordinates.



(6)
(Total 8 marks)

14. The complex numbers $z_1 = 2 - 2\mathrm{i}$ and $z_2 = 1 - \mathrm{i}\sqrt{3}$ are represented by the points A and B respectively on an Argand diagram. Given that O is the origin,

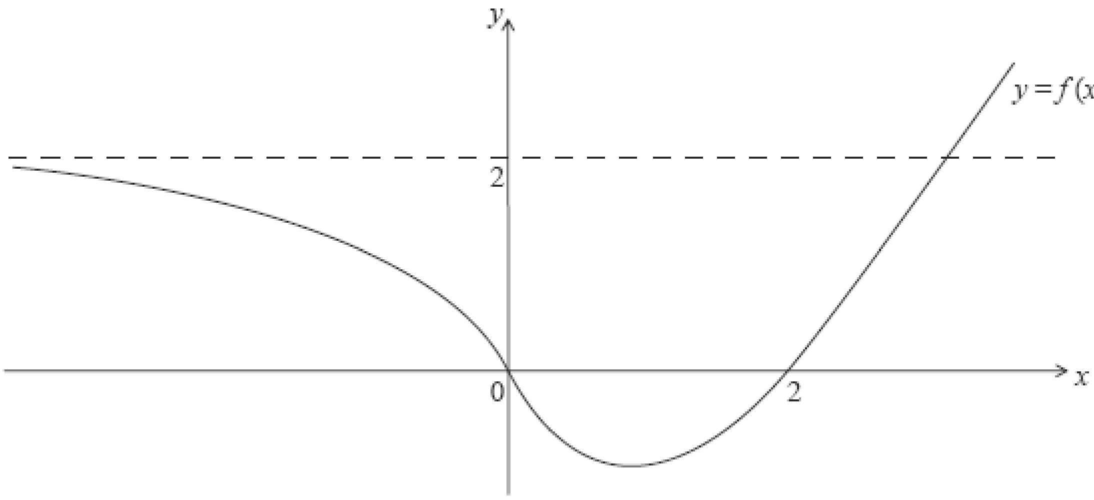
(a) find AB, giving your answer in the form $a\sqrt{b-\sqrt{3}}$, where $a, b \in \mathbb{Z}^+$;

(3)

(b) calculate $\mathrm{A\hat{O}B}$ in terms of π.

(3)
(Total 6 marks)

15. The diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = 2$.



(a) Sketch the graph of $y = \dfrac{1}{f(x)}$.

(3)

(b) Sketch the graph of $y = x\,f(x)$.

(3)
(Total 6 marks)

16. Find the area between the curves $y = 2 + x - x^2$ and $y = 2 - 3x + x^2$.

(Total 7 marks)

17. In triangle ABC, AB = 9 cm, AC =12 cm, and $\hat{B}$ is twice the size of $\hat{C}$.

Find the cosine of $\hat{C}$.

(Total 5 marks)

18. Given that $z_1 = 2$ and $z_2 = 1 + i\sqrt{3}$ are roots of the cubic equation $z^3 + bz^2 + cz + d = 0$ where $b, c, d \in \mathbb{R}$,

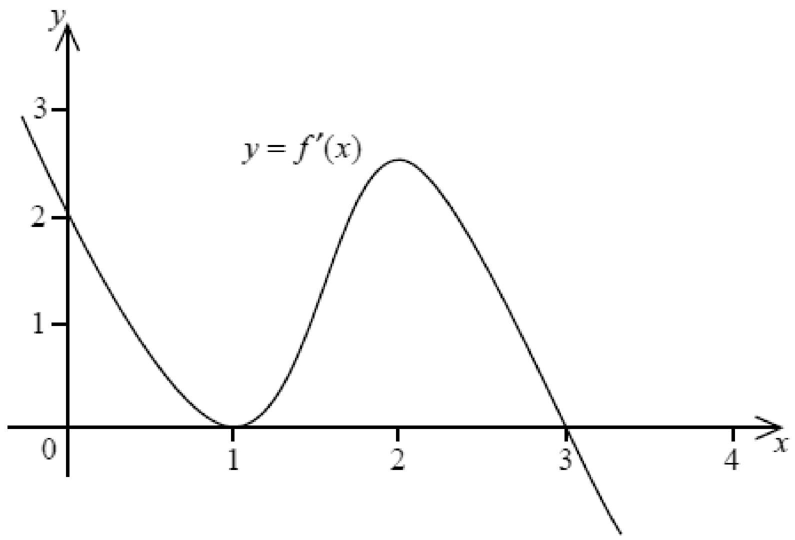
(a) write down the third root, z_3, of the equation;

(1)

(b) find the values of b, c and d;

(4)
(Total 5 marks)

19. The diagram below shows a sketch of the gradient function $f'(x)$ of the curve $f(x)$.



On the graph below, sketch the curve $y = f(x)$ given that $f(0) = 0$. Clearly indicate on the graph any maximum, minimum or inflexion points.

(Total 5 marks)

20. The curve C has equation $y = \frac{1}{8}(9+8x^2-x^4)$.

(a) Find the coordinates of the points on C at which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

(4)

(b) The tangent to C at the point P(1, 2) cuts the x-axis at the point T. Determine the coordinates of T.

(4)

(c) The normal to C at the point P cuts the y-axis at the point N. Find the area of triangle PTN.

(7)
(Total 15 marks)

21. The normal to the curve $xe^{-y} + e^{y} = 1 + x$, at the point $(c, \ln c)$, has a y-intercept $c^2 + 1$.

Determine the value of c.

(Total 7 marks)

22. (a) (i) Sketch the graphs of $y = \sin x$ and $y = \sin 2x$, on the same set of axes, for $0 \le x \le \frac{\pi}{2}$.

(ii) Find the x-coordinates of the points of intersection of the graphs in the domain $0 \le x \le \frac{\pi}{2}$.

(iii) Find the area enclosed by the graphs.

(9)

(b) Find the value of $\int_0^1 \sqrt{\frac{x}{4-x}}\,\mathrm{d}x$ using the substitution $x = 4\sin^2\theta$.

(8

(c) The increasing function f satisfies $f(0) = 0$ and $f(a) = b$, where $a > 0$ and $b > 0$.

(i) By reference to a sketch, show that $\int_0^a f(x)\mathrm{d}x = ab - \int_0^b f^{-1}(x)\mathrm{d}x$.

(ii) **Hence** find the value of $\int_0^2 \arcsin\left(\frac{x}{4}\right)\mathrm{d}x$.

(8)
(Total 25 marks)

23. Jenny goes to school by bus every day. When it is not raining, the probability that the bus is late is $\frac{3}{20}$. When it is raining, the probability that the bus is late is $\frac{7}{20}$. The probability that it rains on a particular day is $\frac{9}{20}$. On one particular day the bus is late. Find the probability that it is not raining on that day.

(Total 5 marks)

24. Given that $Ax^3 + Bx^2 + x + 6$ is exactly divisible by $(x + 1)(x – 2)$, find the value of A and the value of B.

(Total 5 marks)

25. (a) Show that $\frac{3}{x+1}+\frac{2}{x+3}=\frac{5x+11}{x^2+4x+3}$.

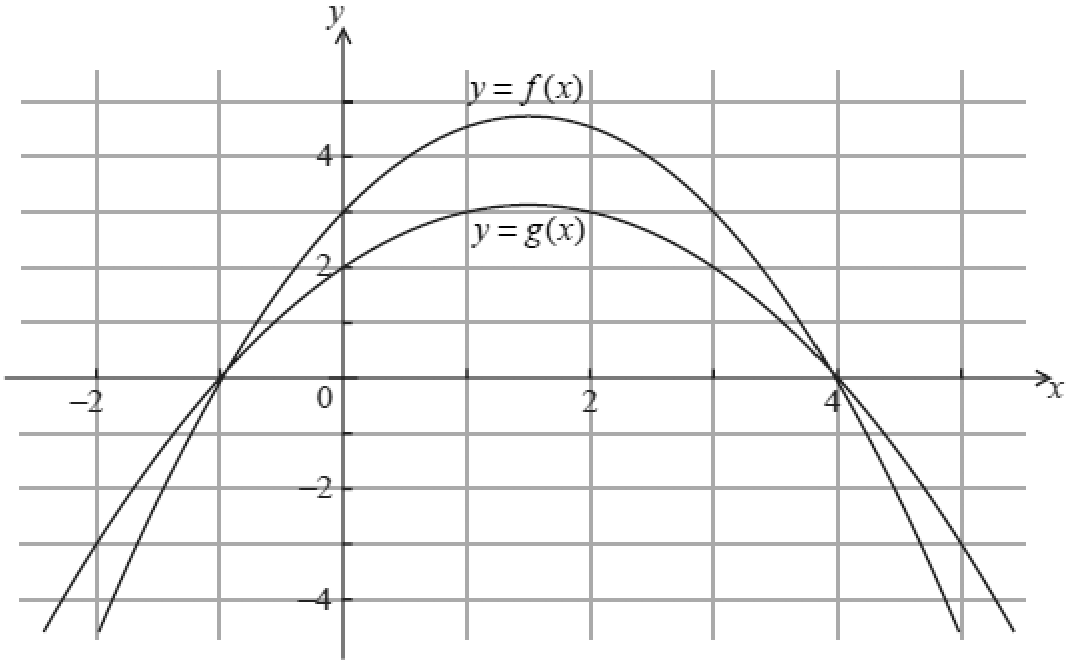
(2)

(b) Hence find the value of k such that $\int_0^2 \frac{5x+11}{x^2+4x+3}\,dx = \ln k$.

(4)

(Total 6 marks)

26. Shown below are the graphs of $y = f(x)$ and $y = g(x)$.



If $(f \circ g)(x) = 3$, find all possible values of x.

(Total 4 marks)

27. (a) Calculate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\sqrt[3]{\tan x}} \mathrm{d}x$.

(6)

(b) Find $\int \tan^3 x \,\mathrm{d}x$.

(3)
(Total 9 marks)

28. The diagram below shows two straight lines intersecting at O and two circles, each with centre O. The outer circle has radius R and the inner circle has radius r.

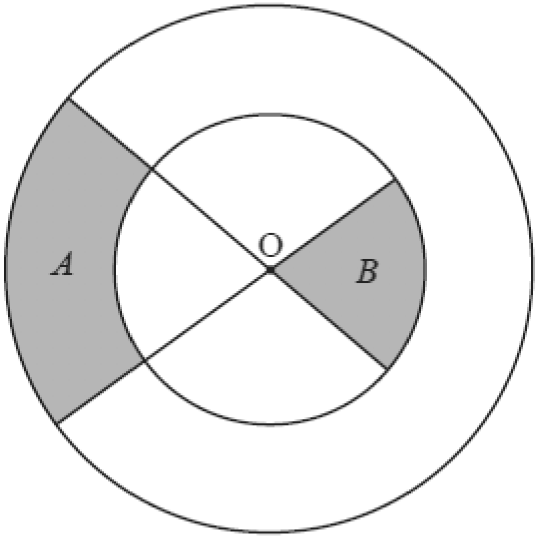


diagram not to scale

Consider the shaded regions with areas A and B. Given that $A : B = 2 : 1$, find the **exact** value of the ratio $R : r$.

(Total 5 marks)

29. If x satisfies the equation $\sin\left(x + \frac{\pi}{3}\right) = 2\sin x \sin\left(\frac{\pi}{3}\right)$, show that $11 \tan x = a + b\sqrt{3}$, where $a, b \in \mathbb{Z}^+$.

(Total 6 marks)

30. The common ratio of the terms in a geometric series is 2^x.

(a) State the set of values of x for which the sum to infinity of the series exists.

(2)

(b) If the first term of the series is 35, find the value of x for which the sum to infinity is 40.

(4)
(Total 6 marks)

31. Consider $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 5x + 4}$.

(a) Find the equations of all asymptotes of the graph of f.

(4)

(b) Find the coordinates of the points where the graph of f meets the x and y axes.

(2)

(c) Find the coordinates of

(i) the maximum point and justify your answer;

(ii) the minimum point and justify your answer.

(10)

(d) Sketch the graph of f, clearly showing all the features found above.

(3)

(e) **Hence**, write down the number of points of inflexion of the graph of f.

(1)
(Total 20 marks)

32. Solve the equation $4^{x-1} = 2^x + 8$.

(Total 5 marks)

33. Two players, A and B, alternately throw a fair six–sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six.

(Total 7 marks)

34. (a) Show that $\arctan\left(\dfrac{1}{2}\right) + \arctan\left(\dfrac{1}{3}\right) = \dfrac{\pi}{4}$.

(2)

(b) Hence, or otherwise, find the value of arctan (2) + arctan (3).

(3)
(Total 5 marks)

35. A function f is defined by $f(x) = \frac{2x-3}{x-1}$, $x \neq 1$.

(a) Find an expression for $f^{-1}(x)$.

(3)

(b) Solve the equation $|f^{-1}(x)| = 1 + f^{-1}(x)$.

(3)
(Total 6 marks)

36. Find the set of values of x for which $|x - 1| > |2x - 1|$.

(Total 4 marks)

37. The region enclosed between the curves $y = \sqrt{x}e^{x}$ and $y = e\sqrt{x}$ is rotated through 2π about the x-axis. Find the volume of the solid obtained.

(Total 7 marks)

38. The function f is defined by $f(x) = e^{x^2-2x-1.5}$.

(a) Find $f'(x)$.

(2)

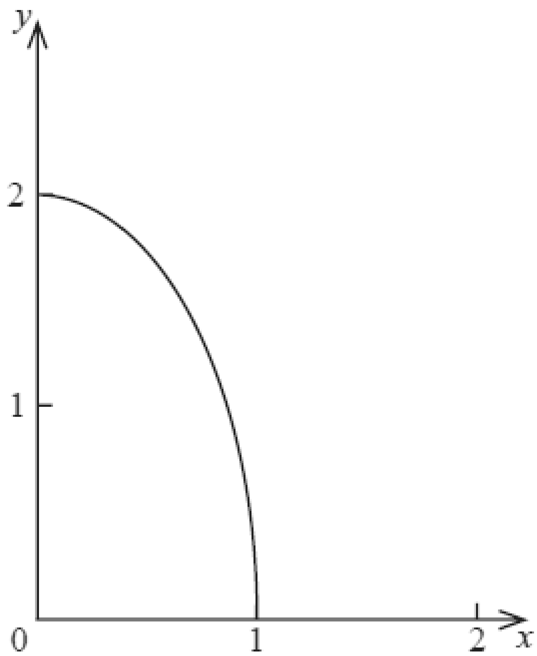
(b) You are given that $y = \frac{f(x)}{x-1}$ has a local minimum at $x = a$, $a > 1$. Find the value of a.

(6)
(Total 8 marks)

39. The mean of the first ten terms of an arithmetic sequence is 6. The mean of the first twenty terms of the arithmetic sequence is 16. Find the value of the 15$^{\text{th}}$ term of the sequence.

(Total 6 marks)

40. Consider the part of the curve $4x^2 + y^2 = 4$ shown in the diagram below.



(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

(3)

(b) Find the gradient of the tangent at the point $\left(\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

(1)

(c) A bowl is formed by rotating this curve through 2π radians about the x-axis.
Calculate the volume of this bowl.

(4)
(Total 8 marks)