AI HL 29.09 [92 marks]

1. [Maximum mark: 6] 18M.1.SL.TZ1.T_2

Each month the number of days of rain in Cardiff is recorded.
The following data was collected over a period of 10 months.

11 13 8 11 8 7 8 14 x 15

For these data the **median** number of days of rain per month is 10.

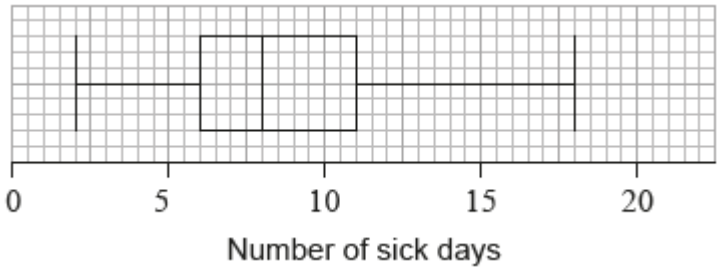
(a) Find the value of x. [2]

(b.i) Find the standard deviation [2]

(b.ii) Find the interquartile range. [2]

2. [Maximum mark: 5] 21M.1.SL.TZ2.5

The number of sick days taken by each employee in a company during a year was recorded. The data was organized in a box and whisker diagram as shown below:



For this data, write down

(a.i) the minimum number of sick days taken during the year. [1]

(a.ii) the lower quartile. [1]

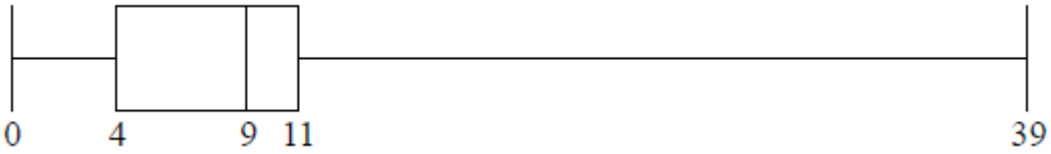
(a.iii) the median. [1]

(b) Paul claims that this box and whisker diagram can be used to infer that the percentage of employees who took fewer than six sick days is smaller than the percentage of employees who took more than eleven sick days.

State whether Paul is correct. Justify your answer. [2]

3. [Maximum mark: 6] 18M.1.SL.TZ1.S_2

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.



(a) Find the value of the interquartile range. [2]

(b) One student sent k text messages, where $k > 11$. Given that k is an outlier, find the least value of k. [4]

4. [Maximum mark: 6] 19N.1.SL.TZ0.T_13

Chicken eggs are classified by grade $(4, 5, 6, 7$ or $8)$, based on weight. A mixed carton contains 12 eggs and could include eggs from any grade. As part of the science project, Rocky buys 9 mixed cartons and sorts the eggs according to their weight.

Grade	Weight, w (grams)	Frequency
4	$40 \leq w < 50$	3
5	$50 \leq w < 60$	30
6	$60 \leq w < 70$	45
7	$70 \leq w < 80$	25
8	$80 \leq w < 90$	5

(a) State whether the weight of the eggs is a continuous or discrete variable. [1]

(b) Write down the modal grade of the eggs. [1]

(c) Use your graphic display calculator to find an estimate for the standard deviation of the weight of the eggs. [2]

(d) The mean weight of these eggs is 64.9 grams, correct to three significant figures.

Use the table and your answer to part (c) to find the **smallest possible** number of eggs that could be within one standard deviation of the mean. [2]

5. [Maximum mark: 6] 19N.1.SL.TZ0.T_6

Galois Airways has flights from Hong Kong International Airport to different destinations. The following table shows the distance, x kilometres, between Hong Kong and the different destinations and the corresponding airfare, y, in Hong Kong dollars (HKD).

Destination	Bali, Indonesia	Sydney, Australia	Bengaluru, India	Singapore	Auckland, New Zealand	Bangkok, Thailand
Distance x, (km)	3400	7400	4000	2600	9200	1700
Airfare y, (HKD)	1550	3600	2800	1300	4000	1400

The Pearson's product–moment correlation coefficient for this data is 0.948, correct to three significant figures.

(a) Use your graphic display calculator to find the equation of the regression line y on x. [2]

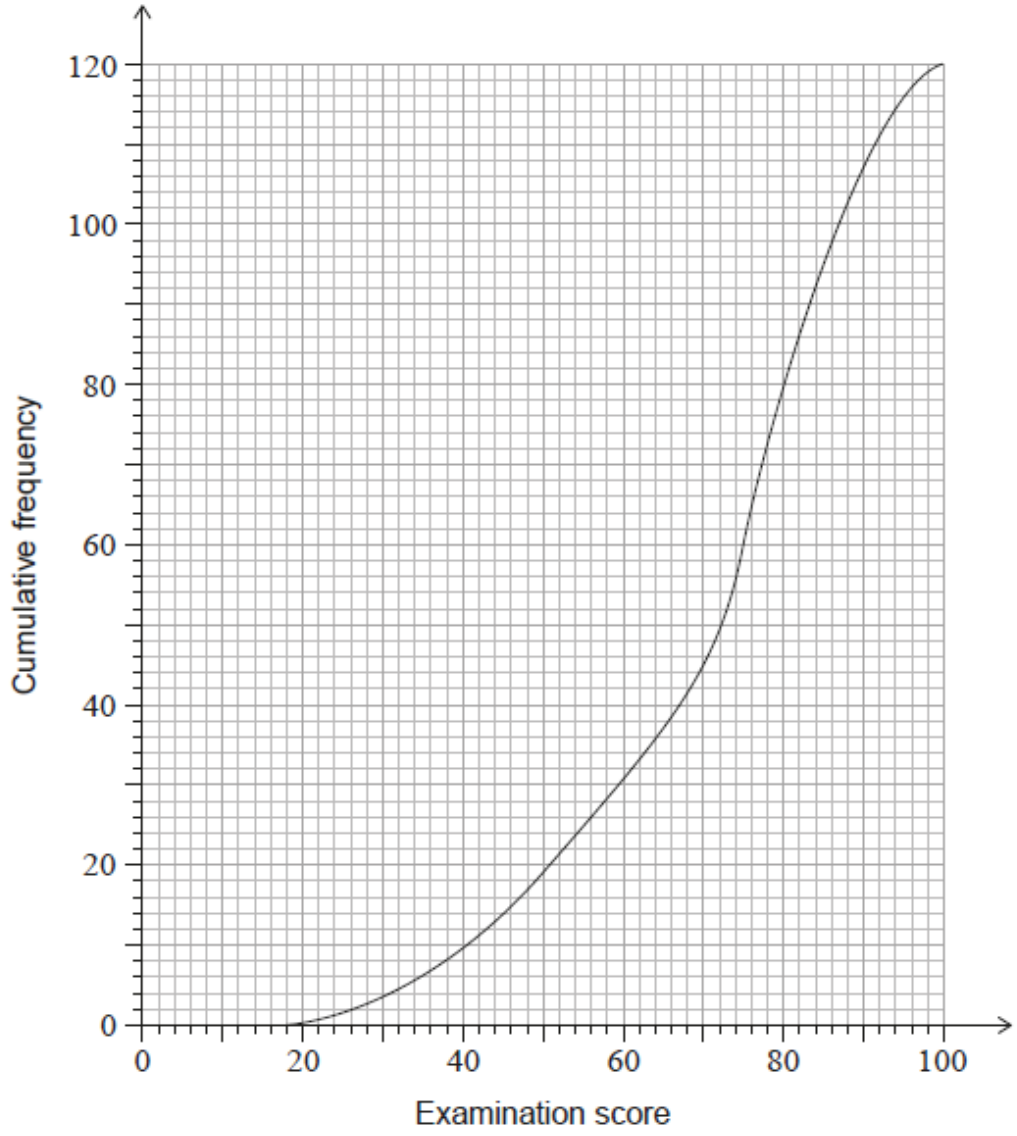
The distance from Hong Kong to Tokyo is 2900 km.

(b) Use your regression equation to estimate the cost of a flight from Hong Kong to Tokyo with Galois Airways. [2]

(c) Explain why it is valid to use the regression equation to estimate the airfare between Hong Kong and Tokyo. [2]

6. [Maximum mark: 8] 21N.1.SL.TZ0.9

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b. [2]

The mean grade for these students is 3.65.

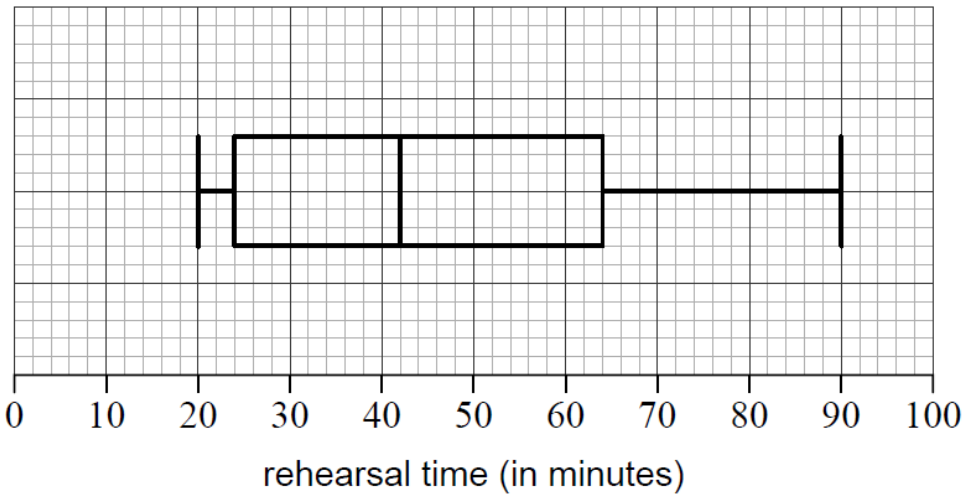
(c.i) Find the number of students who obtained a grade 5. [3]

(c.ii) Find the minimum score needed to obtain a grade 5. [2]

7. [Maximum mark: 6] 19N.1.SL.TZ0.T_15

Stephen was invited to perform a piano recital. In preparation for the event, Stephen recorded the amount of time, in minutes, that he rehearsed each day for the piano recital.

Stephen rehearsed for 32 days and data for all these days is displayed in the following box-and-whisker diagram.



(a) Write down the median rehearsal time. [1]

Stephen states that he rehearsed on each of the 32 days.

(b) State whether Stephen is correct. Give a reason for your answer. [2]

(c) On k days, Stephen practiced exactly 24 minutes.

Find the possible values of k. [3]

8. [Maximum mark: 5] 22M.1.SL.TZ2.7

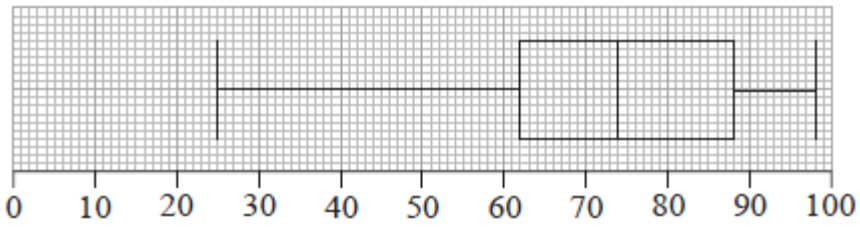
A college runs a mathematics course in the morning. Scores for a test from this class are shown below.

$25 \quad 33 \quad 51 \quad 62 \quad 63 \quad 63 \quad 70 \quad 74 \quad 79 \quad 79 \quad 81 \quad 88 \quad 90 \quad 90 \quad 98$

For these data, the lower quartile is 62 and the upper quartile is 88.

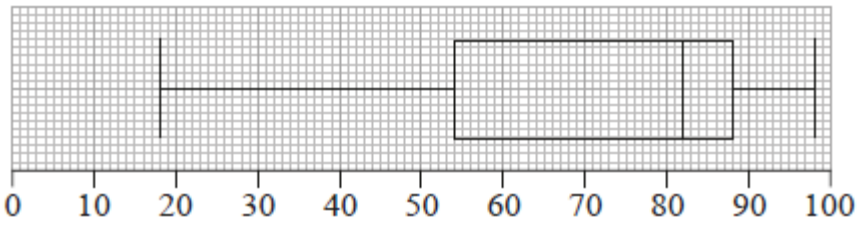
(a) Show that the test score of 25 would not be considered an outlier. [3]

The box and whisker diagram showing these scores is given below.



Test scores

Another mathematics class is run by the college during the evening. A box and whisker diagram showing the scores from this class for the same test is given below.



Test scores

A researcher reviews the box and whisker diagrams and believes that the evening class performed better than the morning class.

(b) With reference to the box and whisker diagrams, state one aspect that may support the researcher's opinion and one

aspect that may counter it. [2]

9. [Maximum mark: 12] 17M.2.SL.TZ1.T_1

The following table shows the average body weight, x, and the average weight of the brain, y, of seven species of mammal. Both measured in kilograms (kg).

Species	Average body weight, x (kg)	Average weight of the brain, y (kg)
Cat	3	0.026
Cow	465	0.423
Donkey	187	0.419
Giraffe	529	0.680
Goat	28	0.115
Jaguar	100	0.157
Sheep	56	0.175

(a) Find the range of the average body weights for these seven species of mammal. [2]

(b.i) For the data from these seven species calculate r, the Pearson's product–moment correlation coefficient; [2]

(b.ii) For the data from these seven species describe the correlation between the average body weight and the average weight of the brain. [2]

(c) Write down the equation of the regression line y on x, in the form $y = mx + c$. [2]

The average body weight of grey wolves is 36 kg.

(d) Use your regression line to estimate the average weight of the brain of grey wolves. [2]

In fact, the average weight of the brain of grey wolves is 0.120 kg.

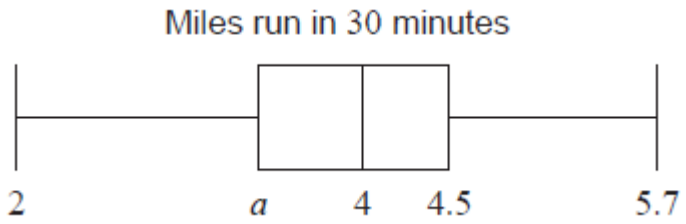
(e) Find the percentage error in your estimate in part (d). [2]

10. [Maximum mark: 15] 20N.1.SL.TZ0.S_8

Each athlete on a running team recorded the distance (M miles) they ran in 30 minutes.

The median distance is 4 miles and the interquartile range is 1.1 miles.

This information is shown in the following box-and-whisker plot.



(a) Find the value of a. [2]

The distance in miles, M, can be converted to the distance in kilometres, K, using the formula $K = \frac{8}{5}M$.

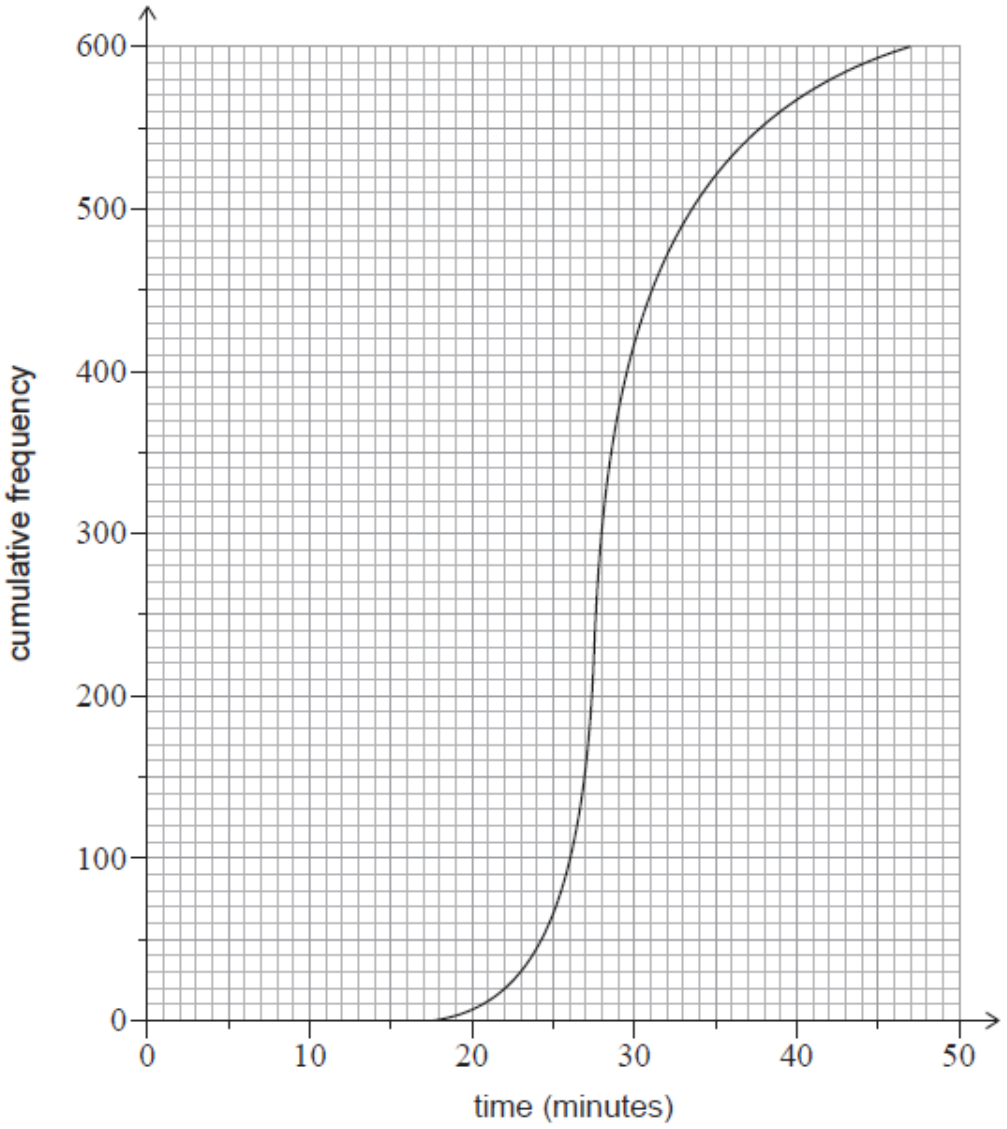
(b) Write down the value of the median distance in kilometres (km). [1]

The variance of the distances run by the athletes is $\frac{16}{9}\ \text{km}^2$.

The standard deviation of the distances is b miles.

(c) Find the value of b. [4]

A total of 600 athletes from different teams compete in a $5\ \text{km}$ race. The times the 600 athletes took to run the $5\ \text{km}$ race are shown in the following cumulative frequency graph.



There were 400 athletes who took between 22 and m minutes to complete the 5 km race.

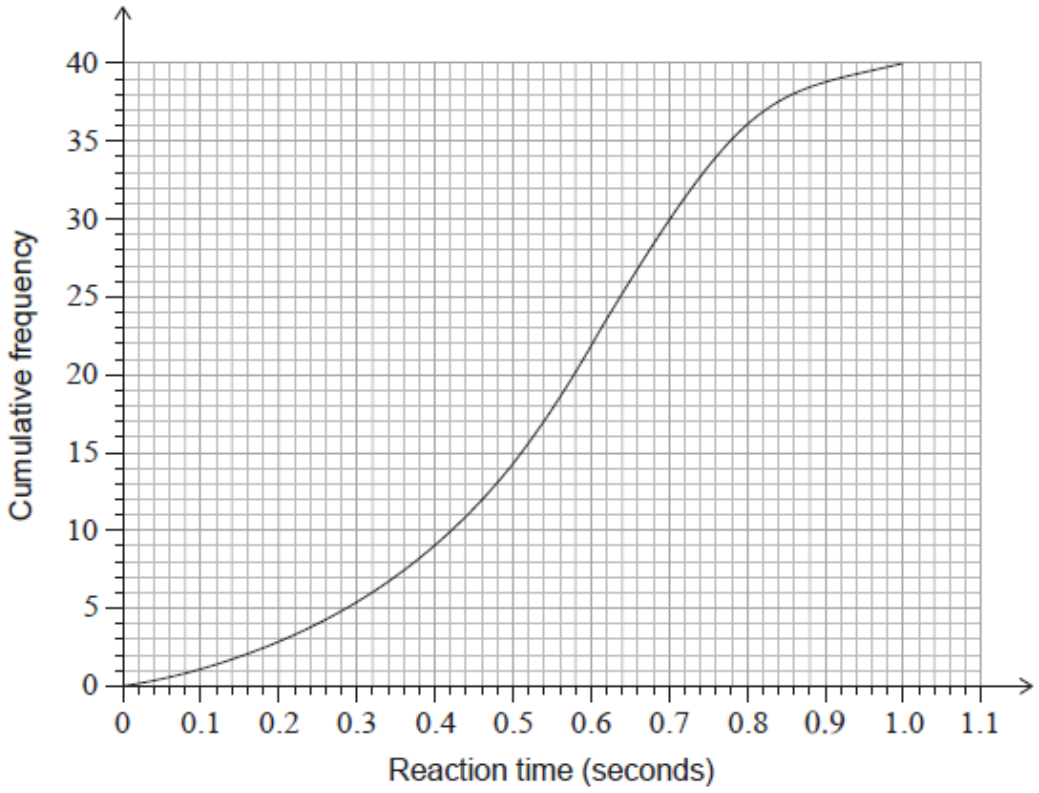
(d) Find m. [3]

(e) The first 150 athletes that completed the race won a prize.

Given that an athlete took between 22 and m minutes to complete the 5 km race, calculate the probability that they won a prize. [5]

11. [Maximum mark: 17] 22M.2.SL.TZ2.1

Mackenzie conducted an experiment on the reaction times of teenagers. The results of the experiment are displayed in the following cumulative frequency graph.



Use the graph to estimate the

(a.i) median reaction time. [1]

(a.ii) interquartile range of the reaction times. [3]

(b) Find the estimated number of teenagers who have a reaction time greater than 0.4 seconds. [2]

(c) Determine the 90th percentile of the reaction times from the cumulative frequency graph. [2]

Mackenzie created the cumulative frequency graph using the following grouped frequency table.

Reaction time, t (s)	Frequency
$0 < t \leq 0.2$	3
$0.2 < t \leq 0.4$	a
$0.4 < t \leq 0.6$	13
$0.6 < t \leq 0.8$	14
$0.8 < t \leq 1.0$	b

(d.i) Write down the value of a. [1]

(d.ii) Write down the value of b. [1]

(e) Write down the modal class from the table. [1]

(f) Use your graphic display calculator to find an estimate of the mean reaction time. [2]

Upon completion of the experiment, Mackenzie realized that some values were grouped incorrectly in the frequency table. Some reaction times recorded in the interval $0 < t \leq 0.2$ should have been recorded in the interval $0.2 < t \leq 0.4$.

(g) Suggest how, if at all, the estimated mean and estimated median reaction times will change if the errors are corrected. Justify your response. [4]