

AI SL 20.09 [113 marks]

1. [Maximum mark: 16]

EXN.2.SL.TZ0.4

A dice manufacturer claims that for a novelty die he produces the probability of scoring the numbers 1 to 5 are all equal, and the probability of a 6 is two times the probability of scoring any of the other numbers.

(a) Find the probability of scoring a six when rolling the novelty die. [3]

(b) Find the probability of scoring more than 2 sixes when this die is rolled 5 times. [4]

To test the manufacture's claim one of the novelty dice is rolled 350 times and the numbers scored on the die are shown in the table below.

Number scored	Frequency
1	32
2	57
3	47
4	58
5	54
6	102

(c.i) Find the expected frequency for each of the numbers if the manufacturer's claim is true. [2]

A χ^2 goodness of fit test is to be used with a 5% significance level.

(c.ii) Write down the null and alternative hypotheses. [2]

(c.iii) State the degrees of freedom for the test. [1]

(c.iv) Determine the conclusion of the test, clearly justifying your answer. [4]

2. [Maximum mark: 9]

EXM.1.SL.TZ0.9

Six coins are tossed simultaneously 320 times, with the following results.

0 tail	5 times
1 tail	40 times
2 tails	86 times
3 tails	89 times
4 tails	67 times
5 tails	29 times
6 tails	4 times

At the 5% level of significance, test the hypothesis that all the coins are fair.

[9]

3. [Maximum mark: 6]

EXM.1.SL.TZ0.2

Kayla wants to measure the extent to which two judges in a gymnastics competition are in agreement. Each judge has ranked the seven competitors, as shown in the table, where 1 is the highest ranking and 7 is the lowest.

Competitor	A	B	C	D	E	F	G
Judge 1	1	2	3	3	5	6	6
Judge 2	2	3	1	4	5	5	7

(a) Calculate Spearman's rank correlation coefficient for this data.

[5]

(b) State what conclusion Kayla can make from the answer in part (a).

[1]

4. [Maximum mark: 5]

21M.1.SL.TZ1.6

Arriane has geese on her farm. She claims the mean weight of eggs from her black geese is less than the mean weight of eggs from her white geese.

She recorded the weights of eggs, in grams, from a random selection of geese. The data is shown in the table.

Weights of eggs from black geese	136	134	142	141	128	126
Weights of eggs from white geese	135	138	141	140	136	134

In order to test her claim, Arriane performs a t -test at a 10% level of significance. It is assumed that the weights of eggs are normally distributed and the samples have equal variances.

- (a) State, in words, the null hypothesis. [1]
- (b) Calculate the p -value for this test. [2]
- (c) State whether the result of the test supports Arriane's claim. Justify your reasoning. [2]

5. [Maximum mark: 6]

21M.1.SL.TZ2.11

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of copies sold	74	97	91	86	112

A goodness of fit test at the 5% significance level is used on this data to determine whether the vendor's model is suitable.

The critical value for the test is 9.49 and the hypotheses are

H_0 : The data satisfies the model.

H_1 : The data does not satisfy the model.

- (a) Find an estimate for how many copies the vendor expects to sell each day. [1]
- (b.i) Write down the degrees of freedom for this test. [1]
- (b.ii) Write down the conclusion to the test. Give a reason for your answer. [4]

6. [Maximum mark: 6]

21M.1.SL.TZ2.8

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

Weight of chinchilla rabbits, kg	4.9	4.2	4.1	4.4	4.3	4.6	4.0	4.7	4.5	4.4
Weight of sable rabbits, kg	4.2	4.1	4.1	4.2	4.5	4.4	4.5	3.9	4.2	4.0

A t -test is to be performed at the 5% significance level.

- (a) Write down the null and alternative hypotheses. [2]
- (b) Find the p -value for this test. [2]
- (c) Write down the conclusion to the test. Give a reason for your answer. [2]

7. [Maximum mark: 18]

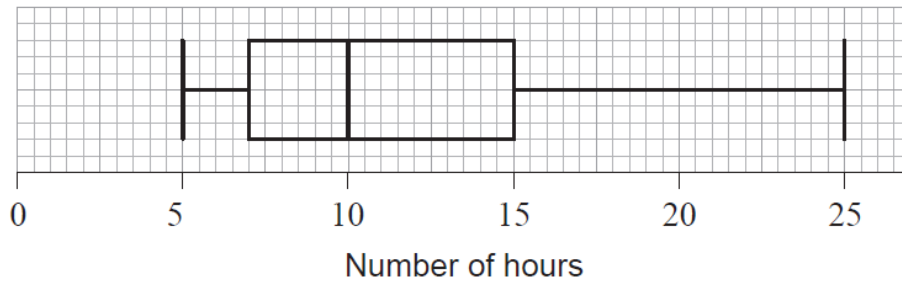
21M.2.SL.TZ1.1

As part of his mathematics exploration about classic books, Jason investigated the time taken by students in his school to read the book *The Old Man and the Sea*. He collected his data by stopping and asking students in the school corridor, until he reached his target of 10 students from **each** of the literature classes in his school.

- (a) State which of the two sampling methods, systematic or quota, Jason has used.

[1]

Jason constructed the following box and whisker diagram to show the number of hours students in the sample took to read this book.



- (b) Write down the median time to read the book.

[1]

- (c) Calculate the interquartile range.

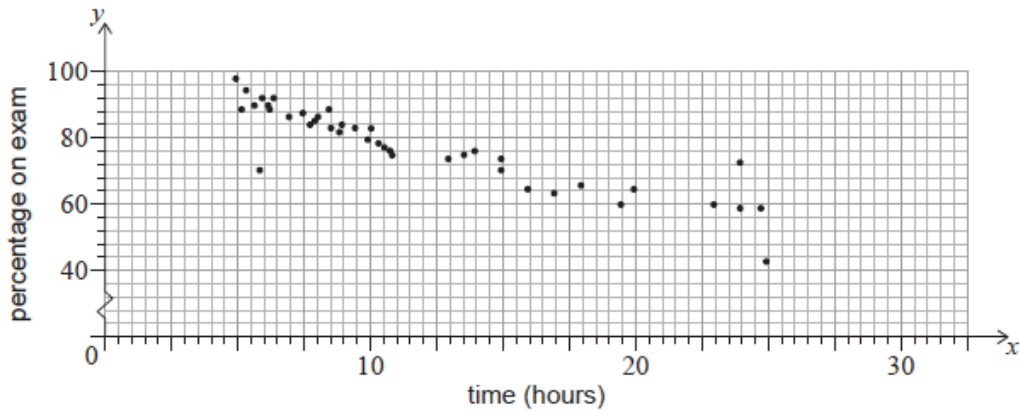
[2]

Mackenzie, a member of the sample, took 25 hours to read the novel. Jason believes Mackenzie's time is not an outlier.

- (d) Determine whether Jason is correct. Support your reasoning.

[4]

For each student interviewed, Jason recorded the time taken to read *The Old Man and the Sea* (x), measured in hours, and paired this with their percentage score on the final exam (y). These data are represented on the scatter diagram.



(e) Describe the correlation. [1]

Jason correctly calculates the equation of the regression line y on x for these students to be

$$y = -1.54x + 98.8.$$

He uses the equation to estimate the percentage score on the final exam for a student who read the book in 1.5 hours.

(f) Find the percentage score calculated by Jason. [2]

(g) State whether it is valid to use the regression line y on x for Jason's estimate. Give a reason for your answer. [2]

Jason found a website that rated the 'top 50' classic books. He randomly chose eight of these classic books and recorded the number of pages. For example, Book H is rated 44th and has 281 pages. These data are shown in the table.

Book	A	B	C	D	E	F	G	H
Number of pages (n)	4215	863	585	1225	366	209	624	281
Top 50 rating (r)	1	2	5	7	13	22	40	44

Jason intends to analyse the data using Spearman's rank correlation coefficient, r_s .

(h) Copy and complete the information in the following table.

Book	A	B	C	D	E	F	G	H
Rank – Number of pages	1							
Rank – Top 50 Rating	1							

[2]

(i.i) Calculate the value of r_s .

[2]

(i.ii) Interpret your result.

[1]

8. [Maximum mark: 13]

21M.2.SL.TZ1.4

The stopping distances for bicycles travelling at 20 km h^{-1} are assumed to follow a normal distribution with mean 6.76 m and standard deviation 0.12 m .

Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at 20 km h^{-1} manages to stop

(a.i) in less than 6.5 m . [2]

(a.ii) in more than 7 m . [1]

1000 randomly selected bicycles are tested and their stopping distances when travelling at 20 km h^{-1} are measured.

Find, correct to four significant figures, the expected number of bicycles tested that stop between

(b.i) 6.5 m and 6.75 m . [2]

(b.ii) 6.75 m and 7 m . [1]

The measured stopping distances of the 1000 bicycles are given in the table.

Measured stopping distance	Number of bicycles
Less than 6.5 m	12
Between 6.5 m and 6.75 m	428
Between 6.75 m and 7 m	527
More than 7 m	33

It is decided to perform a χ^2 goodness of fit test at the 5% level of significance to decide whether the stopping distances of bicycles travelling at 20 km h^{-1} can be modelled by a normal distribution with mean 6.76 m and standard deviation 0.12 m .

- (c) State the null and alternative hypotheses. [2]
- (d) Find the p -value for the test. [3]
- (e) State the conclusion of the test. Give a reason for your answer. [2]

9. [Maximum mark: 6]

20N.1.SL.TZ0.T_10

On 90 journeys to his office, Isaac noted whether or not it rained. He also recorded his journey time to the office, and classified each journey as short, medium or long.

Of the 90 journeys to the office, there were 3 short journeys when it rained, 22 medium journeys when it rained, and 15 long journeys when it rained. There were also 14 short journeys when it did not rain.

Isaac carried out a χ^2 test at the 5% level of significance on these data, looking at the weather and the types of journeys.

- (a) Write down H_0 , the null hypothesis for this test. [1]
- (b) Find the expected number of short trips when it rained. [3]
- (c) The p -value for this test is 0.0206.
State the conclusion to Isaac's test. Justify your reasoning. [2]

10. [Maximum mark: 15]

19N.2.SL.TZ0.T_1

Casanova restaurant offers a set menu where a customer chooses **one** of the following meals: pasta, fish or shrimp.

The manager surveyed 150 customers and recorded the customer's age and chosen meal. The data is shown in the following table.

	Pasta	Fish	Shrimp	Total
Adults	24	25	32	81
Children	20	14	35	69
Total	44	39	67	150

A χ^2 test was performed at the 10% significance level. The critical value for this test is 4.605.

- (a) State H_0 , the null hypothesis for this test. [1]
- (b) Write down the number of degrees of freedom. [1]
- (c) Show that the expected number of children who chose shrimp is 31, correct to two significant figures. [2]

Write down

- (d.i) the χ^2 statistic. [2]
- (d.ii) the p -value. [1]
- (e) State the conclusion for this test. Give a reason for your answer. [2]

A customer is selected at random.

- (f.i) Calculate the probability that the customer is an adult. [2]
- (f.ii) Calculate the probability that the customer is an adult or that the customer chose shrimp.

[2]

(f.iii) Given that the customer is a child, calculate the probability that they chose pasta or fish.

[2]

11. [Maximum mark: 13]

19M.2.SL.TZ1.T_3

A survey was conducted on a group of people. The first question asked how many pets they each own. The results are summarized in the following table.

Number of pets owned	0	1	2	3	4	5
Number of people	20	45	40	30	20	5

- (a) Write down the total number of people, from this group, who are **pet owners**. [1]
- (b) Write down the modal number of pets. [1]
- (c.i) For these data, write down the median number of pets. [1]
- (c.ii) For these data, write down the lower quartile. [1]
- (c.iii) For these data, write down the upper quartile. [1]

The second question asked each member of the group to state their age and preferred pet. The data obtained is organized in the following table.

Preferred pet	Age	
	Teenager	Non-teenager
cat	23	32
dog	35	23
bird	16	13
other	11	7

- (d) Write down the ratio of teenagers to non-teenagers in its simplest form. [1]

A χ^2 test is carried out at the 10 % significance level.

- (e.i) State the null hypothesis. [1]
- (e.ii) State the alternative hypothesis. [1]
- (f) Write down the number of degrees of freedom for this test. [1]
- (g) Calculate the expected number of teenagers that prefer cats. [2]
- (i) State the conclusion for this test. Give a reason for your answer. [2]