Approximations and error

In this presentation we will go through an example of finding the largest possible percentage error. You are encouraged to try to solve the example on your own before looking up the solution. A similar example may appear on the short test.

In this presentation we will go through an example of finding the largest possible percentage error. You are encouraged to try to solve the example on your own before looking up the solution. A similar example may appear on the short test.

Worked Example

The lenghts of two sides of a triangle have been measured to be 4 and 5 metres respectively (measured to the nearest metre). The angle between the two sides is measured to be 110° (correct to 2 significant figures).

(a) Use the above measurements to calculate the area of the triangle.

(b) Express your answer to part (a) in cm^2 in standard form.

(c) Find the lower and upper bounds for the actual area of the triangle and hence find the largest possible percentage error of your answer to part (a).

Worked Example (a)

Easy!

We have the lengths of two sides and the angle between these two sides, so we can simply apply the appropriate formula:

$$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}\cdot 4\cdot 5\cdot \sin 110^\circ = 9.396926...\ m^2 \approx 9.40m^2$$

Worked Example (a)

Easy!

We have the lengths of two sides and the angle between these two sides, so we can simply apply the appropriate formula:

$$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}\cdot 4\cdot 5\cdot \sin 110^\circ = 9.396926...\ m^2 \approx 9.40m^2$$

Worked Example (b)

Again, very easy!

We have:

$1\ m = 100\ cm$, so $1\ m^2 = (100\ cm)^2 = 10000\ cm^2$

This gives:

$$A \approx 9.40\ m^2 = 9.40 \cdot 10000\ cm^2 = 9.40 \cdot 10^4\ cm^2$$

Worked Example (b)

Again, very easy!

We have:
$1\ m = 100\ cm$, so $1\ m^2 = (100\ cm)^2 = 10000\ cm^2$

This gives:

$$A \approx 9.40\ m^2 = 9.40 \cdot 10000\ cm^2 = 9.40 \cdot 10^4\ cm^2$$

Worked Example (c)

Here we need to be careful...

Worked Example (c)

Here we need to be careful...

Worked Example (c)

We have $3.5\ m \leq a < 4.5\ m$ and $4.5\ m \leq b < 5.5\ m$. We also have $105^\circ \leq \gamma < 115^\circ$.

You may now think that we should have:

$$\frac{1}{2} \cdot 3.5 \cdot 4.5 \cdot \sin(105^\circ) \leq A < \frac{1}{2} \cdot 4.5 \cdot 5.5 \cdot \sin(115^\circ)$$

But this is incorrect! Try to figure out why on your own before proceeding.

Worked Example (c)

We have $3.5\ m \leq a < 4.5\ m$ and $4.5\ m \leq b < 5.5\ m$. We also have $105^\circ \leq \gamma < 115^\circ$.

You may now think that we should have:

$$\frac{1}{2} \cdot 3.5 \cdot 4.5 \cdot \sin(105^\circ) \leq A < \frac{1}{2} \cdot 4.5 \cdot 5.5 \cdot \sin(115^\circ)$$

But this is incorrect! Try to figure out why on your own before proceeding.

Worked Example (c)

We have $3.5\ m \leq a < 4.5\ m$ and $4.5\ m \leq b < 5.5\ m$. We also have $105^\circ \leq \gamma < 115^\circ$.

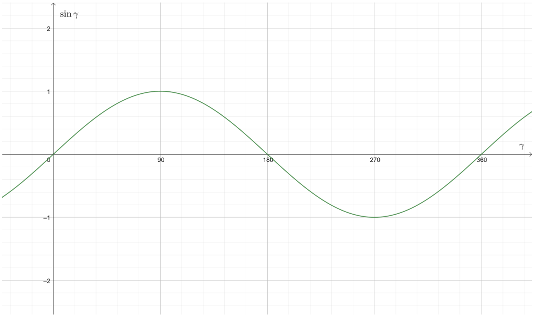
You may now think that we should have:

$$\frac{1}{2} \cdot 3.5 \cdot 4.5 \cdot \sin(105^\circ) \leq A < \frac{1}{2} \cdot 4.5 \cdot 5.5 \cdot \sin(115^\circ)$$

But this is incorrect! Try to figure out why on your own before proceeding.

Worked Example (c)

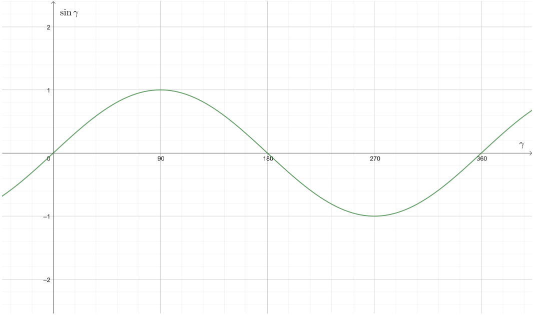
Our angle is in the second quadrant. Let's recall the graph of *sine* function:



$\sin\gamma$ is decreasing in the second quadrant. This means that the greater the value of γ, the smaller the value of the $\sin\gamma$. In other words we will have a smaller area, if the angle is greater. Note that this is only true because we are in the second quadrant!

Worked Example (c)

Our angle is in the second quadrant. Let's recall the graph of *sine* function:



$\sin\gamma$ is decreasing in the second quadrant. This means that the greater the value of γ, the smaller the value of the $\sin\gamma$. In other words we will have a smaller area, if the angle is greater. Note that this is only true because we are in the second quadrant!

Worked Example (c)

The correct bounds are then:

$$\frac{1}{2}\cdot 3.5\cdot 4.5\cdot \sin(115^\circ)\leq A<\frac{1}{2}\cdot 4.5\cdot 5.5\cdot \sin(105^\circ)$$

$$7.13717...m^2\leq A<11.9533...m^2$$

Worked Example (c)

The correct bounds are then:

$$\frac{1}{2} \cdot 3.5 \cdot 4.5 \cdot \sin(115^\circ) \leq A < \frac{1}{2} \cdot 4.5 \cdot 5.5 \cdot \sin(105^\circ)$$

$$7.13717...m^2 \leq A < 11.9533...m^2$$

Worked Example (c)

We now have our approximated area $A_{measured} = 9.396926...\ m^2$.

We also know that the actual (exact) area is in the range:

$$7.13717...m^2 \le A_{exact} < 11.9533...m^2$$

Let us now consider two extreme cases.

Extreme case 1. Overestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 7.13717...|}{7.13717...} \cdot 100\% \approx 31.7\%$$

Extreme case 2. Underestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 11.9533...|}{11.9533...} \cdot 100\% \approx 21.4\%$$

Worked Example (c)

We now have our approximated area $A_{measured} = 9.396926...\ m^2$.

We also know that the actual (exact) area is in the range:

$$7.13717...m^2 \leq A_{exact} < 11.9533...m^2$$

Let us now consider two extreme cases.

Extreme case 1. Overestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 7.13717...|}{7.13717...} \cdot 100\% \approx 31.7\%$$

Extreme case 2. Underestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 11.9533...|}{11.9533...} \cdot 100\% \approx 21.4\%$$

Worked Example (c)

We now have our approximated area $A_{measured} = 9.396926...\ m^2$.

We also know that the actual (exact) area is in the range:

$$7.13717...m^2 \leq A_{exact} < 11.9533...m^2$$

Let us now consider two extreme cases.

Extreme case 1. Overestimating:

$$\epsilon\% = \frac{|9.396926... - 7.13717...|}{7.13717...} \cdot 100\% \approx 31.7\%$$

Extreme case 2. Underestimating:

$$\epsilon\% = \frac{|9.396926... - 11.9533...|}{11.9533...} \cdot 100\% \approx 21.4\%$$

Worked Example (c)

We now have our approximated area $A_{measured} = 9.396926...\ m^2$.

We also know that the actual (exact) area is in the range:

$$7.13717...m^2 \leq A_{exact} < 11.9533...m^2$$

Let us now consider two extreme cases.

Extreme case 1. Overestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 7.13717...|}{7.13717...} \cdot 100\% \approx 31.7\%$$

Extreme case 2. Underestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 11.9533...|}{11.9533...} \cdot 100\% \approx 21.4\%$$

Worked Example (c)

We now have our approximated area $A_{measured} = 9.396926...\ m^2$.

We also know that the actual (exact) area is in the range:

$$7.13717...m^2 \leq A_{exact} < 11.9533...m^2$$

Let us now consider two extreme cases.

Extreme case 1. Overestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 7.13717...|}{7.13717...} \cdot 100\% \approx 31.7\%$$

Extreme case 2. Underestimating:

$$\epsilon_{\%} = \frac{|9.396926... - 11.9533...|}{11.9533...} \cdot 100\% \approx 21.4\%$$

Worked Example (c)

The largest possible percentage error is $\epsilon_{\%} \approx 31.7\%$.

In case of any questions you can email me at t.j.lechowski@gmail.com or message me via Librus or MS Teams.