# Points in 3d space

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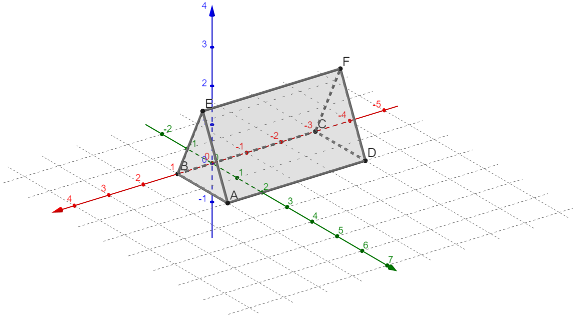
# Worked Example

The diagram below shows a tent in a shape of triangular prism. The XY plane is the level ground. The distances are measured in metres. The base of the tent is formed by rectangle $ABCD$ with $A(1, 2, 0)$, $B(1, 0, 0)$ and $C(-3, 0, 0)$. Points $E$ and $F$ are 2 metres above the midpoints of $AB$ and $CD$ respectively.



# Worked Example

(a) State the coordinates of $D$, $E$ and $F$.

(b) Find the volume and the surface area (including the base) of the tent.

The tent is supported by metal beams $FA$ and $FB$.

(c) Find the angle that the beams make with the base of the tent.

(d) Find the angle between the beams.

# Worked Example (a)

Point $D$ has the same $x$-coordinate as point $C$ and the same $y$-coordinate as point $A$. It also have the $z$-coordinate equal to 0. So $D = (-3, 2, 0)$.

The midpoints of line segments $AB$ and $CD$ are:

$$M_{AB} = (1, 1, 0) \qquad \text{and} \qquad M_{CD} = (-3, 1, 0)$$

Points $E$ and $F$ are two metres above these points, so $E = (1, 1, 2)$ and $F = (-3, 1, 2)$.

# Worked Example (b)

The volume is the area of the cross-section times length.

The cross-section is a triangle with base $|AB| = 2$ and height $|M_{AB}E| = 2$, so its area is $A_{\triangle} = 2\ m^2$.

The length of the tent is 4 metres, so the volume is:

$$V_{tent} = 8\ m^3$$

# Worked Example (b)

The surface area consists of two (congruent) triangles and 3 rectangles. We already have the area of the triangle. The area of the base rectangle is $A_{ABCD} = 2 \cdot 4 = 8\ m^2$.

To calculate the area of rectangle $ADFE$ we need the length of $|AE|$. We can use the distance formula (or simply apply Pythagorean theorem).

$$|AE| = \sqrt{0^2 + 1^2 + 2^2} = \sqrt{5}\ m^2$$

So the total surface area of the tent is:

$$SA_{tent} = 2 \cdot 2\ m^2 + 8\ m^2 + 2 \cdot \sqrt{5}\ m^2 \approx$$

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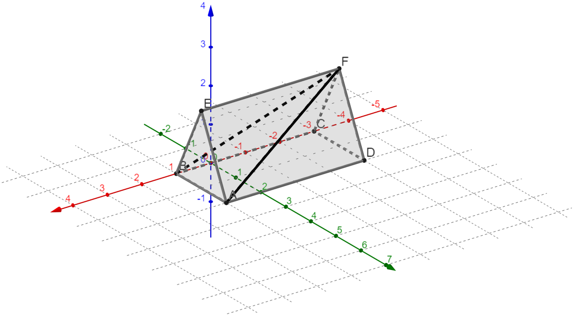
# Worked Example (c)

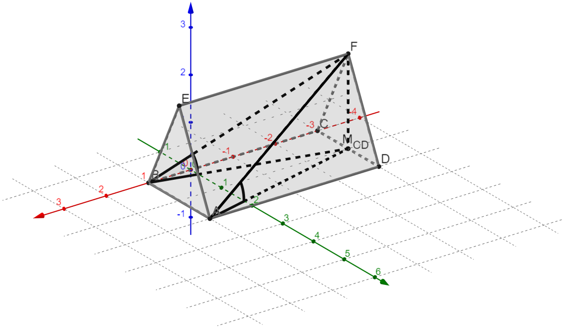
Let's add some line segments to our diagram:




Both beams make the same angle with the base plane, so let's focus on the beam $AF$.

# Worked Example (c)

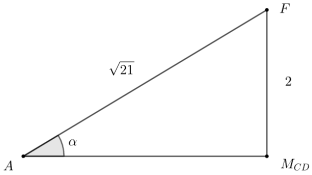
The perpendicular projection of $AF$ onto the base plane is $AM_{CD}$. So the triangle we want to consider is the $\triangle AM_{CD}F$ and the angle in question is the angle $\angle M_{CD}AF$:



# Worked Example (c)

We know that $|FM_{CD}| = 2$, we need to calculate one more side of the triangle - for instance $AF$:

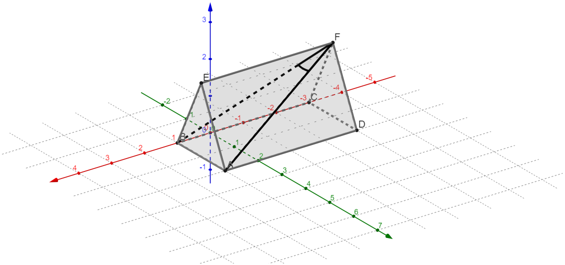
$$|AF| = \sqrt{4^2 + 1^2 + 2^2} = \sqrt{21}$$




We can now calculate the angle $\tan\alpha = \frac{2}{\sqrt{21}}$, so $\alpha = \tan^{-1}\frac{2}{\sqrt{21}} \approx 23.6^\circ$. So the angle the beam makes with the base plane is $23.6^\circ$.

# Worked Example (d)

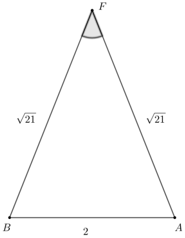
Let us look at the diagram again:



# Worked Example (d)

Now our triangle $ABF$ is not a right triangle, but we already have all its sides:

$$|AB| = 2, \qquad |AF| = |BF| = \sqrt{21}$$




We can now calculate the angle in question using two methods.

# Worked Example (d)

Because our triangle is isosceles, we can divide it into two right triangles and calculate half of our angle using *SOHCAHTOA* and then multiply the result by 2.

We can also calculate the angle directly using the cosine rule. And this is what we will do:

$$\cos\beta = \frac{21+21-4}{2\cdot\sqrt{21}\sqrt{21}} = \frac{19}{21}$$

So we have $\beta = \cos^{-1}\frac{19}{21} \approx 25.2°$ and this is the angle between the support beams.

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In case of any questions you can email me at t.j.lechowski@gmail.com or message me via Librus or MS Teams.