Review of year 1 [94 marks]

Ms Calhoun measures the heights of students in her mathematics class. She is interested to see if the mean height of male students,  $\mu_1$ , is the same as the mean height of female students,  $\mu_2$ . The information is recorded in the table.

Male height (cm)	150	148	143	152	151	149	147	
Female height (cm)	148	152	154	147	146	153	152	150

At the 10 % level of significance, a *t*-test was used to compare the means of the two groups. The data is assumed to be normally distributed and the standard deviations are equal between the two groups.

(a.i) State the null hypothesis.

Markscheme  $\mu_1-\mu_2=0$  A1 Note: Accept equivalent statements in words. [1 mark]

(a.ii) State the alternative hypothesis.

[1]

Markscheme

 $\mu_1-\mu_2
eq 0$  A1

**Note:** Accept equivalent statements in words.

[1 mark]

(b) Calculate the *p*-value for this test.

[2]

Markscheme

[1]

0.296 (0.295739...) **A2** 

[2 marks]

(c) State, giving a reason, whether Ms Calhoun should accept the null hypothesis.

[2]

0.296 > 0.1 *R1* 

fail to reject the null hypothesis, there is no difference between the mean height of male and female students **A1** 

**Note:** Award **(R1)** for a correct comparison of their *p*-value to the test level, award **(A1)** for the correct interpretation from that comparison. Do not award **R0A1**.

[2 marks]

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week					
	0 1-2 3-4 >4					
Students	45	26	18	6		
Professors	15	8	5	12		
Staff and Administration	16	13	10	6		

Jing conducted a  $\chi^2$  test for independence at a 5 % level of significance.

(a) State the null hypothesis.

[1]

Markscheme
number of salad meals per week is independent of a person's position in the university <b>A1</b>
Note: Accept "not associated" instead of independent.
[1 mark]

(b) Calculate the *p*-value for this test.

Markscheme	
0.0201 (0.0201118) <b>A2</b>	
[2 marks]	

(c) State, giving a reason, whether the null hypothesis should be accepted.

Markscheme					
0.0201 < 0.05 <b>R1</b>					
the null hypothesis is rejected A1					
<b>Note:</b> Award <b>(R1)</b> for a correct comparison of their <i>p</i> -value to the test level, award <b>(A1)</b> for the correct interpretation from that comparison.					
Do not award <b>(R0)(A1)</b> .					
[2 marks]					

#### SPM.1.SL.TZ0.3

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

## (a) State whether the data is discrete or continuous.

[1]

Markschei	me			
discrete	A1			
[1 mark]				

The mean number of classes in which a student used the internet is 2.

(b) Find the value of k.

[4]

Markscheme
$rac{24+60+3k+40+15+6}{88+k}=2$ M1A1
<b>Note:</b> Award <i>M1</i> for substitution into the formula for the mean, award <b>A1</b> for a correct equation.
attempt to solve their equation (M1)
k=31 <b>A1</b>
[4 marks]

 (c) It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

Identify the sampling technique used in the survey.

Г	1	1
L		٦

Markscheme	
systematic A1	
[1 mark]	

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question.

[1]

Markscheme	
$rac{6}{15} \left( 0.4, \ rac{2}{5}  ight)$ A1	
[1 mark]	

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times.

[2]

Ν	<i>Narkscheme</i>
Ρ	P(X = 8) (M1)
	<b>Note:</b> Award <i>(M1)</i> for evidence of recognizing binomial probability. eg P( $\chi = 8$ ), $\chi \sim B\left(20, \frac{6}{15}\right)$ .
0	0.180 (0.179705) <b>A1</b>
[2	2 marks]

(c) Find the probability he will choose a male student at most 9 times.

[3]

Markscheme

P(male) =  $\frac{9}{15}$ (0.6) *A*1 P(X ≤ 9) = 0.128 (0.127521...) *(M1)A*1 **Note:** Award *(M1)* for evidence of correct approach *eg*, P(X ≤ 9). *[3 marks]* 

The faces of the die are labelled -3, -1, 0, 1, 2 and 5.

The score for the game, *X*, is the number which lands face up after the die is rolled.

The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
P(X=x)	$\frac{1}{18}$	р	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of *p*.

Markscheme  $\frac{4}{18}\left(\frac{2}{9}\right)$  A1 [1 mark]

Jae Hee plays the game once.

(b) Calculate the expected score.

Markscheme  $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$  (M1) Note: Award (M1) for their correct substitution into the formula for expected value.  $= 1.83 \left( \frac{33}{18} + 83333 \right)$  (M1)

$$= 1.83 \left( rac{33}{18}, 1.83333 \ldots 
ight)$$
 A1

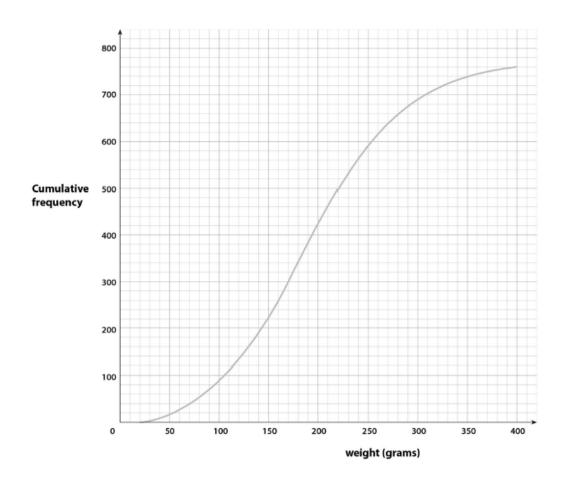
[1]

(c) Jae Hee plays the game twice and adds the two scores together.

Find the probability Jae Hee has a **total** score of -3.

Markscheme
$2 imes rac{1}{18} imes rac{3}{18}$ (M1)(M1)
<b>Note:</b> Award <i>(M1)</i> for $\frac{1}{18} \times \frac{3}{18}$ , award <i>(M1)</i> for multiplying their product by 2.
$=rac{1}{54} \Big( rac{6}{324}, 0.0185185 \ldots, 1.85\%  \Big)$ A1
[3 marks]

A food scientist measures the weights of 760 potatoes taken from a single field and the distribution of the weights is shown by the cumulative frequency curve below.



(a) Find the number of potatoes in the sample with a weight of more than 200 grams.



\* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

760 - 420 = 340 (g) (M1)A1

## [2 marks]

(b.i) Find the median weight.

Markscheme Median = 190 (g) A1 [1 mark]

(b.ii) Find the lower quartile.

MarkschemeLower quartile = 135 - 140 (g) A1[1 mark]

(b.iii) Find the upper quartile.

Markscheme Upper quartile = 242 - 247 (g) A1 [1 mark]

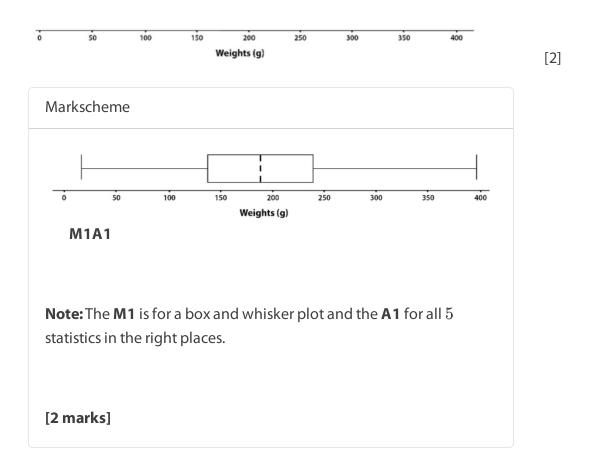
(c) The weight of the smallest potato in the sample is 20 grams and the weight of the largest is 400 grams.

[1]

[1]

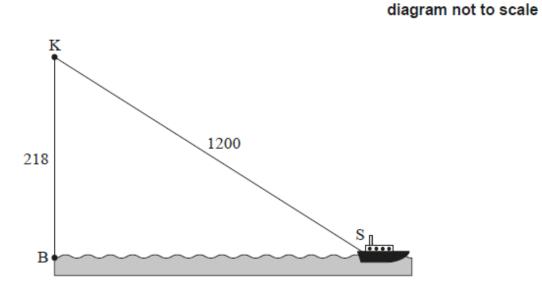
[1]

Use the scale shown below to draw a box and whisker diagram showing the distribution of the weights of the potatoes. You may assume there are no outliers.



Kacheena stands at point K, the top of a 218 m vertical cliff. The base of the cliff is located at point B. A ship is located at point S, 1200 m from Kacheena.

This information is shown in the following diagram.



(a) Find the angle of elevation from the ship to Kacheena.

Markscheme  $\sin\left(B\widehat{S}K\right) = \frac{218}{1200} \text{ OR } \frac{\sin\left(B\widehat{S}K\right)}{218} = \frac{\sin\left(90^{\circ}\right)}{1200} \quad \text{(M1)}$ Note: Award M1 for a correct trig formula. Accept other variables

representing  $\widehat{BSK}$ .

$$\left( {
m B\widehat{S}K} = 
ight) 10.5 \,^\circ \, \left( 10.4668 \ldots 
ight)$$
 At

Note: Award A1 for the radian answer, 0. 182681 . . . . Award M1A0 if

the candidate finds the correct angle of elevation but then uses it to find a complementary angle as their final answer.

#### [2 marks]

## (b) Find the horizontal distance from the base of the cliff to the ship.

Markscheme  

$$SB^{2} + 218^{2} = 1200^{2} \text{ OR } \cos(10.4468...) = \frac{SB}{1200} \text{ OR} \\ \tan(10.4468...) = \frac{218}{SB} \text{ OR } \frac{BS}{\sin(79.5331...^{\circ})} = \frac{1200}{\sin(90^{\circ})} \quad (M1)$$

$$1800 \text{ (m) } \left(\sqrt{1392476}, 1180.03...\right) \qquad A1$$
[2 marks]

(c) Write down your answer to part (b) in the form  $a imes 10^k$  where  $1\leq a<10$  and  $k\in\mathbb{Z}.$ 

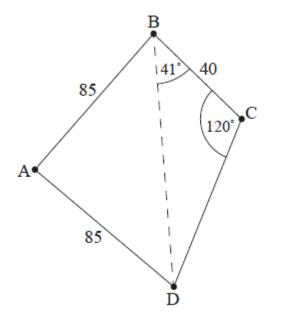
Markscheme	
$1.18 imes 10^3$	A1A1
	1.18 ed answer to part (b). wers of the type: $11.8 imes 10^2$

[2 marks]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

 $AB = 85 \text{ m}, \text{ AD} = 85 \text{ m}, \text{ BC} = 40 \text{ m}, \text{ C}\widehat{B}D = 41^{\circ}, \text{ B}\widehat{C}D = 120^{\circ}$ 

## diagram not to scale



(a.i) Write down the value of angle BDC.

Markscheme

[1]

Markscheme	
19°	A1
[1 mark]	

(a.ii) Hence use triangle BDC to find the length of path BD.

$$\frac{BD}{\sin 120^{\circ}} = \frac{40}{\sin 19^{\circ}} \qquad (M1)(A1)$$
Note: Award M1 for substituted sine rule for BCD, A1 for their correct substitution.
$$(BD =) 106 \text{ m} (106.401...) \qquad A1$$
[3 marks]

(b) Calculate the size of angle  $B\widehat{A}D$ , correct to five significant figures.

Markscheme	
METHOD 1 (cosine rule)	
$\cos BAD = \frac{85^2 + 85^2 - 106.401^2}{2 \times 85 \times 85}$ (M1)(A1)	
<b>Note:</b> Award <i>M1</i> for substituted cosine rule, <i>A1</i> for their correct substitution.	
77.495 <b>A1</b>	
<b>Note:</b> Accept an answer of $77.149$ from use of $3$ sf answer from (a). The final answer must be correct to five significant figures.	ı part
METHOD 2 (right angled trig/isosceles triangles)	
$\sin\left(\frac{\text{BAD}}{2}\right) = \frac{53.2008}{85} \tag{A1)(M1)}$	

Note: Award A1 for 53. 2008 . . . seen. Award M1 for correctlysubstituted trig ratio. Follow through from part (a).77. 495 . . .A1

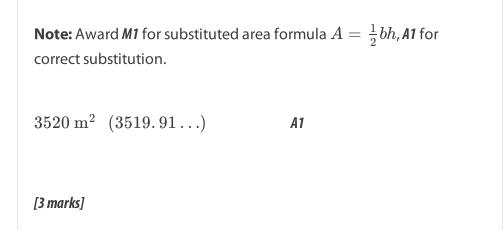
**Note:** Use of 3 sf answer from part (a), results in 77. 149.

[3 marks]

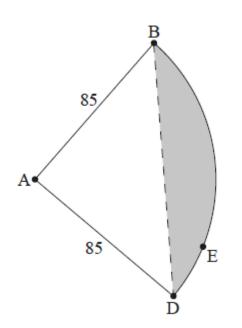
The size of angle  $B\widehat{A}D$  rounds to  $77\degree$ , correct to the nearest degree. Use  $B\widehat{A}D=77\degree$  for the rest of this question.

(c)	Find the area bounded by the path ${ m BD}$ , and fences ${ m AB}$ and ${ m AD}.$	[3]
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Markscheme	
EITHER	
$({ m Area}=) \ rac{1}{2}  imes 85  imes 85  imes \sin(77\degree)$	(M1)(A1)
<b>Note:</b> Award <i>M1</i> for substituted area formula, <i>A1</i> fo substitution. Award at most <i>(M1)(A1)A0</i> if an angle o used.	
OR	
$( ext{Area}=) rac{1}{2}  imes (2  imes 85  imes \sin(38.5^\circ))  imes (85  imes$ (M1)(A1)	$\cos(38.5^\circ))$



A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.



# diagram not to scale

(d) Write down the distance from A to E.

[1]

Markscheme			
$85~{ m m}$	A1		

(e) Find the perimeter of the proposed park, ABED.

Markscheme $85 + 85 + \frac{77}{360} \times 2\pi \times 85$ (M1)(M1)Note: Award M1 for correctly substituted into  $\frac{\theta}{360} \times 2\pi \times r$ , M1 for<br/>addition of AB and AD.284 m (284.231...)A1[3 marks]

(f) Find the area of the shaded region in the proposed park.

[3]

Markscheme $\frac{77}{360} \times \pi \times (85)^2 - 3519.91...$ (M1)(M1)Note: Award M1 for correctly substituted area of sector formula, M1 for subtraction of their area from part (c). $1330 \text{ m}^2$  (1334.93...)A1[3 marks]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

## Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a.i) Find the repayment made each quarter.

Markscheme N = 24 1% = 14 PV = -14000 FV = 0 P/Y = 4 C/Y = 4 (M1)(A1) Note: Award M1 for an attempt to use a financial app in their technology, award A1 for all entries correct. Accept PV = 14000. ( $\in$ )871.82 A1 [3 marks]

(a.ii) Find the total amount paid for the car.

[2]

Markscheme	
4 × 6 × 871.82	(M1)
(€) 20923.68	A1

[2 marks]

(a.iii) Find the interest paid on the loan.

Markscheme			
20923.68 – 14000	(M1)		
(€) 6923.68 <b>A1</b>			
[2 marks]			

#### Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of  $\in$  250.

(b.i) Find the amount to be borrowed for this option.

 $0.9 \times 14000 (= 14000 - 0.10 \times 14000)$  M1

(€) 12600.00 **A1** 

Markscheme

[2 marks]

(b.ii) Find the annual interest rate, r.

Markscheme N = 72 PV = 12600PMT = -250 [2]

[2]

FV = 0 P/Y = 12  $C/Y = 12 \quad (M1)(A1)$ Note: Award M1 for an attempt to use a financial app in their technology, award A1 for all entries correct. Accept PV = -12600 provided PMT = 250.  $12.56(\%) \qquad A1$ [3 marks]

(c) State which option Bryan should choose. Justify your answer.

Markscheme
EITHER
Bryan should choose Option A <b>A1</b>
no deposit is required <b>R1</b>
<b>Note:</b> Award <b><i>R1</i></b> for stating that no deposit is required. Award <b><i>A1</i></b> for the correct choice from that fact. Do not award <b><i>R0A1</i></b> .
OR
Bryan should choose Option B <b>A1</b>
cost of Option A (6923.69) > cost of Option B (72 × 250 – 12600 = 5400) <b>R1</b>
<b>Note:</b> Award <b><i>R1</i></b> for a correct comparison of costs. Award <b><i>A1</i> for the correct choice from that comparison. Do not award <b><i>R0A1</i></b>.</b>
[2 marks]

(d) Bryan's car depreciates at an annual rate of 25 % per year.

Find the value of Bryan's car six years after it is purchased.

[3]

Markscheme  $14\,000 \Big(1-rac{25}{100}\Big)^6$  (M1)(A1) **Note:** Award *M1* for substitution into compound interest formula. Award **A1** for correct substitutions. = (€)2491.70 **A1** OR N = 6 1% = -25 $PV = \pm 14\,000$ P/Y = 1C/Y = 1 (A1)(M1) **Note:** Award *A1* for  $PV = \pm 14000$ , *M1* for other entries correct. (€)2491.70 **A1** [3 marks]

*Give your answers to this question correct to two decimal places.* 

Gen invests \$2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

(a) Calculate the value of her savings after 10 years.

[2]

Markscheme	
$2400{(1.04)}^{10}=\$3552.59$	M1A1
[2 marks]	

(b) The rate of inflation during this 10 year period is 1.5% per year.

Calculate the real value of her savings after 10 years.

Markscheme
real interest rate = $4-1.5=2.5\%$ A1
$2400(1.025)^{10} = \$3072.20$ M1A1
[3 marks]

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of \$4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

(a) Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal.

[3]

Markscheme	
Use of finance solver M1	
<i>N</i> = 360, <i>I</i> = 5%, Pmt = 4000, FV = 0, PpY = 12, CpY = 1	A1
\$755000 (correct to 3 s.f.) <b>A1</b>	
[3 marks]	

(b) Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund.

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

[3]

Markscheme N = 384, l = 5%, PV = 0, FV = 754638, PpY = 12, CpY = 1 M1A1 \$817 per month (correct to 3 s.f.) A1 [3 marks]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

(a) Find the number of trees to be planted in the  $15 \mathrm{th}$  month.

[3]

Markscheme	
use of the $n^{ m th}$ term of an arithmetic sequence formula	(M1)
$u_{15}=85+(15-1){ imes}30$ (A1)	
505 <b>A1</b>	
[3 marks]	

(b) Find the total number of trees to be planted in the first 15 months.

[2]

Markscheme

use of the sum of *n* terms of an arithmetic sequence formula *(M1)* 

$$S_{15} = rac{15}{2}(85+505)$$
 OR  $rac{15}{2}(2 imes 85+(15-1) imes 30)$ 

4430 (4425) A1 [2 marks]

(c) Find the mean number of trees planted per month during the first  $15\,$  months.

[2]

Markscheme
$rac{4425}{15}$ OR $85+(8-1){ imes}30$ (M1)
295 A1
<b>Note:</b> Accept $295.333$ from use of 3sf value from part (b).
[2 marks]

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