

## Test 1 - revision questions [61 marks]

1. [Maximum mark: 18]

SPM.2.AHL.TZ0.3

**In this question, give all answers to two decimal places.**

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

**Finance option A:**

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a.i) Find the repayment made each quarter.

[3]

Markscheme

$$N = 24$$

$$I \% = 14$$

$$PV = -14000$$

$$FV = 0$$

$$P/Y = 4$$

$$C/Y = 4 \quad (M1)(A1)$$

**Note:** Award *M1* for an attempt to use a financial app in their technology, award *A1* for all entries correct. Accept  $PV = 14000$ .

$$(\text{€})871.82 \quad A1$$

**[3 marks]**

(a.ii) Find the total amount paid for the car.

[2]

Markscheme

$$4 \times 6 \times 871.82 \quad (M1)$$

$$(\text{€}) 20923.68 \quad A1$$

*[2 marks]*

(a.iii) Find the interest paid on the loan.

[2]

Markscheme

$$20923.68 - 14000 \quad (M1)$$

$$(\text{€}) 6923.68 \quad A1$$

*[2 marks]*

### Finance option B:

A 6 year loan at a nominal annual interest rate of  $r\%$  **compounded monthly**.  
Terms of the loan require a 10% deposit and monthly repayments of €250.

(b.i) Find the amount to be borrowed for this option.

[2]

Markscheme

$$0.9 \times 14000 (= 14000 - 0.10 \times 14000) \quad M1$$

$$(\text{€}) 12600.00 \quad A1$$

*[2 marks]*

(b.ii) Find the annual interest rate,  $r$ .

[3]

Markscheme

$$N = 72$$

$$PV = 12600$$

$$PMT = -250$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12 \quad (M1)(A1)$$

**Note:** Award *M1* for an attempt to use a financial app in their technology, award *A1* for all entries correct. Accept  $PV = -12600$  provided  $PMT = 250$ .

$$12.56(\%) \quad A1$$

*[3 marks]*

(c) State which option Bryan should choose. Justify your answer.

[2]

Markscheme

***EITHER***

Bryan should choose Option A *A1*

no deposit is required *R1*

**Note:** Award *R1* for stating that no deposit is required. Award *A1* for the correct choice from that fact. Do not award *ROA1*.

***OR***

Bryan should choose Option B *A1*

cost of Option A (6923.69) > cost of Option B ( $72 \times 250 - 12600 = 5400$ )

*R1*

**Note:** Award *R1* for a correct comparison of costs. Award *A1* for the correct choice from that comparison. Do not award *ROA1*.

*[2 marks]*

- (d) Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

If they invest it in an account paying 0.4 % interest per month and inflation is 0.1 % per month, calculate the real amount of money the car dealership has received by the end of the 6 year period.

[4]

#### Markscheme

real interest rate is  $0.4 - 0.1 = 0.3\%$  **(M1)**

value of other payments  $250 + 250 \times 1.003 + \dots + 250 \times 1.003^{71}$

use of sum of geometric sequence formula or financial app on a GDC  
**(M1)**

= 20 058.43

value of deposit at the end of 6 years

$1400 \times (1.003)^{72} = 1736.98$  **(A1)**

Total value is (€) 21 795.41 **A1**

**Note:** Both **M** marks can awarded for a correct use of the GDC's financial app:

$N = 72$  ( $6 \times 12$ )

$I \% = 3.6$  ( $0.3 \times 12$ )

$PV = 0$

$PMT = -250$

$FV =$

$P/Y = 12$

$C/Y = 12$

**OR**

$$N = 72 (6 \times 12)$$

$$I \% = 0.3$$

$$PV = 0$$

$$PMT = -250$$

$$FV =$$

$$P/Y = 1$$

$$C/Y = 1$$

***[4 marks]***

2. [Maximum mark: 15]

EXM.2.SL.TZ0.2

Sophie is planning to buy a house. She needs to take out a mortgage for \$120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay \$1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

(a.i) Calculate the monthly repayment using option 1.

[2]

Markscheme

evidence of using Finance solver on GDC *M1*

Monthly payment = \$785 (\$784.60) *A1*

*[2 marks]*

(a.ii) Calculate the total amount Sophie would pay, using option 1.

[2]

Markscheme

$240 \times 785 = \$188000$  *M1A1*

*[2 marks]*

(b.i) Calculate the number of months it will take to repay the mortgage using option 2.

[3]

Markscheme

$N = 180.7$  *M1A1*

It will take 181 months *A1*

**[3 marks]**

(b.ii) Calculate the total amount Sophie would pay, using option 2. [2]

Markscheme

$$181 \times 1000 = \$ 181000 \quad \mathbf{M1A1}$$

**[2 marks]**

Give a reason why Sophie might choose

(c.i) option 1. [1]

Markscheme

The monthly repayment is lower, she might not be able to afford \$1000 per month. **R1**

**[1 mark]**

(c.ii) option 2. [1]

Markscheme

the total amount to repay is lower. **R1**

**[1 mark]**

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

(d.i) Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years.



[2]

Markscheme

\$74400 (accept \$74300) **M1A1**

**[2 marks]**

(d.ii) Hence calculate her monthly repayment for the final 10 years.

[2]

Markscheme

Use of finance solver with  $N=120$ ,  $PV = \$74400$ ,  $I = 7\%$  **A1**

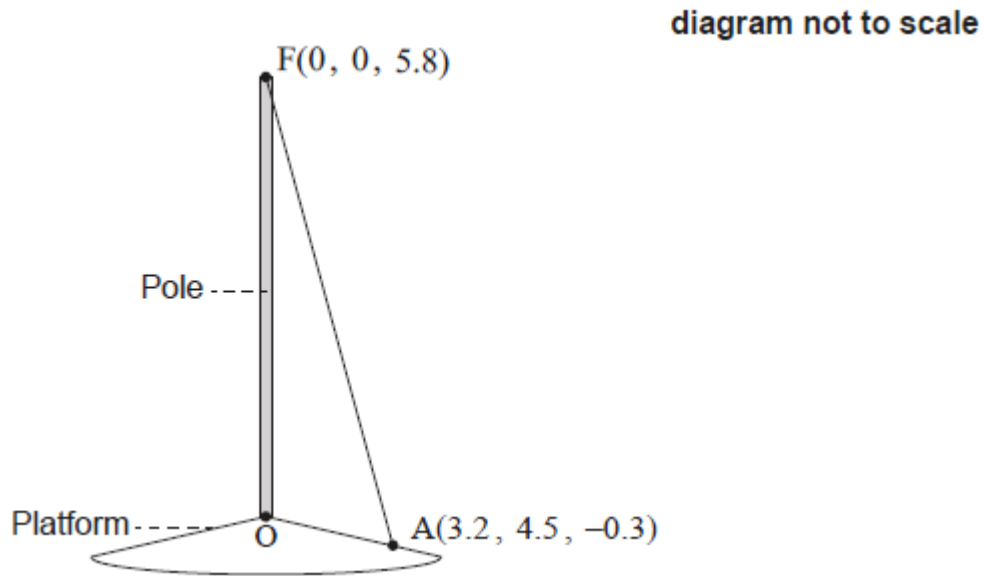
\$855 (accept \$854 – \$856) **A1**

**[2 marks]**

3. [Maximum mark: 8]

22M.1.AHL.TZ1.6

A vertical pole stands on a sloped platform. The bottom of the pole is used as the origin,  $O$ , of a coordinate system in which the top,  $F$ , of the pole has coordinates  $(0, 0, 5.8)$ . All units are in metres.



The pole is held in place by ropes attached at  $F$ .

One of these ropes is attached to the platform at point  $A(3.2, 4.5, -0.3)$ . The rope forms a straight line from  $A$  to  $F$ .

(a) Find  $\overrightarrow{AF}$ .

[1]

Markscheme

$$\begin{pmatrix} -3.2 \\ -4.5 \\ 6.1 \end{pmatrix} \quad A1$$

[1 mark]

(b) Find the length of the rope.

[2]

Markscheme

$$\sqrt{(-3.2)^2 + (-4.5)^2 + 6.1^2} \quad (M1)$$

$$8.22800 \dots \approx 8.23 \text{ m} \quad A1$$

[2 marks]

(c) Find  $\hat{F}\hat{A}\hat{O}$ , the angle the rope makes with the platform.

[5]

Markscheme

**EITHER**

$$\vec{AO} = \begin{pmatrix} -3.2 \\ -4.5 \\ 0.3 \end{pmatrix} \quad A1$$

$$\cos \theta = \frac{\vec{AO} \cdot \vec{AF}}{|\vec{AO}| |\vec{AF}|}$$

$$\vec{AO} \cdot \vec{AF} = (-3.2)^2 + (-4.5)^2 + (0.3 \times 6.1) (= 32.32) \quad (A1)$$

$$\cos \theta = \frac{32.32}{\sqrt{3.2^2 + 4.5^2 + 0.3^2} \times 8.22800 \dots} \quad (M1)$$

$$= 0.710326 \dots \quad (A1)$$

**Note:** If  $\vec{OA}$  is used in place of  $\vec{AO}$  then  $\cos \theta$  will be negative.

Award **A1(A1)(M1)(A1)** as above. In order to award the final **A1**, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

**OR**

$$AO = \sqrt{3.2^2 + 4.5^2 + 0.3^2} (= 5.52991\dots) \quad (A1)$$

$$\cos \theta = \frac{8.22800\dots^2 + 5.52991\dots^2 - 5.8^2}{2 \times 8.22800\dots \times 5.52991\dots} \quad (M1)(A1)$$

$$= 0.710326\dots \quad (A1)$$

**THEN**

$$\theta = 0.780833\dots \approx 0.781 \text{ OR } 44.7384\dots^\circ \approx 44.7^\circ \quad A1$$

**[5 marks]**

4. [Maximum mark: 4]

21M.1.SL.TZ1.1

Katya approximates  $\pi$ , correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6 + \frac{13}{16}}$$

- (a) Calculate Katya's approximation of  $\pi$ , correct to four decimal places.

[2]

Markscheme

$$\begin{aligned}\pi &\approx 3 + \frac{1}{6 + \frac{13}{16}} \\ &= 3.14678\dots \left( \frac{343}{109}, 3\frac{16}{109} \right) \quad (A1) \\ &= 3.1468 \quad A1\end{aligned}$$

**Note:** Award **A1** for correct rounding to 4 decimal places. Follow through within this part.

[2 marks]

- (b) Calculate the percentage error in using Katya's four decimal place approximation of  $\pi$ , compared to the exact value of  $\pi$  in your calculator.

[2]

Markscheme

$$\left| \frac{3.1468 - \pi}{\pi} \right| \times 100 \quad (M1)$$

**Note:** Award *M1* for substitution of their final answer in part (a) into the percentage error formula. Candidates should use the exact value of  $\pi$  from their GDC.

$$= 0.166 (\%) (0.165754\dots) \quad \mathbf{A1}$$

*[2 marks]*

5. [Maximum mark: 6]

21M.1.SL.TZ2.10

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

- (a) Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places.

[3]

Markscheme

**METHOD 1**

$$\begin{array}{ll} N = 5 & \text{OR} & N = 10 \\ I\% = 2.75 & & I\% = 2.75 \\ PV = -1500 & & PV = -1500 \\ PMT = 0 & & PMT = 0 \\ P/Y = 1 & & P/Y = 2 \\ C/Y = 2 & & C/Y = 2 \quad (M1)(A1) \end{array}$$

**Note:** Award *M1* for an attempt to use a financial app in their technology, *A1* for all entries correct.

**METHOD 2**

$$1500 \left( 1 + \frac{2.75}{2 \times 100} \right)^{2 \times 5} \quad (M1)(A1)$$

$$1719.49 \text{ euro} \quad A1$$

[3 marks]

- (b) Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5

years. Assume the account pays a nominal annual interest of  $r\%$  **compounded quarterly**.

Determine the value of  $r$ .

[3]

### Markscheme

#### METHOD 1

$$\begin{array}{ll} N = 5 & \text{OR} & N = 20 \\ PV = \pm 1500 & & PV = \pm 1500 \\ FV = \mp 2250 & & FV = \mp 2250 \\ PMT = 0 & & PMT = 0 \\ P/Y = 1 & & P/Y = 4 \\ C/Y = 4 & & C/Y = 4 \quad (M1)(A1) \end{array}$$

**Note:** Award *M1* for an attempt to use a financial app in their technology, *A1* for all entries correct. *PV* and *FV* must have opposite signs.

#### METHOD 2

$$1500 \left(1 + \frac{r}{4 \times 100}\right)^{4 \times 5} = 2250 \quad \text{OR} \quad \left(1 + \frac{r}{4 \times 100}\right)^{4 \times 5} = 1.5 \quad (M1)$$

(A1)

**Note:** Award *M1* for substitution in compound interest formula, *A1* for correct substitution and for equating to 2250 (if using LHS equation) or to 1.5 (if using RHS equation).

$$r = 8.19 \quad (8.19206 \dots) \quad A1$$

**Note:** Accept  $r = 8.19\%$ .



Accept a trial and error method which leads to  $r = 8.19$ .

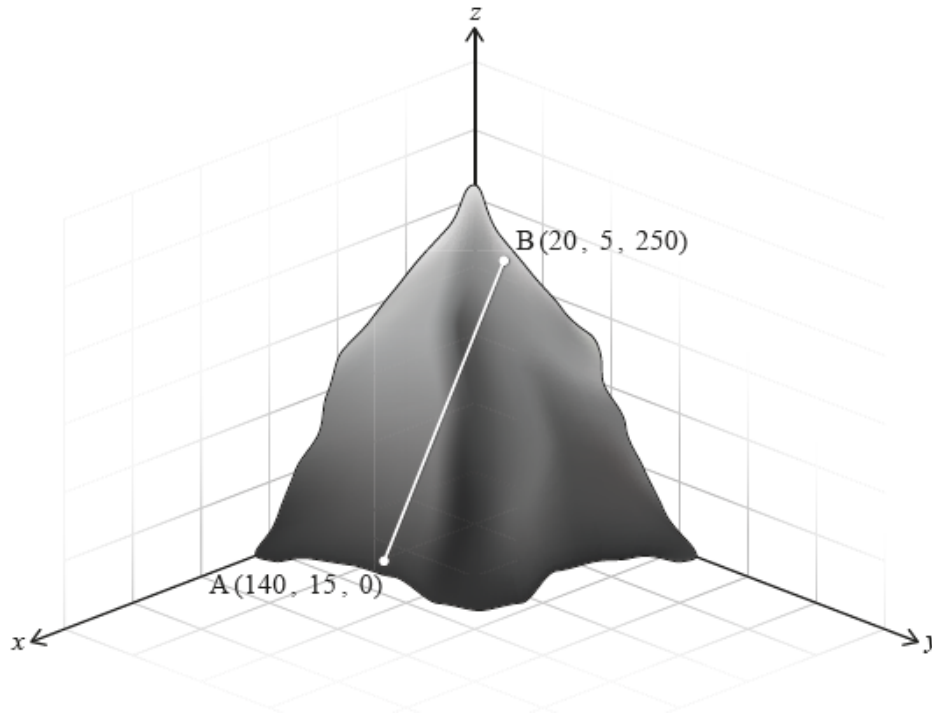
*[3 marks]*

6. [Maximum mark: 5]

21M.1.SL.TZ2.2

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.

diagram not to scale



The locations of the stations on the railway can be described by coordinates in reference to  $x$ ,  $y$ , and  $z$ -axes, where the  $x$  and  $y$  axes are in the horizontal plane and the  $z$ -axis is vertical.

The ground level station A has coordinates  $(140, 15, 0)$  and station B, located near the top of the hill, has coordinates  $(20, 5, 250)$ . All coordinates are given in metres.

(a) Find the distance between stations A and B.

[2]

Markscheme

attempt at substitution into 3D distance formula (M1)

$$AB = \sqrt{(140 - 20)^2 + (15 - 5)^2 + 250^2} \quad (= \sqrt{77\,000})$$

$$= 277 \text{ m } (10\sqrt{770}, 277.488\dots) \quad \mathbf{A1}$$

**[2 marks]**

Station M is to be built halfway between stations A and B.

(b) Find the coordinates of station M.

[2]

Markscheme

attempt at substitution in the midpoint formula **(M1)**

$$\left( \frac{140+20}{2}, \frac{15+5}{2}, \frac{0+250}{2} \right)$$

$$(80, 10, 125) \quad \mathbf{A1}$$

**[2 marks]**

(c) Write down the height of station M, in metres, above the ground.

[1]

Markscheme

125 m **A1**

**[1 mark]**

7. [Maximum mark: 5]

21M.1.AHL.TZ2.5

Roger buys a new laptop for himself at a cost of £495. At the same time, he buys his daughter Chloe a higher specification laptop at a cost of £2200.

It is anticipated that Roger's laptop will depreciate at a rate of 10% per year, whereas Chloe's laptop will depreciate at a rate of 15% per year.

(a) Estimate the value of Roger's laptop after 5 years.

[2]

Markscheme

$$£495 \times 0.9^5 = £292 \text{ (} £292.292 \dots \text{)} \quad (M1)A1$$

[2 marks]

Roger and Chloe's laptops will have the same value  $k$  years after they were purchased.

(b) Find the value of  $k$ .

[2]

Markscheme

$$£495 \times 0.9^k = 2200 \times 0.85^k \quad (M1)$$

$$k = 26.1 \text{ (} 26.0968 \dots \text{)} \quad A1$$

**Note:** Award *M1A0* for  $k - 1$  in place of  $k$ .

[2 marks]

(c) Comment on the validity of your answer to part (b).

[1]

## Markscheme

depreciation rates unlikely to be constant (especially over a long time period) **R1**

**Note:** Accept reasonable answers based on the magnitude of  $k$  or the fact that “value” depends on factors other than time.

**[1 mark]**