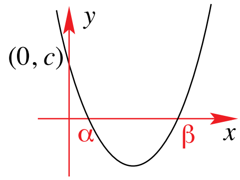
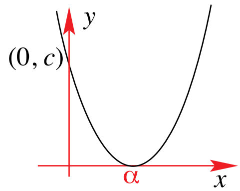
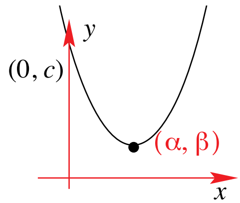
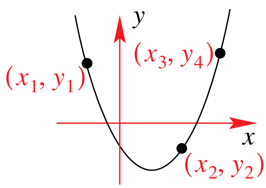
Finding the equation from a graph

If sufficient information is provided on a graph, then it is possible to obtain the equation that corresponds to that graph. When dealing with quadratics there are some standard approaches that can be used (depending on the information provided).

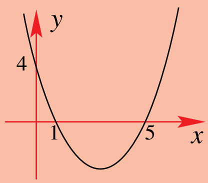
Information provided	Process
Graph cuts the x–axis at two points: 	Use the function $f(x) = k(x-\alpha)(x-\beta)$ and then use the point $(0, c)$ to solve for k.
Graph touches the x–axis at $x = \alpha$: 	Use the function $f(x) = k(x-\alpha)^2$ and then use the point $(0, c)$ to solve for k.
Graph does not meet the x–axis: 	Use the function $f(x) = k(x-\alpha)^2+\beta$ and then use the point $(0, c)$ to solve for k.
Three arbitrary points are given: 	Use the function $f(x) = ax^2+bx+c$ and then set up and solve the system of simultaneous equations by substituting each coordinate into the function: $ax_1^2+bx_1+c = y_1$ $ax_2^2+bx_2+c = y_2$ $ax_3^2+bx_3+c = y_3$

Note: the process is identical for a downward concave parabola.

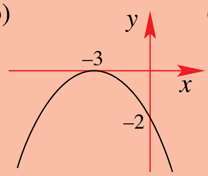
EXAMPLE 2.29

Find the equation defining each of the following graphs.

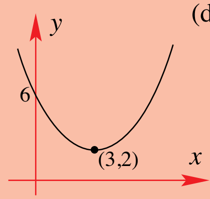
(a)



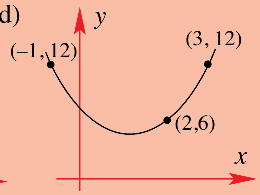
(b)



(c)



(d)



Solution

(a) Using the form $f(x) = k(x-\alpha)(x-\beta)$ we have $f(x) = k(x-1)(x-5)$.

Next, when $x = 0$, $y = 4$, therefore, $4 = k(0-1)(0-5) \Leftrightarrow 4 = 5k \Leftrightarrow k = \frac{4}{5}$.

Therefore, $f(x) = \frac{4}{5}(x-1)(x-5)$.

(b) Graph touches x–axis at $x = -3$, therefore use $f(x) = k(x+3)^2$.

As graph passes through $(0, -2)$, we have $-2 = k(0+3)^2 \Leftrightarrow -2 = 9k \Leftrightarrow k = -\frac{2}{9}$.

Therefore, $f(x) = -\frac{2}{9}(x+3)^2$.

(c) Graph shows turning point and another point, so use the form $f(x) = k(x-\alpha)^2 + \beta$.

So we have, $f(x) = k(x-3)^2 + 2$.

Then, as graph passes through $(0, 6)$, we have $6 = k(0-3)^2 + 2 \Leftrightarrow 4 = 9k \Leftrightarrow k = \frac{4}{9}$.

Therefore, $f(x) = \frac{4}{9}(x-3)^2 + 2$.

(d) As we are given three arbitrary points, we use the general equation $f(x) = ax^2 + bx + c$.

From $(-1, 12)$ we have $12 = a(-1)^2 + b(-1) + c$ i.e., $12 = a - b + c$ – (1)

From $(2, 6)$ we have $6 = a(2)^2 + b(2) + c$ i.e., $6 = 4a + 2b + c$ – (2)

From $(3, 12)$ we have $12 = a(3)^2 + b(3) + c$ i.e., $12 = 9a + 3b + c$ – (3)

Solving for a, b and c we have: (2) – (1): $-6 = 3a + 3b$

i.e., $-2 = a + b$ – (3)

(3) – (2): $6 = 5a + b$ – (4)

(4) – (3): $8 = 4a \Leftrightarrow a = 2$

Substitute $a = 2$ into (3): $-2 = 2 + b \Leftrightarrow b = -4$.

Substitute results into (1): $12 = 2 - (-4) + c \Leftrightarrow c = 6$.

Therefore function is $f(x) = 2x^2 - 4x + 6$

EXERCISES 2.4.2

1. Express the following functions in turning point form and hence sketch their graphs.

(a)	$y = x^2 - 2x + 1$	(b)	$y = x^2 + 4x + 2$	(c)	$y = x^2 - 4x + 2$
(d)	$y = x^2 + x - 1$	(e)	$y = x^2 - x - 2$	(f)	$y = x^2 + 3x + 1$
(g)	$y = -x^2 + 2x + 1$	(h)	$y = -x^2 - 2x + 2$	(i)	$y = 2x^2 - 2x - 1$
(j)	$y = -\frac{1}{2}x^2 + 3x - 2$	(k)	$y = -\frac{x^2}{3} + x - 2$	(l)	$y = 3x^2 - 2x + 1$