Name:
Result:

1. *(3 points)*

You are given the following quantities: $x = 300$, $\alpha = 10°$, $y = 2000$ and $z = 0.023$. Given that x and α are rounded to 1 significant figure and y and z are rounded to 2 significant figures, find the range of possible values of W, where:

$$W = \frac{y \cdot \tan\alpha}{z} - x$$

Let's fist establish the bounds for the given quantities:

$$250 \leq x < 350$$

$$9.5° \leq \alpha < 15°$$

$$1950 \leq y < 2050$$

$$0.0225 \leq z < 0.0235$$

Let's explain α. The least possible value for α is $9.5°$ as 9.5 rounded to 1 significant figure gives us 10. We are rounding to 1 significant figure, not the nearest 10 degrees, so for example 5 or 9.4 would be rounded to 5 (it already was rounded to 1 s.f.) and 9 respectively. The upper bound is 15 (and not 10.5), for example 14.9 would be rounded to 10.

Now W is greatest when y and α are the greatest and z and x are the least. Similarly W is the least when y and α are the least and z and x are the greatest. So we have:

$$\frac{1950 \cdot \tan(9.5°)}{0.0235} - 350 \leq W < \frac{2050 \cdot \tan(15°)}{0.0225} - 250$$

Which gives:

$$24163 \leq W < 13536$$

Here I rounded the bounds to the nearest integer. I accept other correct levels of accuracy.

2. *(7 points)*

Two particles start moving from the same point at the same time. The angle between their paths was measured to be 30° (correct to the nearest degree). The speeds of the particles were measured to be 5 $\frac{m}{s}$ and 7 $\frac{m}{s}$ (correct to 1 s.f.) respectively.

(a) Use the above measurements to calculate the distance between the two particles after exactly 4 s. Give your answer in cm correct to 3 significant figures and in the standard form. [3]

(b) Find the maximal percentage error of your answer to part (a). [4]

(a) We apply the *cosine rule*:

$$d_{approximated} = \sqrt{20^2 + 28^2 - 2 \cdot 20 \cdot 28 \cdot \cos(30°)} \approx 14.6\ m$$

Which gives $d_{approximated} \approx 1460\ cm = 1.46 \cdot 10^3\ cm$. And this rounded answer is our answer to part (a). In part (b) we want to use this answer.

(b) We need to find the range of possible values of the distance between the two particles. The distance will be greatest when the speed is the greatest and the angle is as closest to 180°. We have

$$\sqrt{18^2 + 26^2 - 2 \cdot 18 \cdot 26 \cdot \cos(29.5°)} \leq d_{exact} < \sqrt{22^2 + 30^2 - 2 \cdot 22 \cdot 30 \cdot \cos(30.5°)}$$

Which gives:

$$13.61422... \leq d_{exact} < 15.705079...$$

Now the percentage error is the absolute error divided by the exact value and expressed as a percentage:

$$\epsilon_{\%} = \frac{|d_{exact} - d_{approximated}|}{d_{exact}} \cdot 100\%$$

If we use the lower bound for d_{exact} (so our approximation is overapproximation) we get:

$$\epsilon_{\%} = \frac{|13.61422... - 14.6|}{13.61422...} \cdot 100\% \approx 7.24\%$$

Now if we use the upper bound for d_{exact} (so our approximation is underapproximation) we get:

$$\epsilon_{\%} = \frac{|15.705079... - 14.6|}{15.705079...} \cdot 100\% \approx 7.04\%$$

This means that the maximal percentage error of our answer to part (a) is approximately 7.24%.