Linear equations

In this presentation we will go through the process of solving linear equations.

Example 1

Solve:

$$\frac{2x+1}{3} - \frac{x-5}{2} = \frac{3x-1}{4}$$

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Remember: solving an equation for x means finding all values of x for which this equation is true. For example we can see that $x = 1$ is **not** a solution to the above equation, because if we substitute 1 for x we get 3 on the left hand side and $\frac{1}{2}$ on the right hand side.

Example 1

We will start by multiplying both sides of the equation by 12 in order to get rid of the denominators:

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This gives:

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Now we expand the brackets and add and subtract like terms to get:

$$8x + 4 - 6x + 30 = 9x - 3$$

$$2x + 34 = 9x - 3$$

Example 1

Finally we move terms containing x to one side and all the remaining terms to the other side:

$$37 = 7x$$

Divide by 7 to get $x = \dfrac{37}{7} = 5\dfrac{2}{7}$

Practice 1

Solve the following equations:
(a)

$$\frac{4x+2}{3} - \frac{2x+1}{5} = x$$

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Answer: $x = 7$

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(b)

$$\frac{5x-1}{2} - \frac{x+6}{8} = \frac{2x-1}{4}$$

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Answer: $x = \frac{8}{15}$

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Answer: $x = \frac{8}{15}$

(c)

$$\frac{x-1}{5} - \frac{x+1}{2} = \frac{2x-3}{3}$$

Answer: $x = \frac{9}{29}$

Practice 1 continued

Solve the following equations:
(d)
$$\frac{2x+3}{2} - \frac{3x+1}{5} = \frac{2x+7}{5}$$

Practice 1 continued

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Answer: contradiction, no solutions.

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$$\frac{2x+3}{2} - \frac{3x+1}{5} = \frac{2x+7}{5}$$

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(e)

$$\frac{3x+5}{3} - \frac{x+4}{2} = \frac{3x-2}{6}$$

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Answer: $x \in \mathbb{R}$, all real numbers satisfy this equation.

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(f)

$$\frac{5x-1}{2} - \frac{x+2}{4} = \frac{3x-5}{5}$$

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Answer: $x \in \mathbb{R}$, all real numbers satisfy this equation.

(f)

$$\frac{5x-1}{2} - \frac{x+2}{4} = \frac{3x-5}{5}$$

Answer: $x = 0$

Important formulae

In the next examples we will use the following formulae:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)(a + b) = a^2 - b^2$$

Example 2

Solve:

$$(2x + 3)^2 - (x - 4)^2 = 3(x - 5)(x + 5)$$

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We start by applying the formulae to expand the brackets:

$$4x^2 + 12x + 9 - (x^2 - 8x + 16) = 3(x^2 - 25)$$

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$$(2x + 3)^2 - (x - 4)^2 = 3(x - 5)(x + 5)$$

We start by applying the formulae to expand the brackets:

$$4x^2 + 12x + 9 - (x^2 - 8x + 16) = 3(x^2 - 25)$$

Now we simplify:

$$4x^2 + 12x + 9 - x^2 + 8x - 16 = 3x^2 - 75$$

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We start by applying the formulae to expand the brackets:

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Now we simplify:

$$4x^2 + 12x + 9 - x^2 + 8x - 16 = 3x^2 - 75$$

$$3x^2 + 20x - 7 = 3x^2 - 75$$

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$$3x^2 + 20x - 7 = 3x^2 - 75$$

And finally we have:

$$20x = -68$$

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$$4x^2 + 12x + 9 - x^2 + 8x - 16 = 3x^2 - 75$$

$$3x^2 + 20x - 7 = 3x^2 - 75$$

And finally we have:

$$20x = -68$$

So $x = -\frac{68}{20} = -3.4$.

Practice 2

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(a)
$$(x + 2)^2 = (x - 4)(x + 4)$$

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Solve the following equations:

(a)

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(b)

$$(3x + 1)^2 - (3x - 2)(3x + 2) = 5$$

Practice 2

Solve the following equations:

(a)

$$(x + 2)^2 = (x - 4)(x + 4)$$

Answer: $x = -5$

(b)

$$(3x + 1)^2 - (3x - 2)(3x + 2) = 5$$

Answer: $x = 0$

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Solve the following equations:

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Answer: $x = -5$

(b)
$$(3x + 1)^2 - (3x - 2)(3x + 2) = 5$$

Answer: $x = 0$

(c)
$$(2x - 1)^2 + (x + 5)^2 = 5(x - 1)(x + 1)$$

Practice 2

Solve the following equations:

(a)

$$(x + 2)^2 = (x - 4)(x + 4)$$

Answer: $x = -5$

(b)

$$(3x + 1)^2 - (3x - 2)(3x + 2) = 5$$

Answer: $x = 0$

(c)

$$(2x - 1)^2 + (x + 5)^2 = 5(x - 1)(x + 1)$$

Answer: $x = -\frac{31}{6} = -5\frac{1}{6}$

Practice 1 continued

Solve the following equations:
(d)
$$(x+4)^2 + (x-3)^2 = 2(x-2)(x+2)$$

Practice 1 continued

Solve the following equations:
(d)
$$(x+4)^2 + (x-3)^2 = 2(x-2)(x+2)$$

Answer: $x = -\frac{33}{2} = -16\frac{1}{2}$

Practice 1 continued

Solve the following equations:

(d)

$$(x+4)^2 + (x-3)^2 = 2(x-2)(x+2)$$

Answer: $x = -\frac{33}{2} = -16\frac{1}{2}$

(e)

$$(4x-3)^2 - (3x+1)^2 = 7(x-1)(x+1)$$

Practice 1 continued

Solve the following equations:
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$$(x+4)^2 + (x-3)^2 = 2(x-2)(x+2)$$

Answer: $x = -\frac{33}{2} = -16\frac{1}{2}$

(e)
$$(4x-3)^2 - (3x+1)^2 = 7(x-1)(x+1)$$

Answer: $x = \frac{1}{2}$

Practice 1 continued

Solve the following equations:

(d)
$$(x+4)^2 + (x-3)^2 = 2(x-2)(x+2)$$

Answer: $x = -\frac{33}{2} = -16\frac{1}{2}$

(e)
$$(4x-3)^2 - (3x+1)^2 = 7(x-1)(x+1)$$

Answer: $x = \frac{1}{2}$

(f)
$$(x+3)(x-1) + (x+2)(x-2) = 2(x+3)^2$$

Practice 1 continued

Solve the following equations:

(d)
$$(x+4)^2 + (x-3)^2 = 2(x-2)(x+2)$$

Answer: $x = -\frac{33}{2} = -16\frac{1}{2}$

(e)
$$(4x-3)^2 - (3x+1)^2 = 7(x-1)(x+1)$$

Answer: $x = \frac{1}{2}$

(f)
$$(x+3)(x-1) + (x+2)(x-2) = 2(x+3)^2$$

Answer: $x = -\frac{5}{2} = -2.5$

If you have any questions or doubts email me at T.J.Lechowski@gmail.com